Input Filter Interactions with Switching Regulators

Christophe Basso – Technical Fellow

IEEE Senior Member

Course Agenda

- ❑ A Switching Regulator as a Load
- ❑ EMI Filter Impact
- ❑ An Introduction to FACTs
- ❑ Buck Converter Input/Output Impedances
- ❑ Filtering the Input Current
- ❑ Damping the Filter
- ❑ Optimum Component Selection
- ❑ A Practical Case Study
- ❑ Cascading Converters

Rev. 0.1

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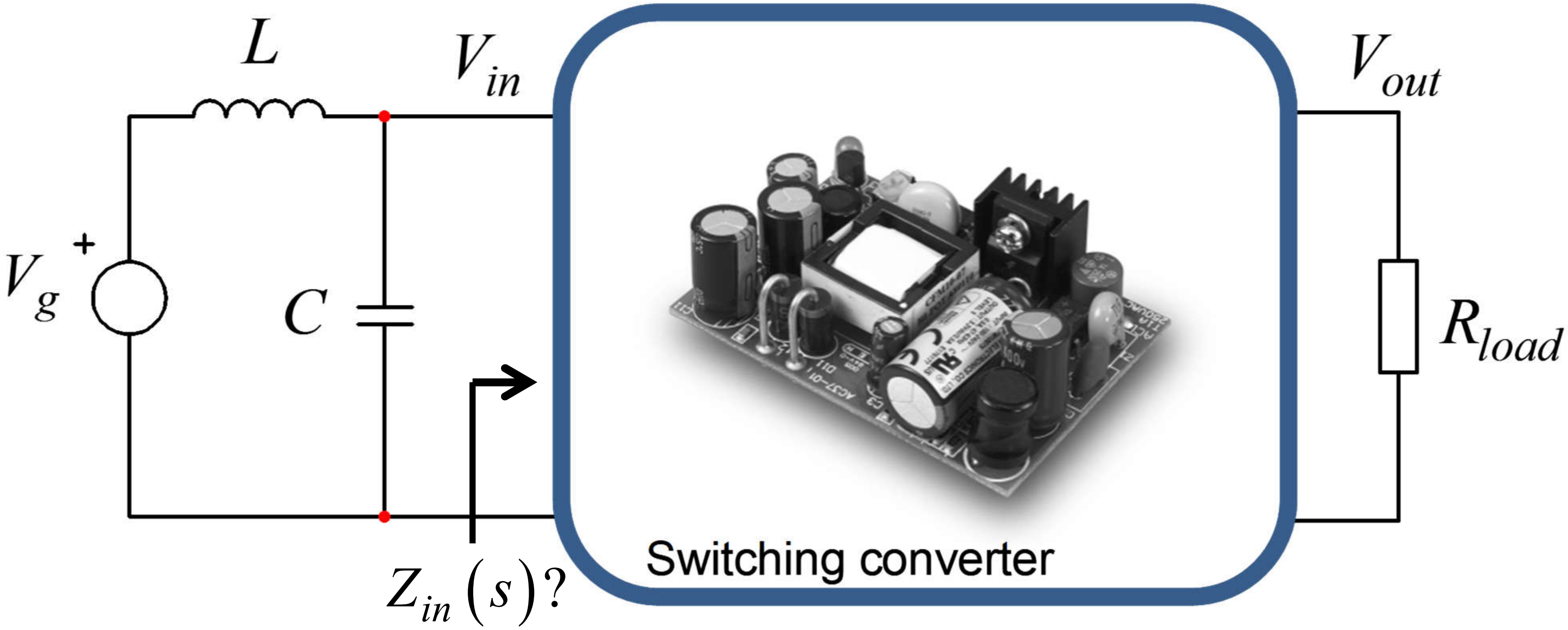
EMI Filter Interaction

- A LC filter is inserted to prevent input line pollution



- What load does the converter offer?

C. Basso, "Designing Control Loops for Linear and Switching Power Supplies", Artech House, 2012

A Negative Incremental Resistance

- Assume a 100%-efficient converter

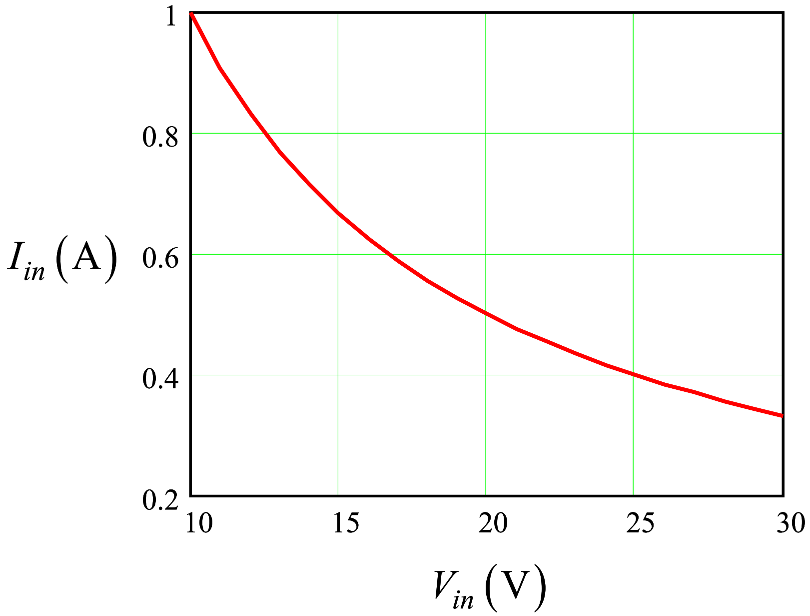
$$P_{out} = P_{in} \longrightarrow I_{in} V_{in} = I_{out} V_{out}$$

Infinite rejection

- In closed-loop operation, P_{out} is constant, no link to V_{in}

$$\longrightarrow I_{in}\left(V_{in}\right) = \frac{P_{out}}{V_{in}}$$

- For a constant P_{out} , if V_{in} increases, I_{in} drops



$$\frac{dI_{in}\left(V_{in}\right)}{dV_{in}} = \frac{d\left(\frac{P_{out}}{V_{in}}\right)}{dV_{in}} = -\frac{P_{out}}{V_{in}^{\ 2}}$$

The incremental input resistance is negative

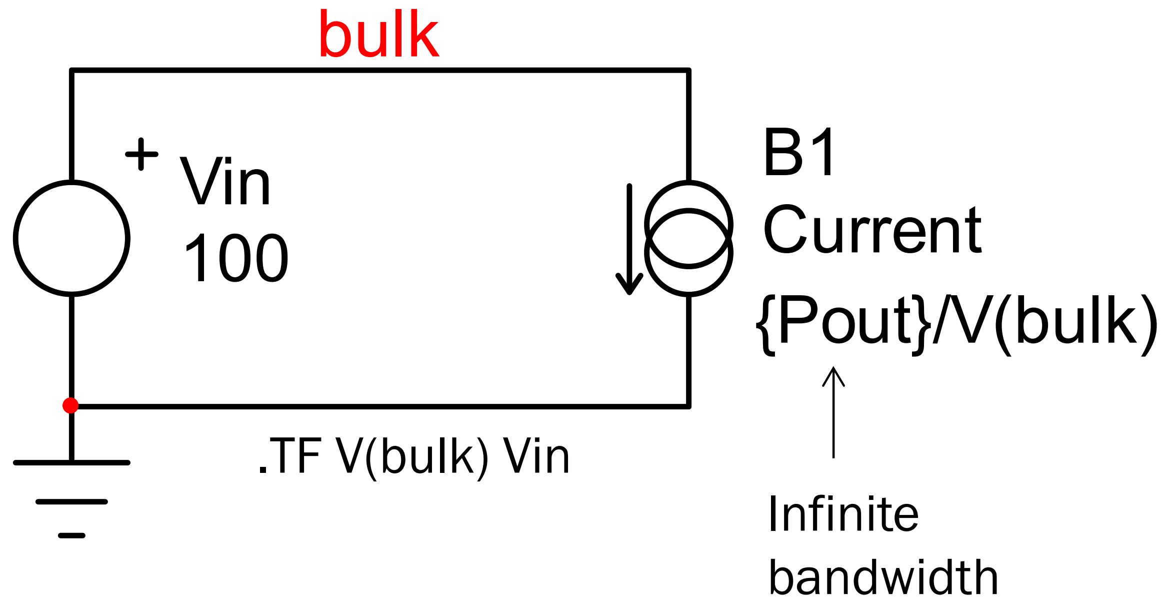
$$\boxed{R_{in} = -\frac{V_{in}^{\ 2}}{P_{out}}}$$

A Simple SPICE Simulation

- A constant-power current source shows the negative resistance

parameters

Pout=50W

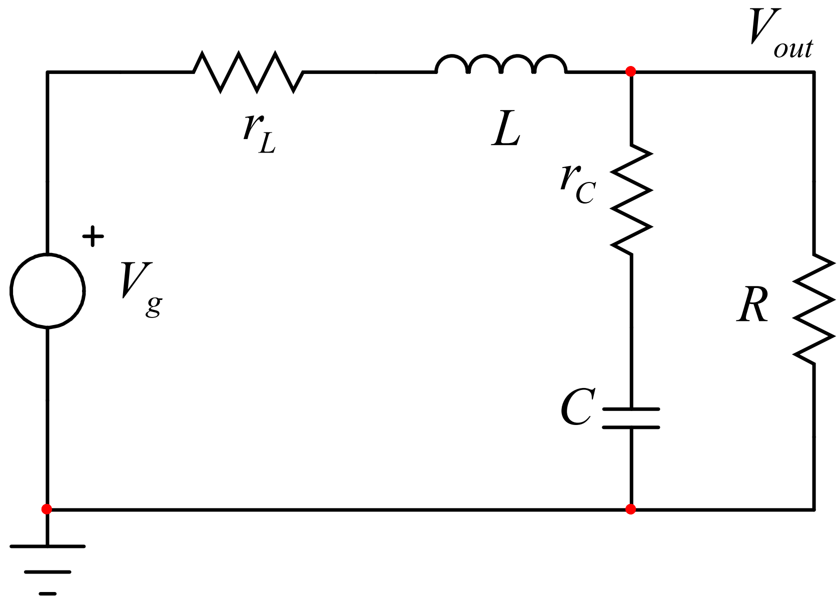


***** SMALL SIGNAL DC TRANSFER FUNCTION

output_impedance_at_V(bulk)	0.000000e+000	
vin#Input_impedance	-2.00000e+002	← Neg. resistance
Transfer_function	1.000000e+000	

A Simple LC Filter

- The low-pass filter is built with L and C elements:



$$H(s) = H_0 \frac{1 + s/\omega_{z_1}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{r_L + R}{r_C + R}} \qquad \omega_{z_1} = \frac{1}{r_C C}$$

$$Q = \frac{LC\omega_0 (r_C + R)}{L + C(r_L r_C + r_L R + r_C R)}$$

$$\left.\begin{array}{c} L + C(r_L r_C + r_L R + r_C R) = 0 \\ \Downarrow \\ \text{If } R = -\dfrac{L + r_C r_L C}{C(r_C + r_L)} \end{array}\right\} Q \to \infty$$

A negative resistance cancels losses: poles become imaginary

$\Rightarrow$ Sustained oscillations

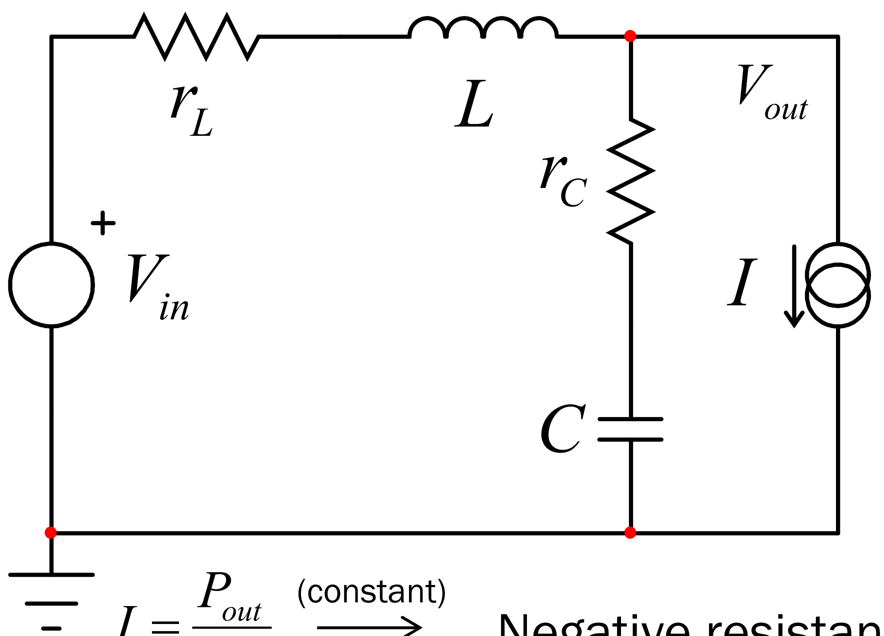
A Negative Resistance Oscillator

- If losses are compensated, the damping ratio is zero

$$H(s) = H_0 \frac{1 + s/\omega_z}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1} \qquad Q = \frac{1}{2\boxed{\zeta}}$$

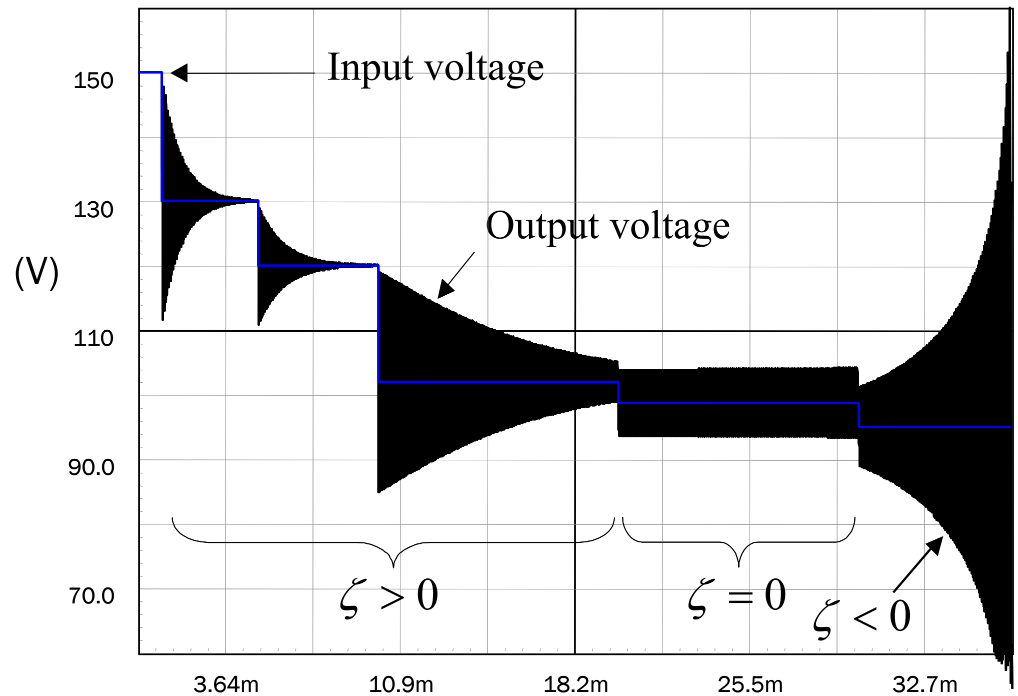
→ If ohmic losses are gone, the damping ratio is zero, Q is infinite.

- Without precautions, instability can happen!



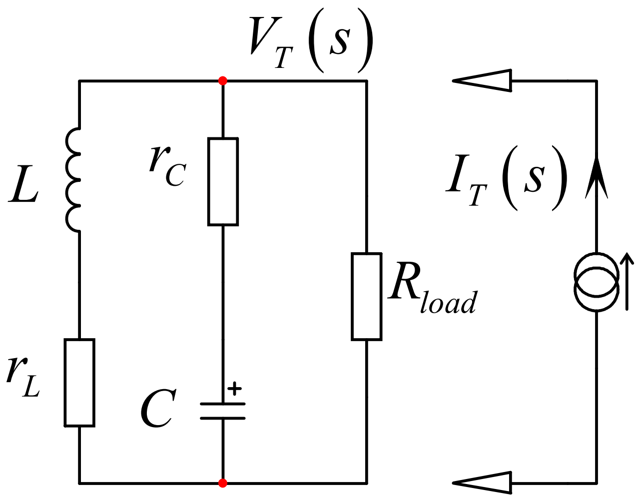
$$I = \frac{P_{out}}{V_{out}} \xrightarrow{\text{(constant)}}$$ Negative resistance

Infinite bandwidth



Filter Output Impedance

- What is the output impedance of an LC filter?



It follows the form

$$Z_{th}(s) = R_0 \boxed{\frac{N(s)}{D(s)}}$$

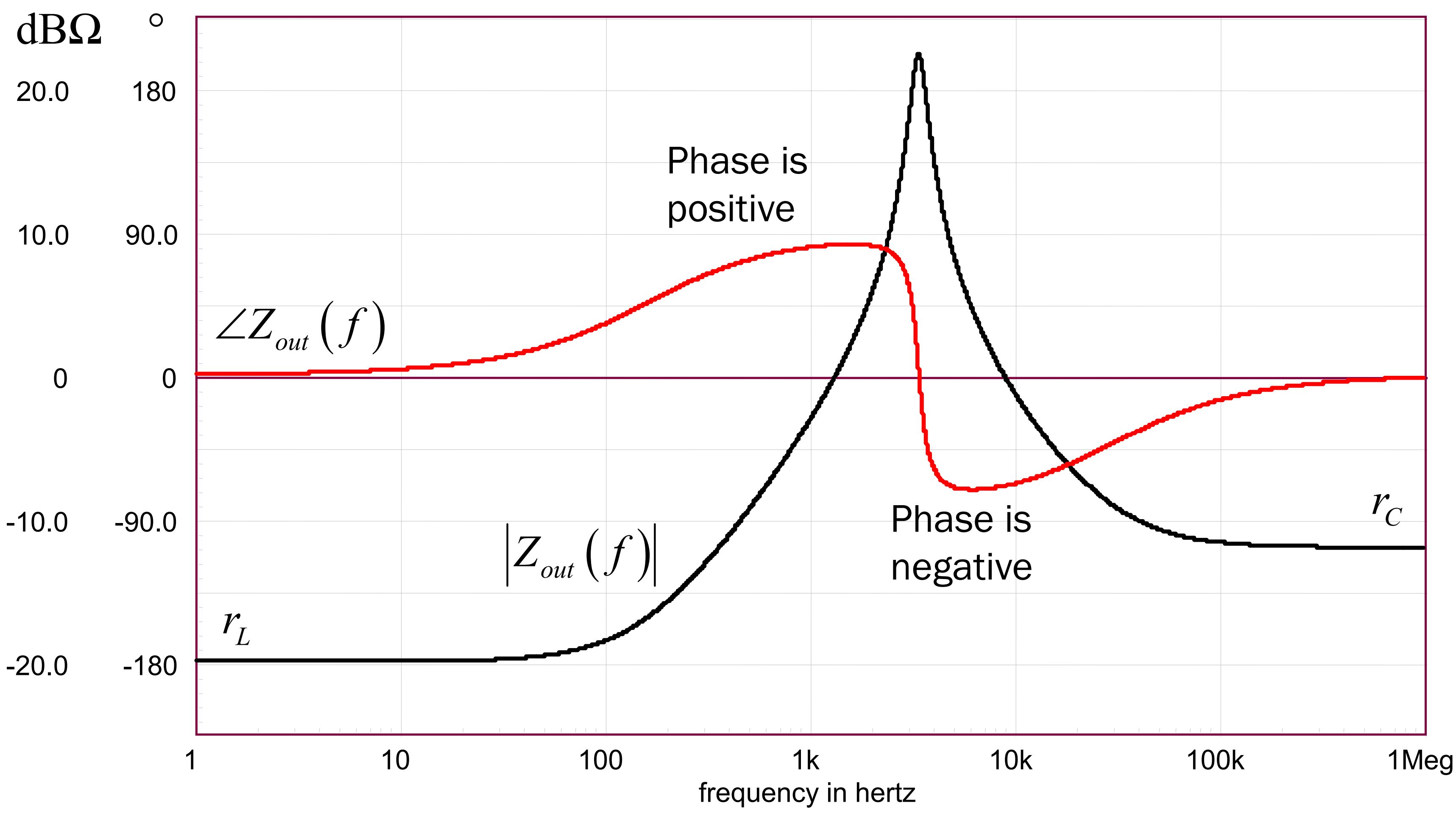
Ω Ω No dimension

$$Z_{th}(s) = (r_L \parallel R_{load}) \frac{\left(1 + s\frac{L}{r_L}\right)(1 + s r_C C_{out})}{1 + s\left(\frac{L}{r_L + R_{load}} + C[r_L \parallel R_{load} + r_C]\right) + s^2\left(LC\frac{r_C + R_{load}}{r_L + R_{load}}\right)}$$

- All losses (ohmic, iron etc.) help decreasing Q

Typical Filter Output Impedance

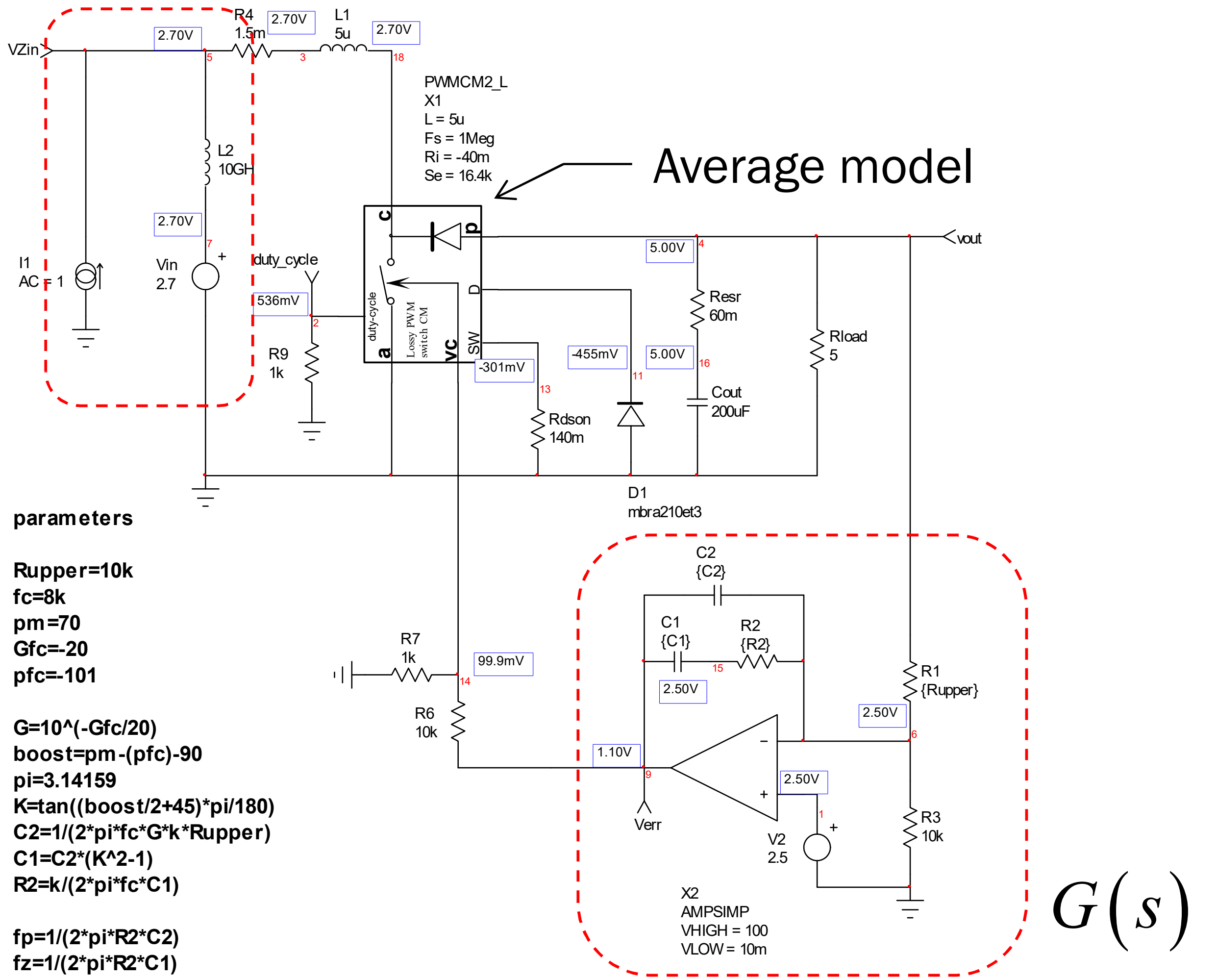
- At low frequency, the inductive ohmic loss dominates



Negative Resistance at Low Frequency

- Neg. resistance exists because of feedback (P_{out} = constant)

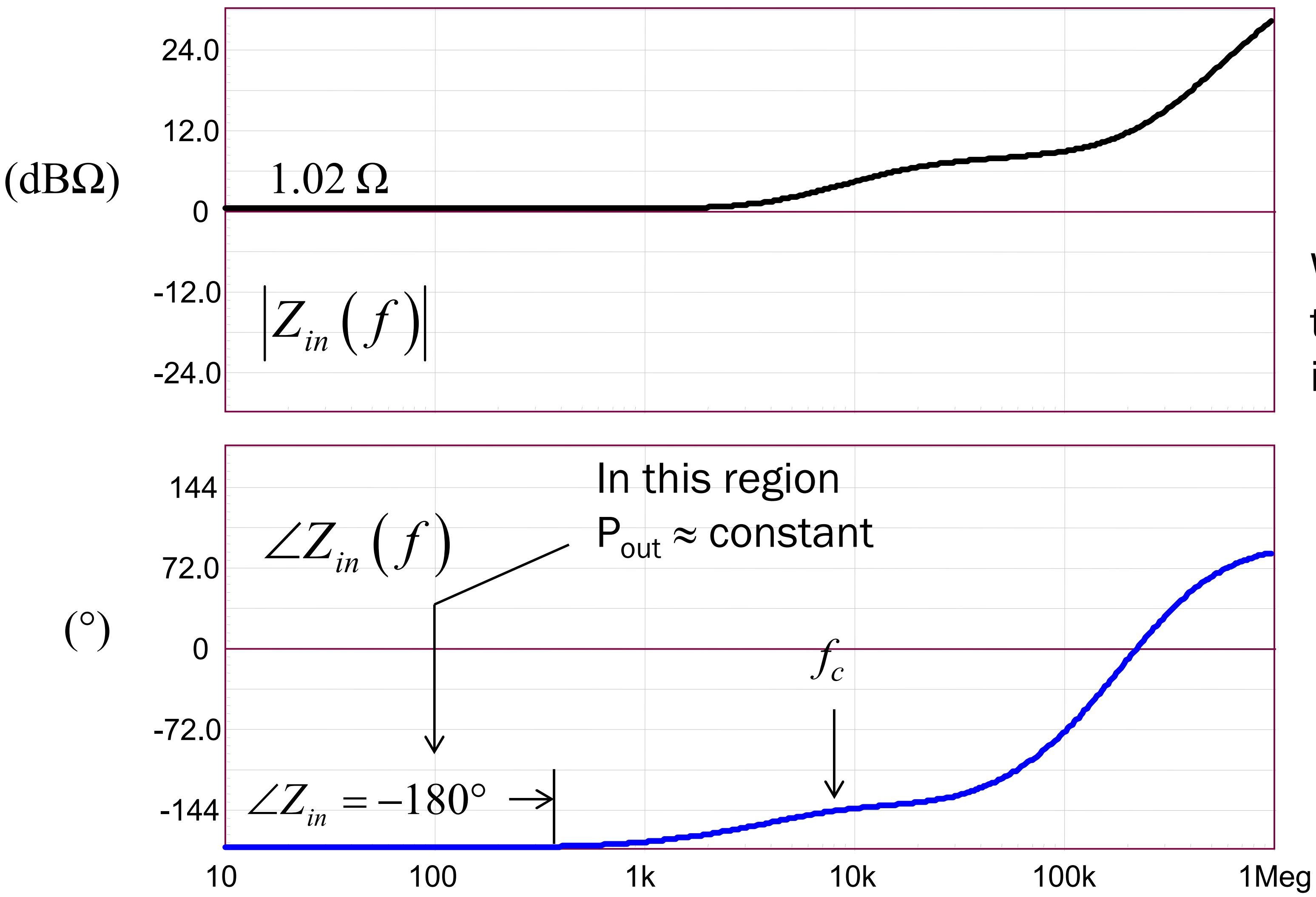
Impedance measurement setup



SwitchMode Power Supplies: SPICE Simulations and Practical Designs
Christophe Basso - McGraw-Hill, 2014

Negative Resistance at Low Frequency

- The resistance is truly negative up to 200 Hz



Well before crossover, the -180° argument is gone.

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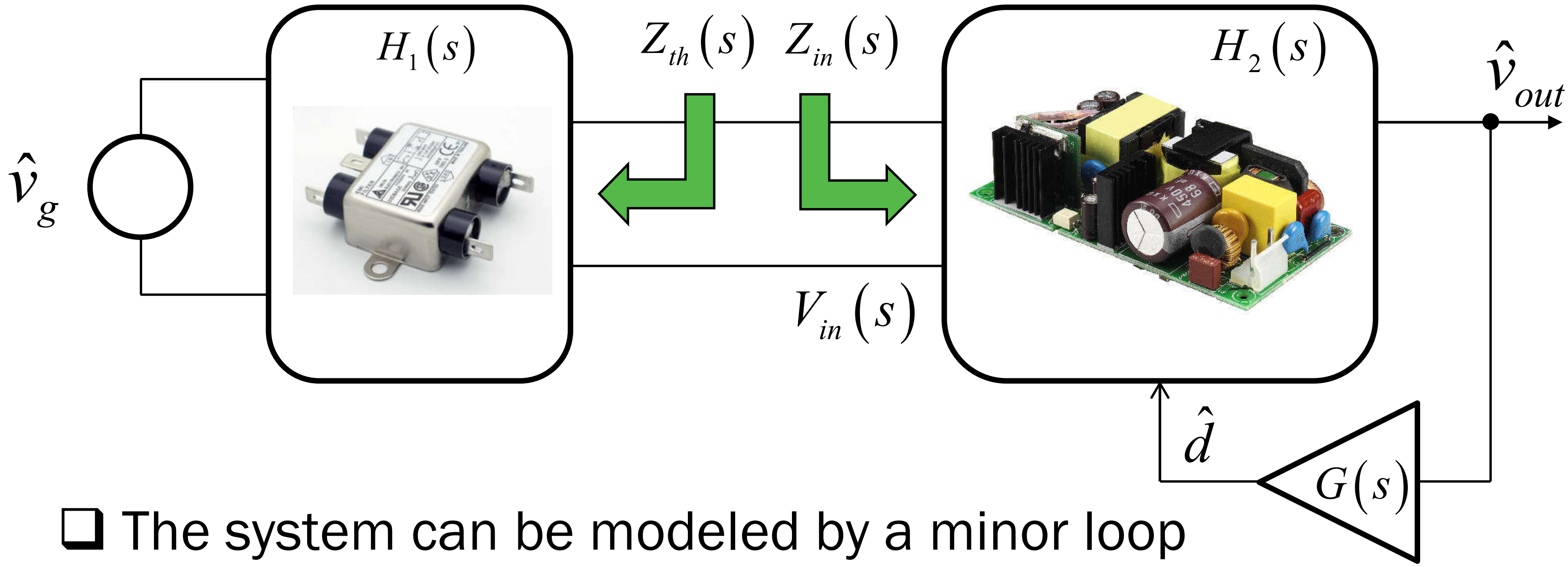
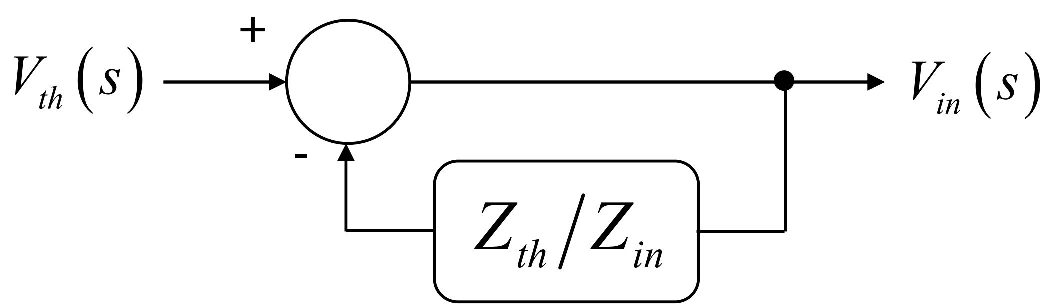
A Filter Affects the Line Output Impedance

- The line impedance driving the converter is no longer 0



- The system can be modeled by a minor loop

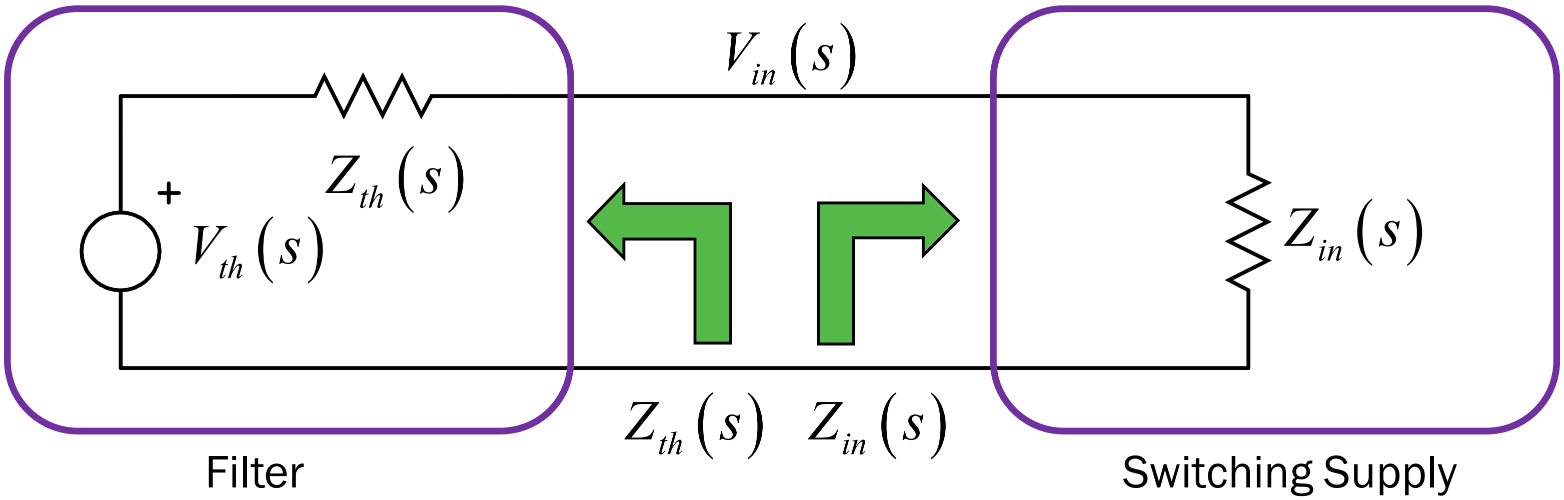
$$V_{in}(s) = V_{th}(s)\frac{1}{1+\underbrace{\boxed{\dfrac{Z_{th}(s)}{Z_{in}(s)}}}_{\text{gain}}}$$



"Input Filter Considerations in Design and Application of Switching Regulators", R. D. Middlebrook, IEEE Proceedings, 1976

Impedance Interactions

- Stability can be at stake when inserting the filter



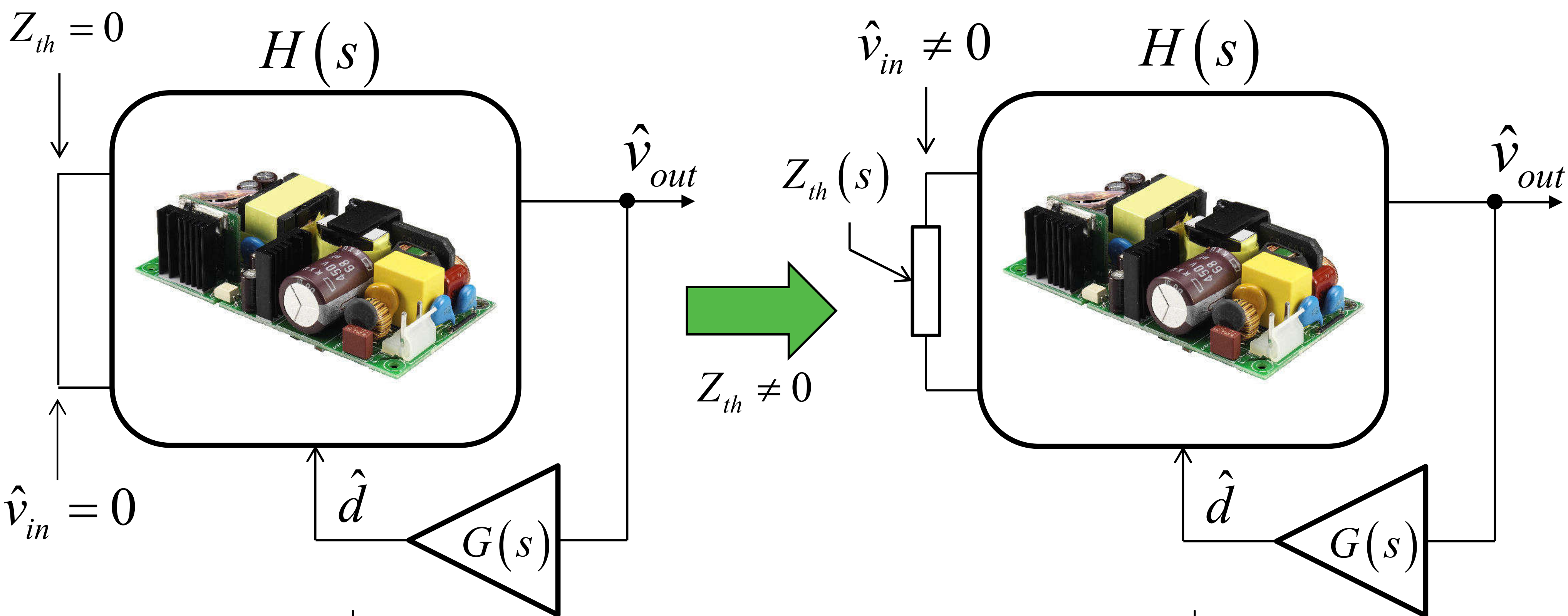
- The Nyquist criterion applies

$$V_{in}(s) = V_{th}(s)\frac{1}{1+\dfrac{Z_{th}(s)}{Z_{in}(s)}} \quad \left\} \begin{array}{l} \dfrac{Z_{th}(s)}{Z_{in}(s)} = -1 \\ \\ \left|\dfrac{Z_{th}(s)}{Z_{in}(s)}\right| = 1 \text{ and } \angle\dfrac{Z_{th}(s)}{Z_{in}(s)} = -180° \end{array}\right.$$

→ Conditions for oscillations

A Filter Modifies Converter's Dynamics

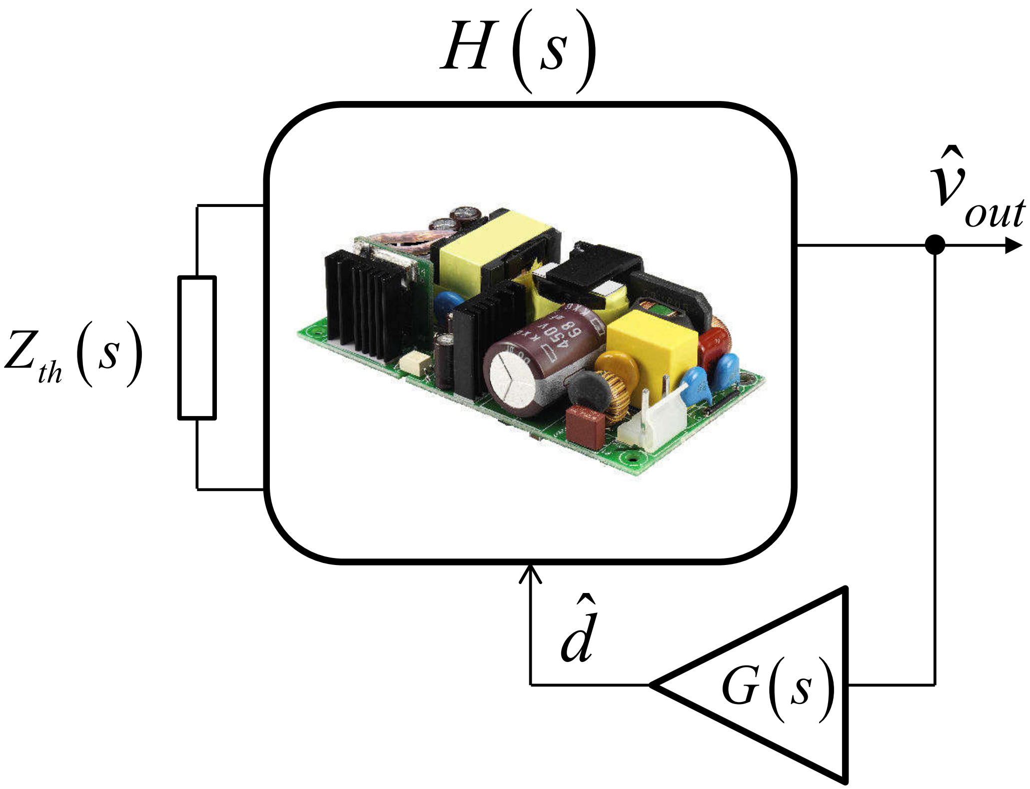
- The EMI filter output impedance makes $Z_{th}(s) \neq 0$
- The converter input voltage is no longer zero in ac analysis



$$H(s) = \left. \frac{V_{out}(s)}{D(s)} \right|_{V_{in}(s)=0} \qquad H(s) = \left. \frac{V_{out}(s)}{D(s)} \right|_{V_{in}(s)\neq 0}$$

Extra Element Theorem to Help

- It can be shown how an EMI filter affects the open-loop gain



$$H(s) = \left.\frac{V_{out}(s)}{D(s)}\right|_{Z_{th}(s)=0}$$ Gain without the filter

$Z_{th}(s)$ Filter output impedance

Extra-Element correction factor

$$H(s) = \left.\frac{V_{out}(s)}{D(s)}\right|_{Z_{th}(s)\neq 0} = \left.\frac{V_{out}(s)}{D(s)}\right|_{Z_{th}(s)=0} \frac{1+\frac{Z_{th}(s)}{Z_N(s)}}{1+\frac{Z_{th}(s)}{Z_D(s)}}$$

≈ 1

$$|Z_{th}(s)| << |Z_N(s)|$$
$$|Z_{th}(s)| << |Z_D(s)|$$

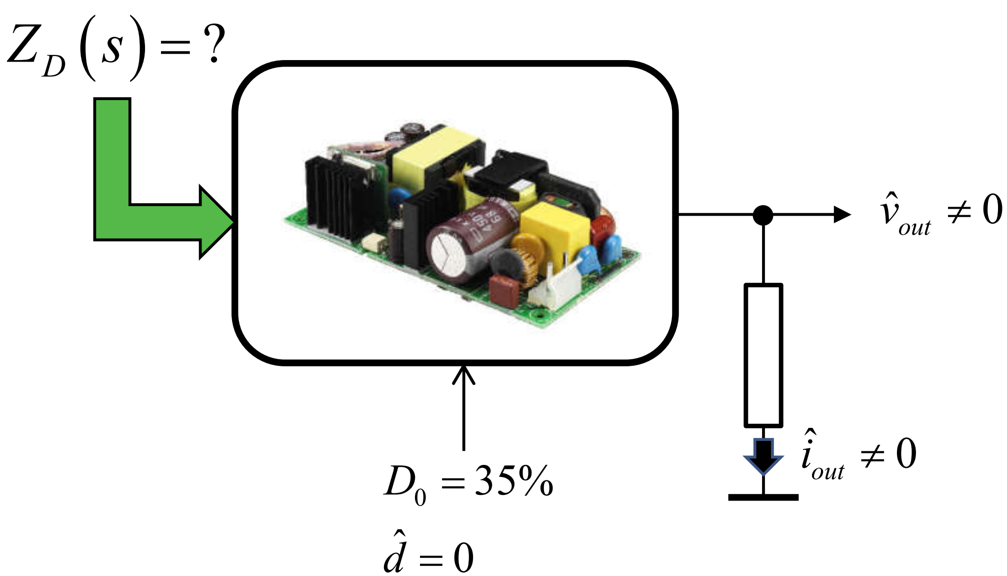
What are Z_D and Z_N?

- Z_D and Z_N come from the Extra-Element Theorem, EET

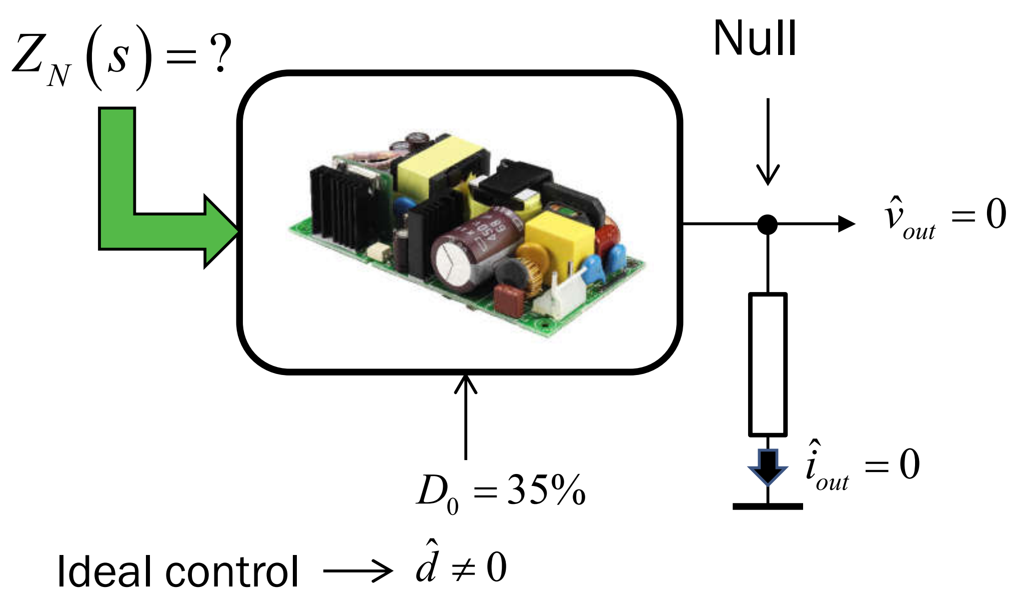
$$Z_D(s) = Z_i(s)\Big|_{D(s)=0}$$ ⇒ Open-loop input impedance

$$Z_N(s) = Z_i(s)\Big|_{\hat{v}_{out}=0}$$ ⇒ Input impedance for a nulled output

Ideal input rejection



Open-loop input impedance



Input impedance for $\hat{v}_{out} = 0$

Consider Closed-Loop Input Impedance

- These values have already been derived

$Z_e(s)$ is the input impedance for a shorted output

Converter	$Z_N(s)$	$Z_D(s)$	$Z_e(s)$
Buck	$-\frac{R}{D^2}$	$\frac{R}{D^2}\left(\frac{1+s\frac{L}{R}+s^2LC}{1+sRC}\right)$	$\frac{sL}{D^2}$
Boost	$-D'^2R\left(1-\frac{sL}{D'^2R}\right)$	$D'^2R\left(\frac{1+s\frac{L}{D'^2R}+s^2\frac{LC}{D'^2}}{1+sRC}\right)$	sL
Buck-Boost	$-\frac{D'^2R}{D^2}\left(1-\frac{sDL}{D'^2R}\right)$	$\frac{D'^2R}{D^2}\left(\frac{1+s\frac{L}{D'^2R}+s^2\frac{LC}{D'^2}}{1+sRC}\right)$	$\frac{sL}{D^2}$

- A converter closed-loop input impedance follows the form

$$\frac{1}{Z_{in}(s)}=\frac{1}{Z_N(s)}\frac{T(s)}{1+T(s)}+\frac{1}{Z_D(s)}\frac{1}{1+T(s)}$$

Loop gain

"Fundamentals of Power Electronics", R. Erickson, D. Maksimovic, Springer, 2001

The Output Impedance is Affected

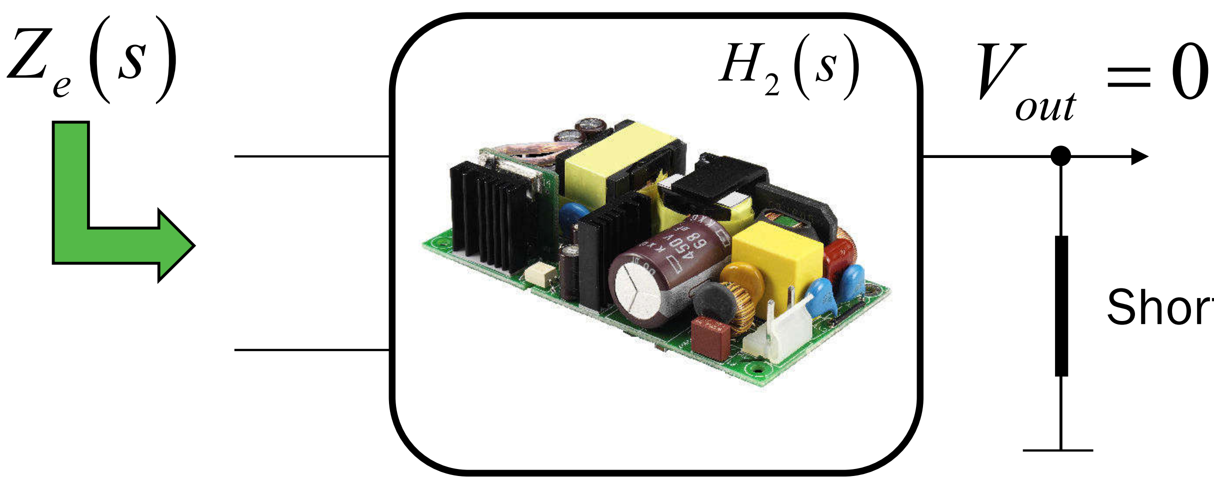
- The filter degrades the closed-loop output impedance

$$Z_{out,CL}(s)\Big|_{Z_{th}(s)\neq 0} = Z_{out,CL}(s)\Big|_{Z_{th}(s)=0} \frac{1+\dfrac{Z_{th}(s)}{Z_e(s)}}{1+\dfrac{Z_{th}(s)}{Z_{in,CL}(s)}}$$

No impact if

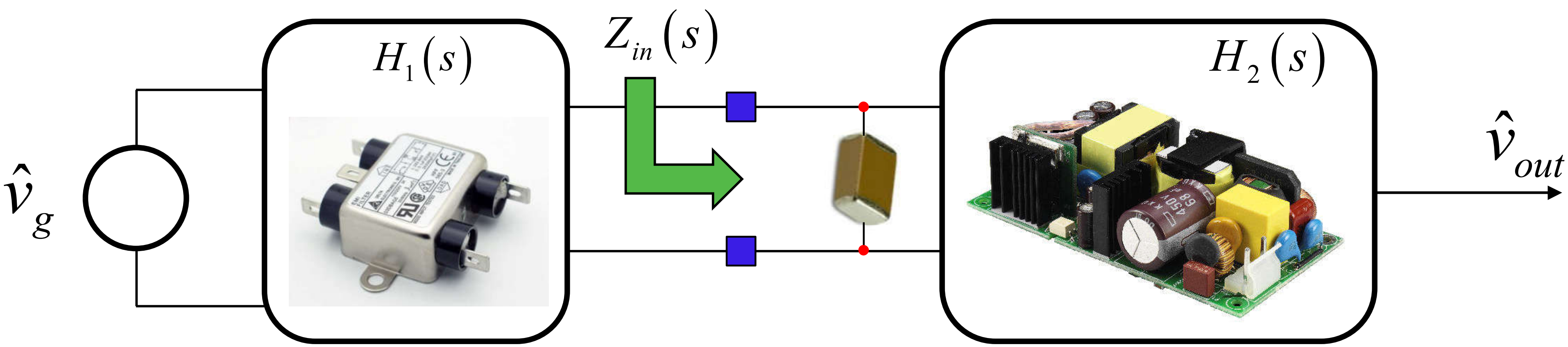
$$Z_{th}(s) << Z_e(s)$$

- $Z_e(s)$ is the converter input impedance with a shorted output

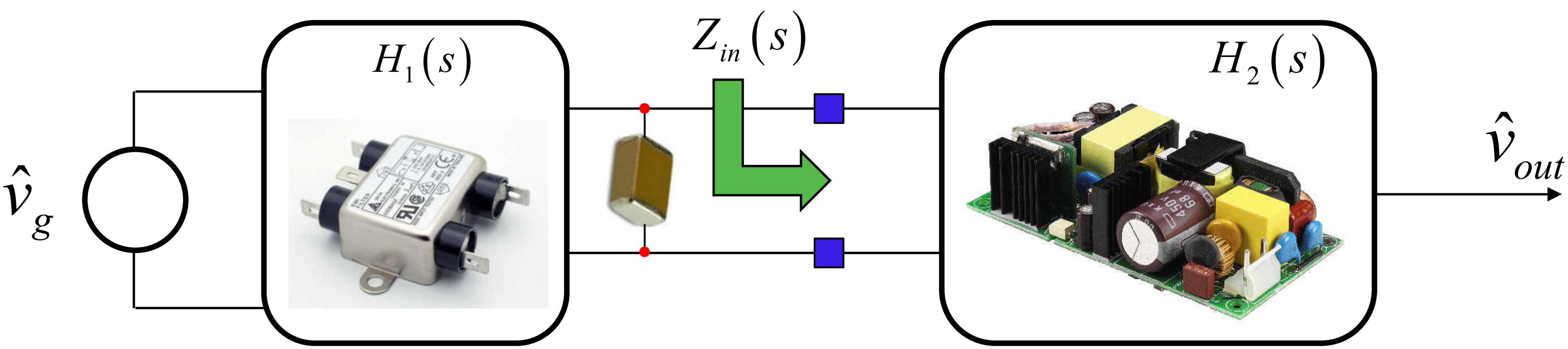


Applying Dr. Middlebrook Criteria

- Previous equations consider switching cell alone
  - Do not include the decoupling capacitor!



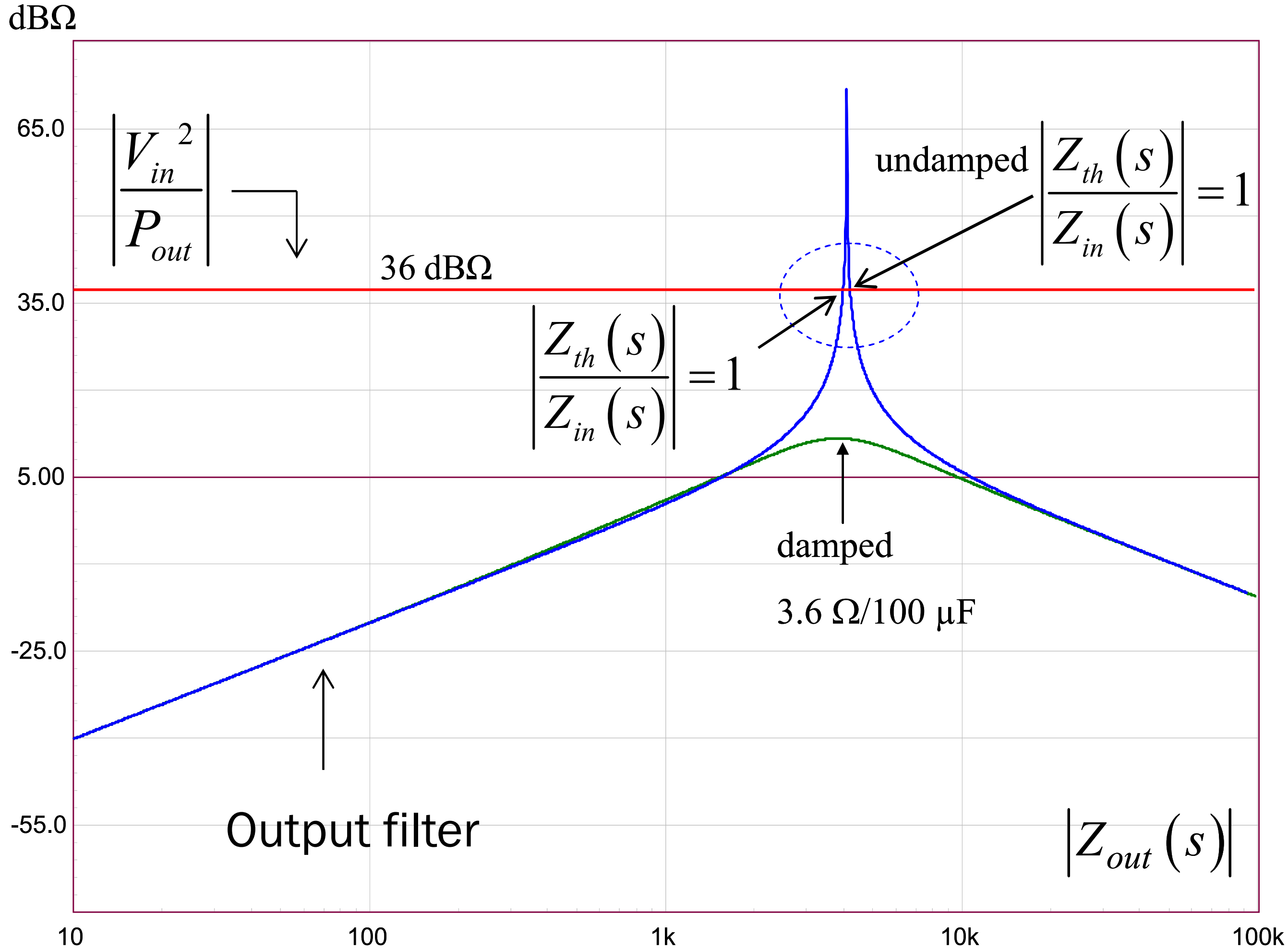
- If possible, move the capacitor to the filter side



"Input Impedance and Filter Interactions Part I", R. Ridley, ridleyengineering.com

At 1st-Order Check Crossover Points

- Plot the converter input resistance value
- Plot the filter output impedance: check for overlaps: $\left|Z_{th}(s)\right| = \left|Z_{in}(s)\right|$



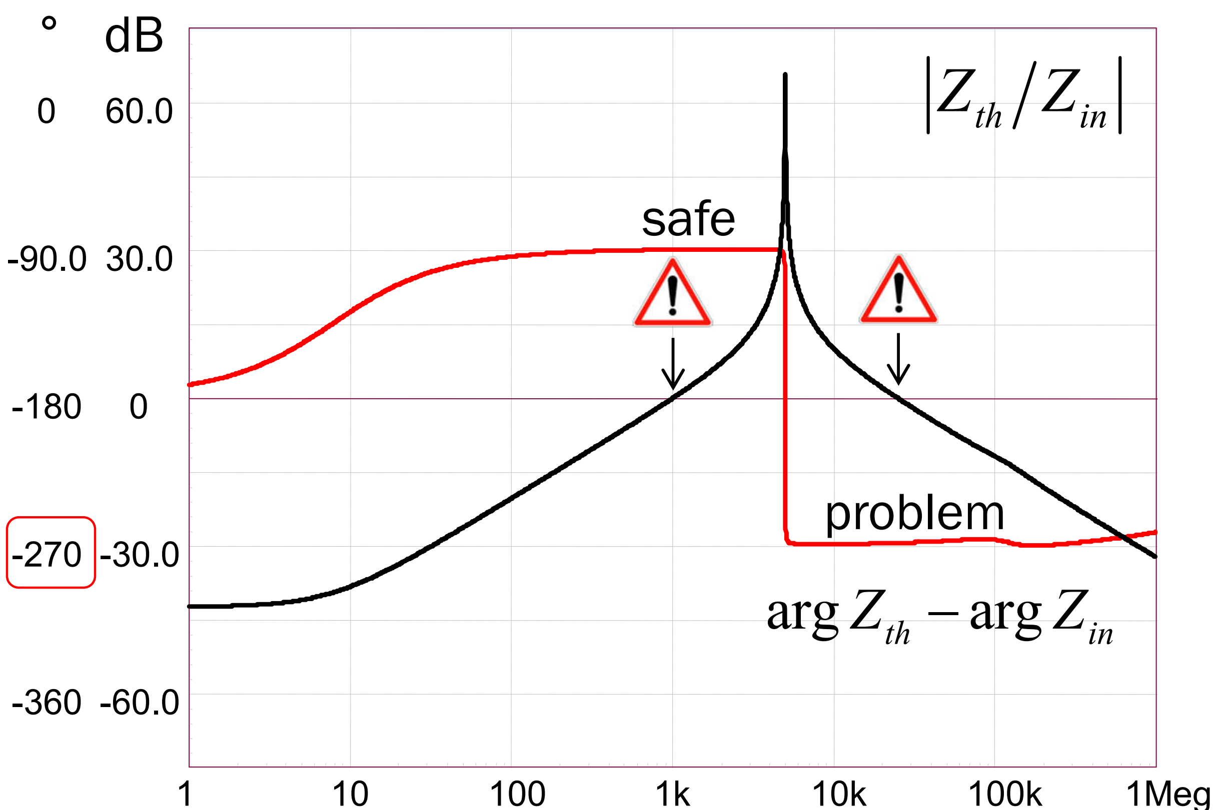
Overlap occurs?

Watch the argument of Z_{th}/Z_{in}

Condition for Oscillations

- Plot the ratio of magnitude $\left|Z_{th}/Z_{in}\right|$
- Plot the difference of arguments $\arg Z_{th} - \arg Z_{in}$

Check phase margin at 0-dB points if any



Conditions for oscillations

$$\left|\frac{Z_{th}(s)}{Z_{in}(s)}\right| = 1$$

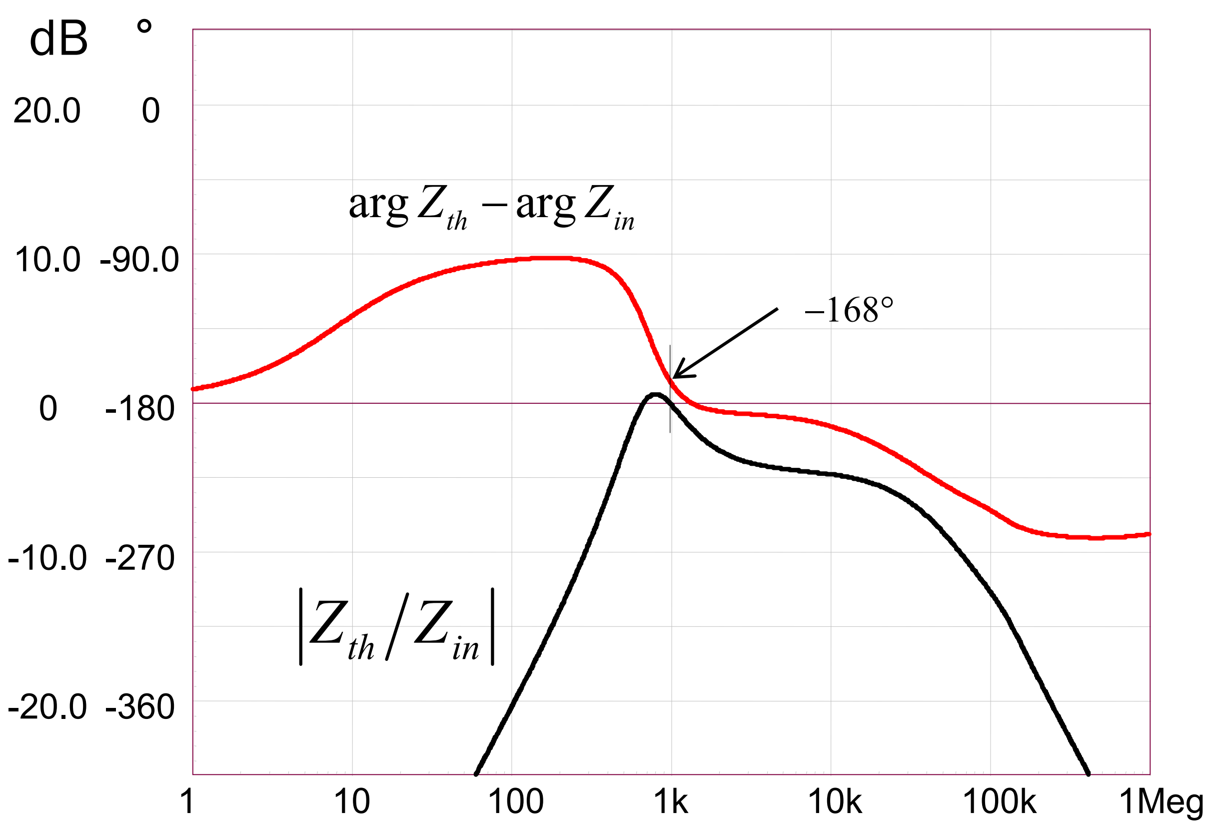
and

$$\angle \frac{Z_{th}(s)}{Z_{in}(s)} = -180°$$

Damping the Filter to Build Margin

- Damping the filter can provide phase margin at 0 dB
- The phase margin in this example is a bare 12°

Avoid magnitude overlaps by working on Z_{th} and Z_{in}!



Preliminary Conclusion

- You design the input filter together with the SMPS

When a filter is installed:

$$\left|Z_{th}(s)\right| << \left|Z_N(s)\right|$$
$$\left|Z_{th}(s)\right| << \left|Z_D(s)\right|$$

Converter loop gain T is unaffected

and

$$\left|Z_{th}(s)\right| << \left|Z_e(s)\right|$$

V_{out} shorted, open loop

Converter output impedance is unaffected

- You design the input filter alone

$$\left|\frac{Z_{th}(s)}{Z_{in}(s)}\right| \text{ and } \angle\frac{Z_{th}(s)}{Z_{in}(s)}$$

Don't care about phase anymore

If damping the filter guarantees $\left|Z_{th}(s)\right| << \left|Z_{in}(s)\right|\Big|_{closed\ loop}$ OK

System is stable but overall performance may be altered

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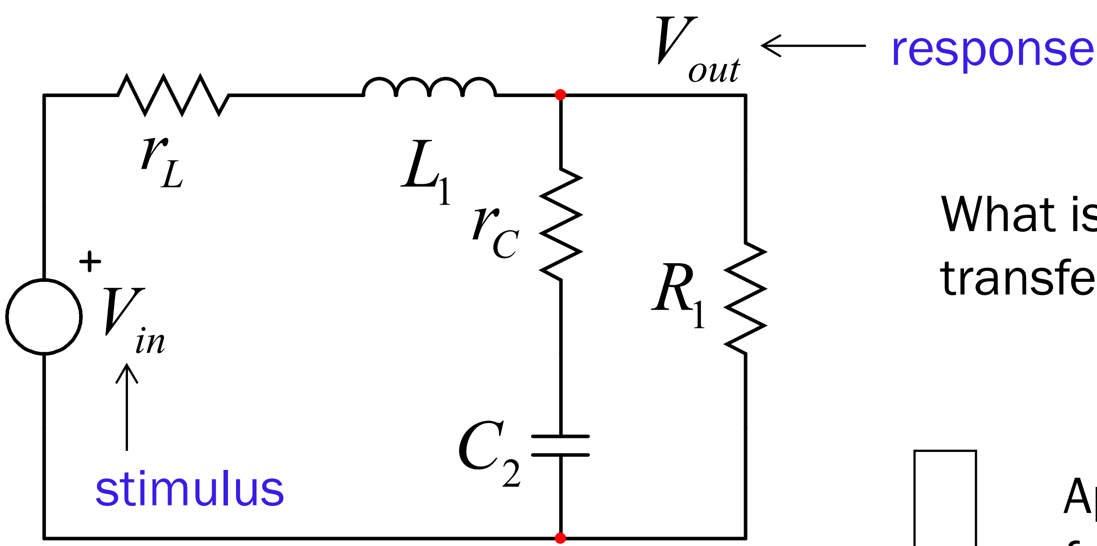
Classical Circuits Analysis Techniques

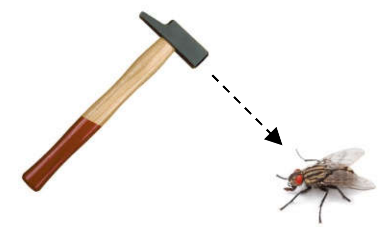
- ❑ Apply classical Kirchhoff's voltage and current laws
- ❖ The expression is correct but disorganized: *high-entropy* form



What is the transfer function?

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}$$

Apply brute force algebra



$$H(s)=\frac{\left(\frac{1}{sC_2}+r_C\right)\|R_1}{\left(\frac{1}{sC_2}+r_C\right)\|R_1+r_L+sL_1}=\frac{R_1+sR_1r_CC_2}{R_1+r_L+sL_1+sC_2R_1r_C+C_2R_1r_Ls+C_2r_Cr_Ls+C_2L_1R_1s^2+C_2L_1r_Cs^2}$$

No insight: poles? Zeros? Dc gain?

Fast Analytical Circuits Techniques

- FACTs describe a set of tools to quickly write transfer functions

$$H(s) = \boxed{\frac{R_1}{R_1 + r_L}} \frac{1 + s r_C C_2}{1 + \boxed{s\left[\frac{L_1}{r_L + R_1} + C_2\left(r_C + r_L \,||\, R_1\right)\right]} + \boxed{s^2 L_1 C_2 \frac{r_C + R_1}{r_L + R_1}}}$$

Dc gain

- Naturally showing gains, poles and zeros…

$$H(s) = \boxed{H_0} \frac{1 + \frac{s}{\omega_z}}{1 + \boxed{\frac{s}{\omega_0 Q}} + \boxed{\left(\frac{s}{\omega_0}\right)^2}}$$

$$\omega_z = \frac{1}{r_C C_2} \qquad \omega_0 = \frac{1}{\sqrt{L_1 C_2}} \sqrt{\frac{r_C + R_1}{r_L + R_1}}$$

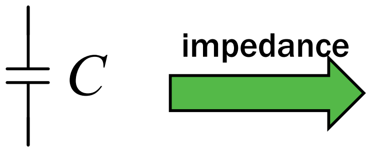
$$Q = \frac{r_L + R_1}{L_1 + C_2\left[r_C r_L + R_1\left(r_C + r_L\right)\right]} \frac{1}{\omega_0}$$

This is a *low-entropy* expression

R. D. Middlebrook, “Methods of Design-Oriented Analysis: Low Entropy Expressions”, New Approaches to Undergraduate Education, July 1992

Two Different Stages

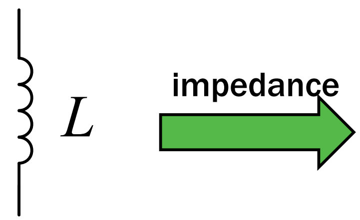
- Consider dc and high-frequency states for L and C



$$Z_C = \frac{1}{sC}$$

Dc state $Z_C = \infty$ Cap. is an open circuit

HF state $Z_C = 0$ Cap. is a short circuit



$$Z_L = sL$$

Dc state $Z_L = 0$ Inductor is a short circuit

HF state $Z_L = \infty$ Inductor is an open circuit

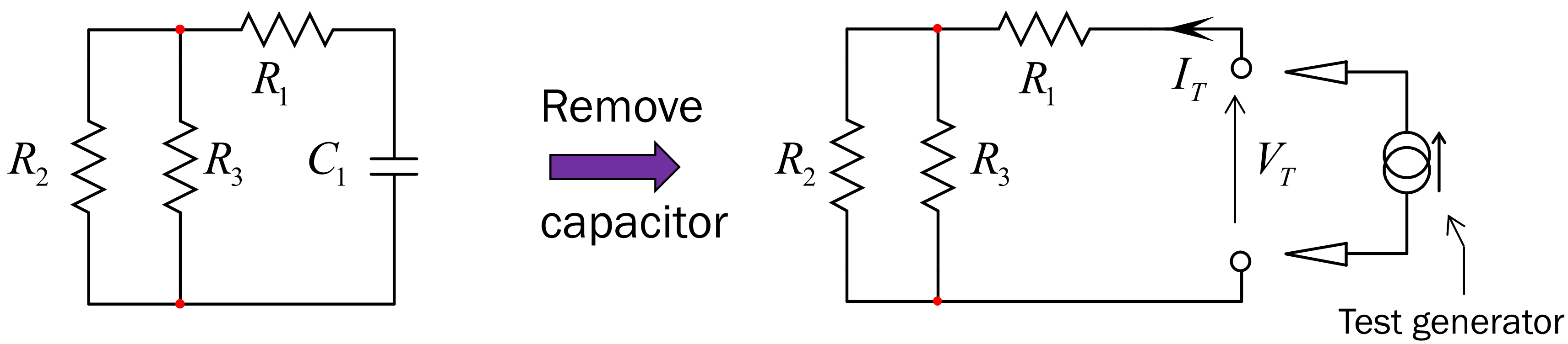
- Change the circuit depending on s

Principles of FACTs: Time Constants

- Determine time constants in two different conditions

1. The excitation is set to zero (no excitation)
2. The output is nulled (no response)

How do you determine a time constant?



Remove L or C and look into its terminals: $R = \frac{V_T}{I_T} = R_2 \| R_3 + R_1$

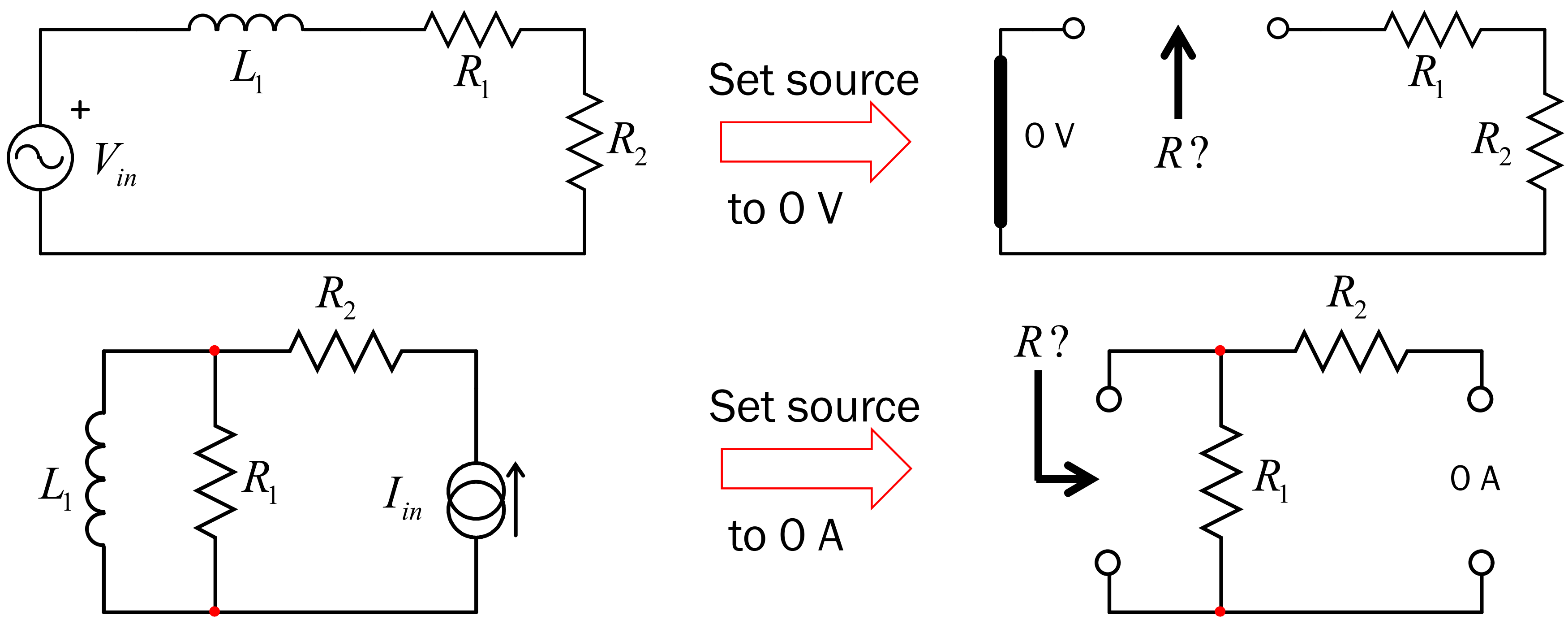
$$\tau = \left(R_2 \| R_3 + R_1\right) C_1$$

In your head, imagine an ohm-meter placed across C_1's terminals

Excitation is set to 0

Turning the Excitation off – The Pole

- Turning the excitation off means
  - A 0-V source becomes a short circuit
  - A 0-A generator is an open circuit and disappears



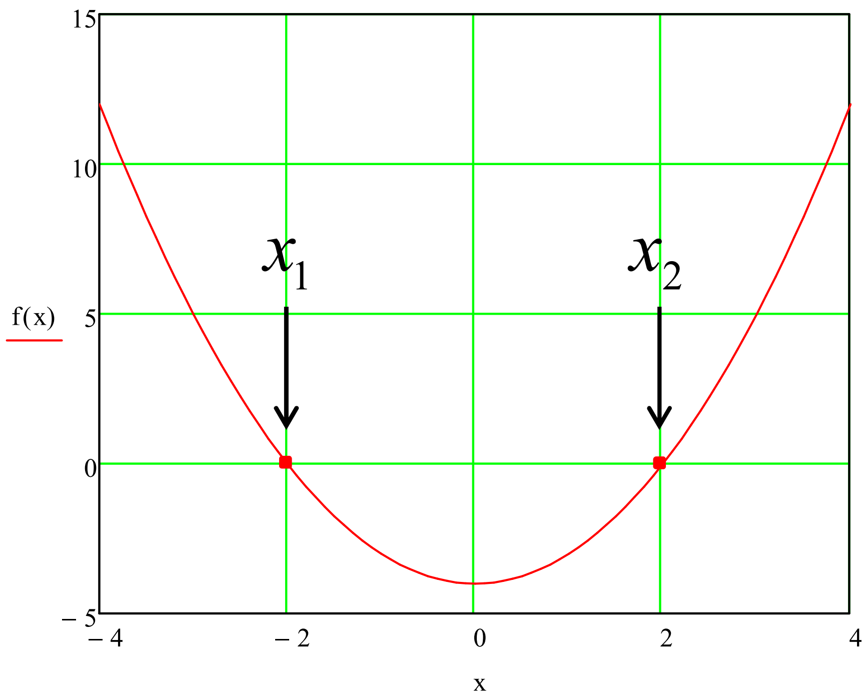
$$\tau = \frac{L_1}{R_1 + R_2}$$

$$\tau = \frac{L_1}{R_1}$$

- The inverse of the time constant in this case is a pole: $\omega_p = \frac{1}{\tau}$

Mathematical Definition of a Zero

- A zero is the root of the equation $f(x)=0$



$$f(x)=x^2-4$$

$$f(x)=0$$

$$\downarrow$$

$$x_1=-2$$

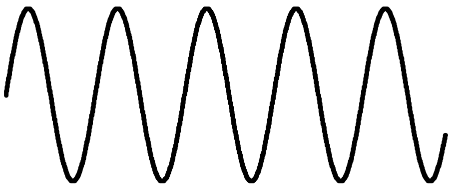
$$x_2=2$$

- Transfer function zeros are the numerator roots

$$N(s)=0 \longrightarrow s_{z_1}, s_{z_2} \dots$$

Nulling the Response

- ❑ If the numerator is 0, then the response is theoretically 0



$\hat{v}_{out} = 0$

Complex excitation

Complex response

$$s = s_z$$

$$N(s_z) = 0$$

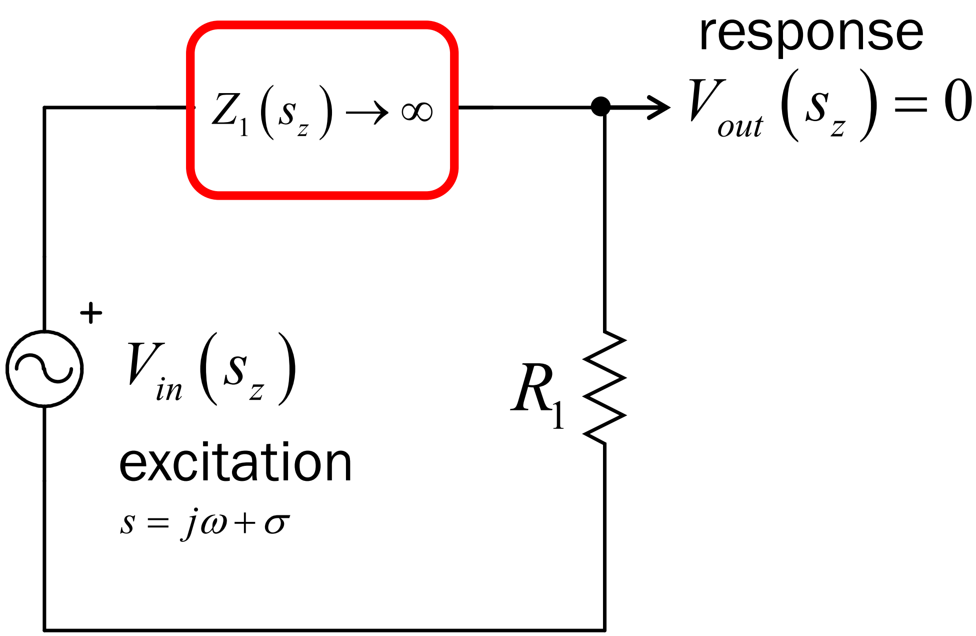
- ❑ What is happening in the box when $s = s_z$?

➡ The excitation cannot reach the output: the response is nulled

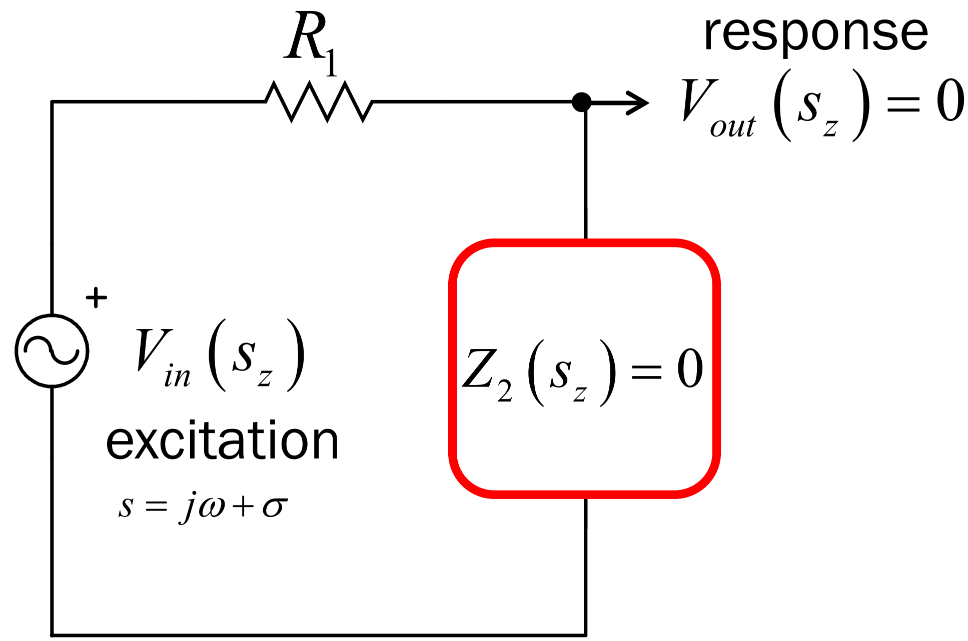
$$V_{out}(s_z) = 0 \Leftrightarrow \hat{v}_{out}(s_z) = 0$$

How Does the Response Disappear?

- The signal is lost in the *transformed* network



A series impedance
becomes infinite.

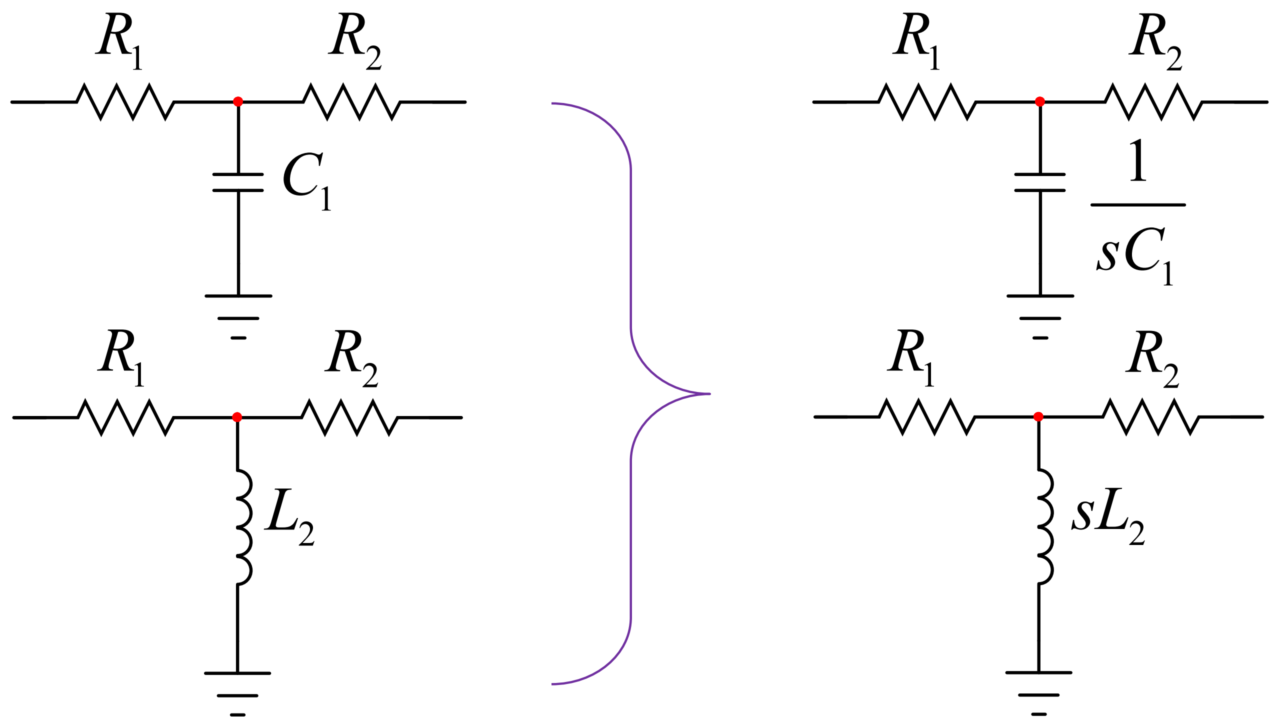


A parallel impedance
shorts the path to ground

- What is a *transformed* network?

The Transformed Network

- Reactances are replaced by their Laplace expression

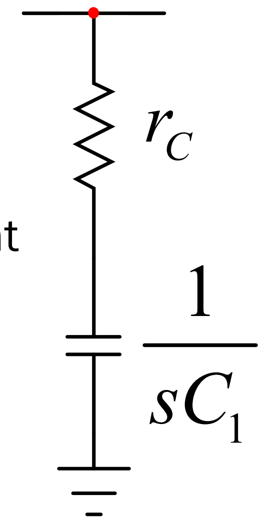


- The circuit is then observed at the zero angular frequency

Considering a Negative Angular Frequency

- For $s = s_{z1}$, the RC impedance is a short circuit

shunt

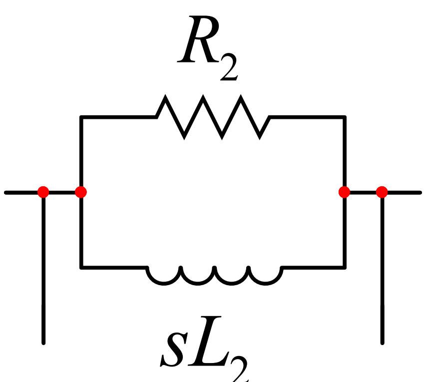


$$Z_1(s) = \frac{\underbrace{1 + sr_C C_1}_{=0}}{sC_1}$$

$$Z_1(s_{z_1}) = 0\ \Omega$$

$$s_{z_1} = -\frac{1}{r_C C_1}$$

- For $s = s_{z2}$, the RL impedance is infinite



series

$$Z_2(s) = \frac{sL_2R_2}{\underbrace{R_2 + sL_2}_{=0}}$$

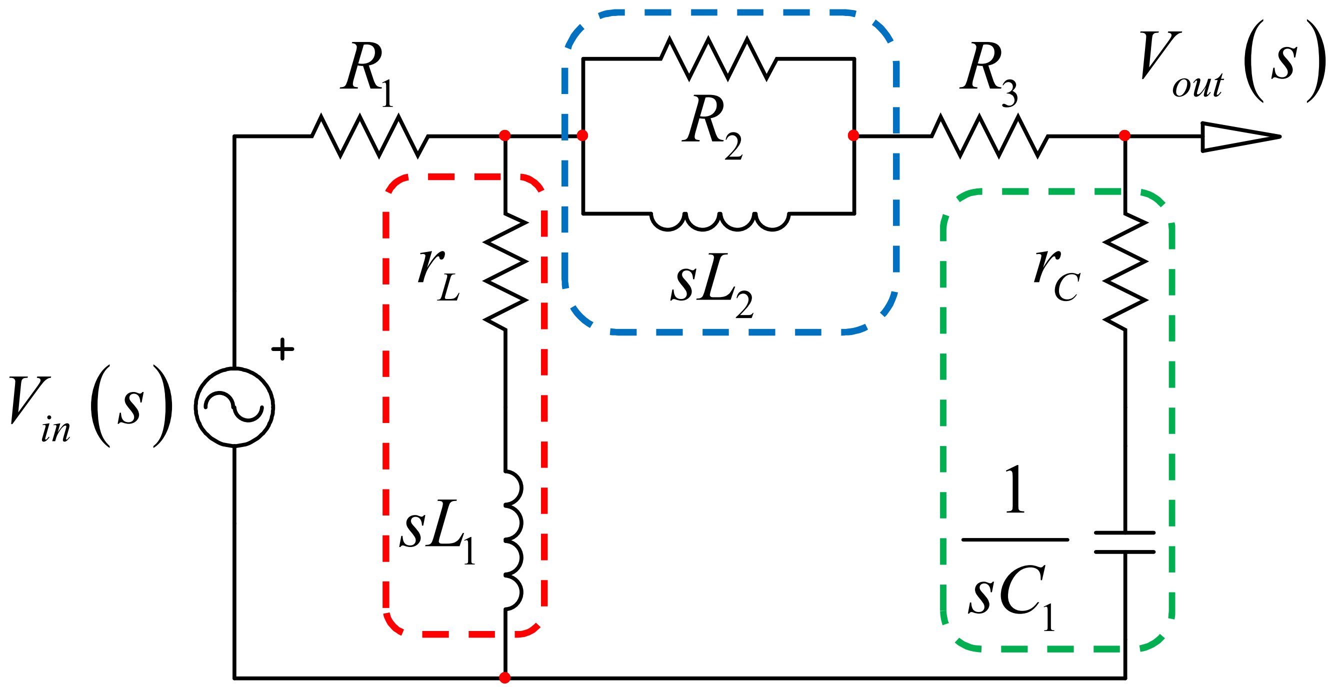
Poles of the *RL* network become zeros of the transfer function.

$$Z_2(s_{z_2}) \to \infty\ \Omega$$

$$s_{z_2} = -\frac{R_2}{L_2}$$

Zeros by Inspection: Fastest Option!

- Identify *transformed* open circuits/short circuits



$$s_{z_1} = -\frac{r_L}{L_1} \qquad \omega_{z_1} = \frac{r_L}{L_1}$$

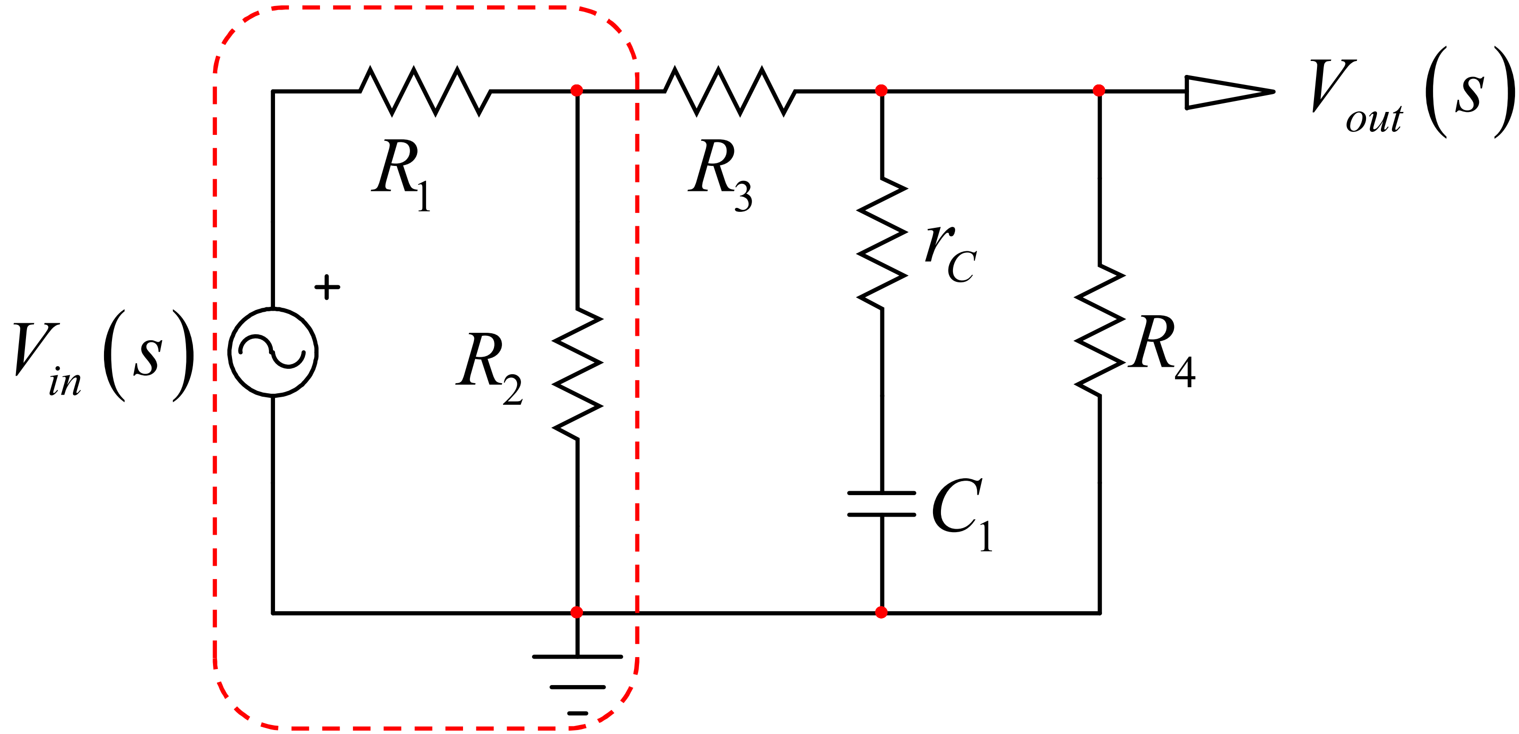
$$s_{z_2} = -\frac{R_2}{L_2} \qquad \omega_{z_2} = \frac{R_2}{L_2}$$

$$s_{z_3} = -\frac{1}{r_C C_1} \qquad \omega_{z_3} = \frac{1}{r_C C_1}$$

$$N(s) = \left(1 + \frac{s}{\omega_{z_1}}\right)\left(1 + \frac{s}{\omega_{z_2}}\right)\left(1 + \frac{s}{\omega_{z_3}}\right)$$

FACTs at Work in an Example

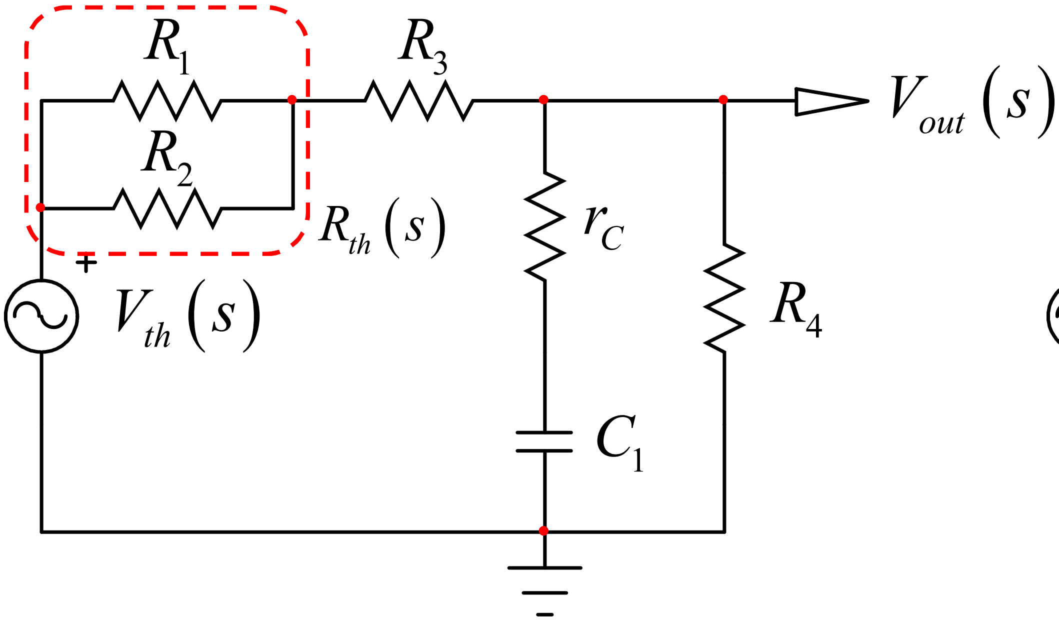
- How would you calculate V_{out} / V_{in}?

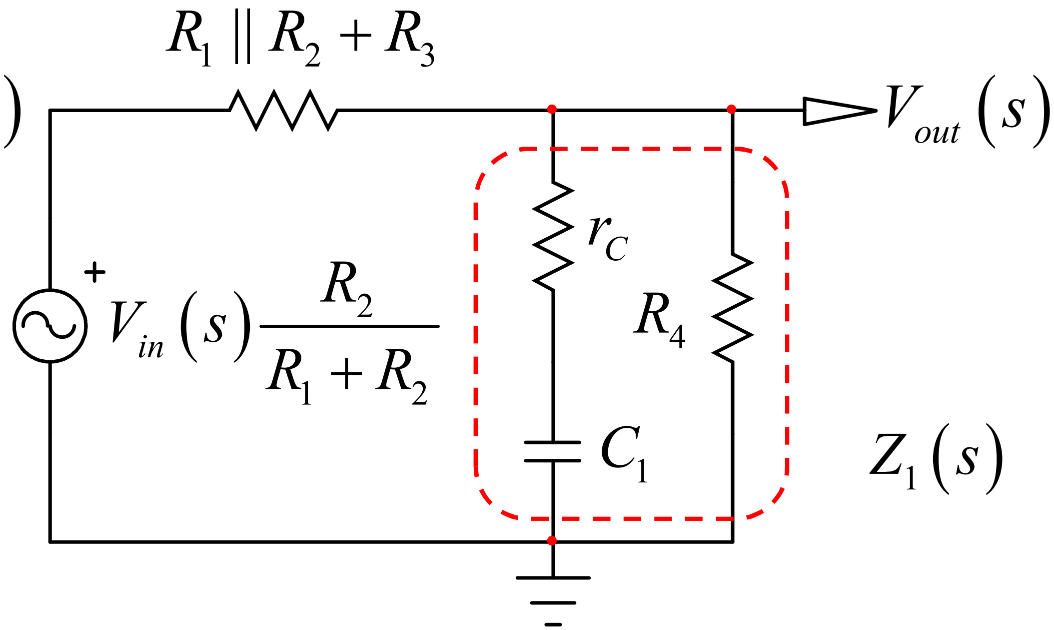


1. Transform the circuit with a Thévenin generator
2. Apply impedance divider involving C_1

Apply Impedance Divider

- Reduce circuit complexity with Thévenin





- Apply impedance divider involving Z_1 and R_{th}

$$\left.\begin{aligned} R_{th}(s) &= R_1 \| R_2 + R_3 \\ Z_1(s) &= R_4 \| \left(r_C + \frac{1}{sC_1}\right) \end{aligned}\right\} \quad H(s) = \frac{Z_1(s)}{Z_1(s) + R_{th}(s)} \frac{R_2}{R_1 + R_2}$$

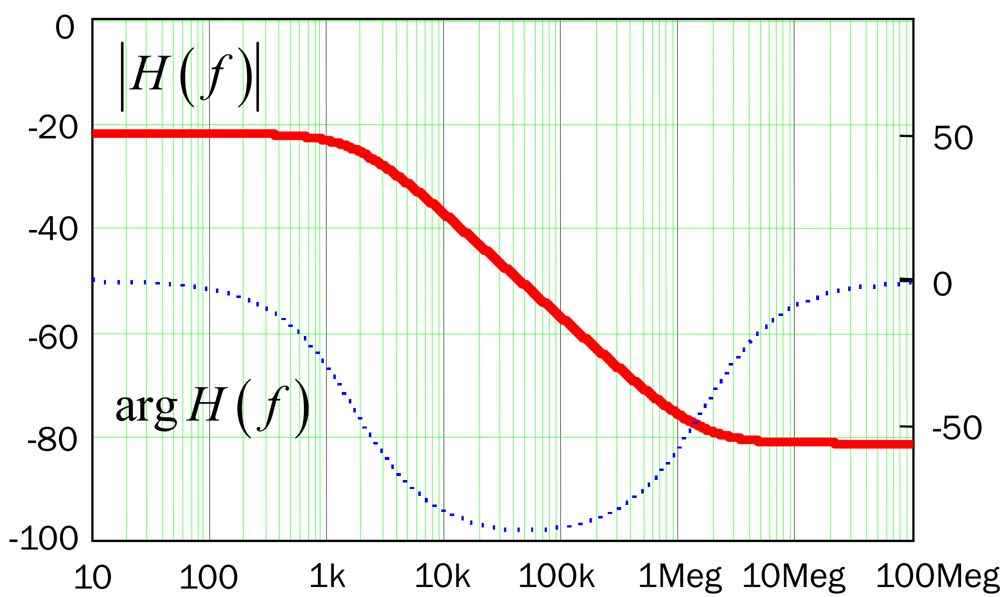
"Who you gonna call?"

High-Entropy Expression

- How do you make use of this result?

$$H_2(s) := \frac{R_2 \cdot R_4 \cdot (C_1 \cdot r_C \cdot s + 1)}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4 + C_1 \cdot R_1 \cdot R_2 \cdot R_4 \cdot s + C_1 \cdot R_1 \cdot R_3 \cdot R_4 \cdot s + C_1 \cdot R_2 \cdot R_3 \cdot R_4 \cdot s + C_1 \cdot R_1 \cdot R_2 \cdot r_C \cdot s + C_1 \cdot R_1 \cdot R_3 \cdot r_C \cdot s + C_1 \cdot R_1 \cdot R_4 \cdot r_C \cdot s + C_1 \cdot R_2 \cdot R_3 \cdot r_C \cdot s + C_1 \cdot R_2 \cdot R_4 \cdot r_C \cdot s}$$

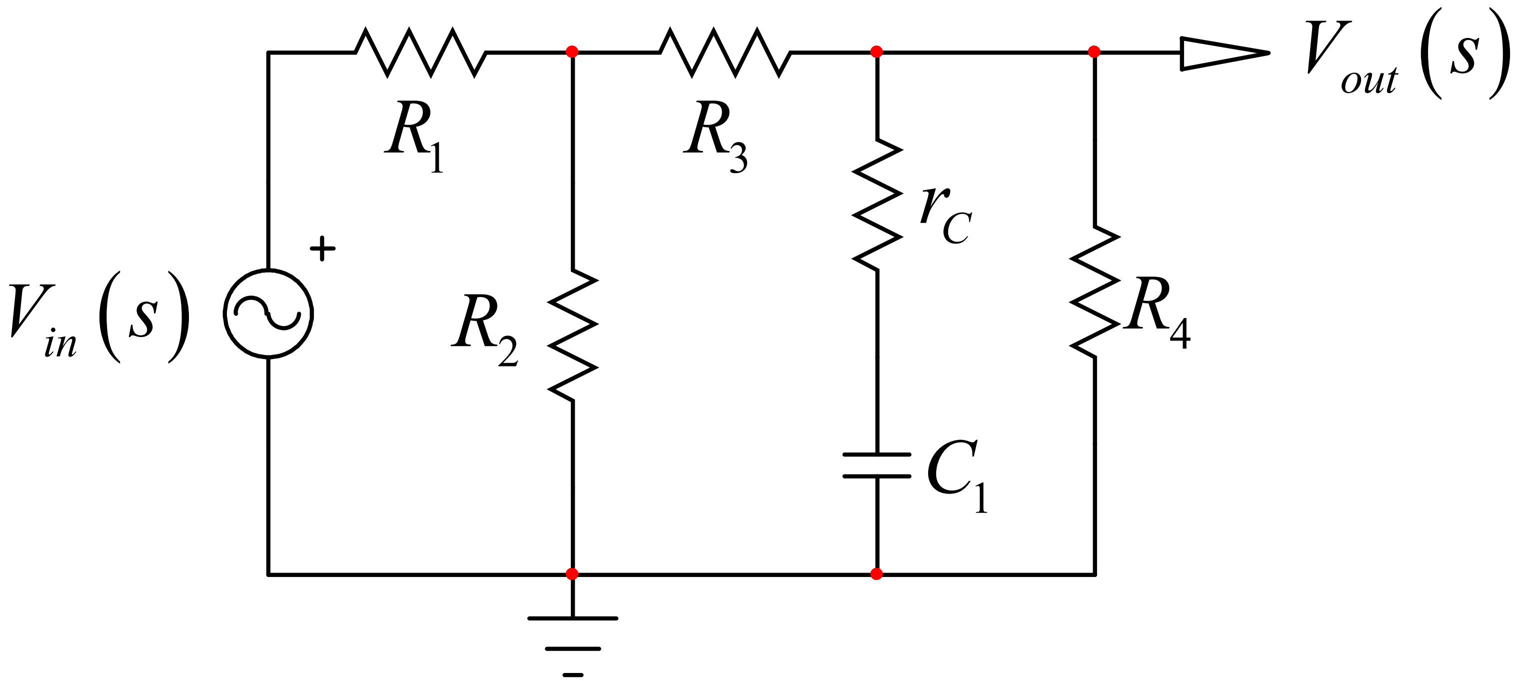
- what is the pole/zero position?
- what affects the quasi-static gain for s = 0?



You can plot the ac response but it yields no insight on what drives poles and zeros!

Applying FACTs Now

- What is the gain when V_{in} is a dc voltage?



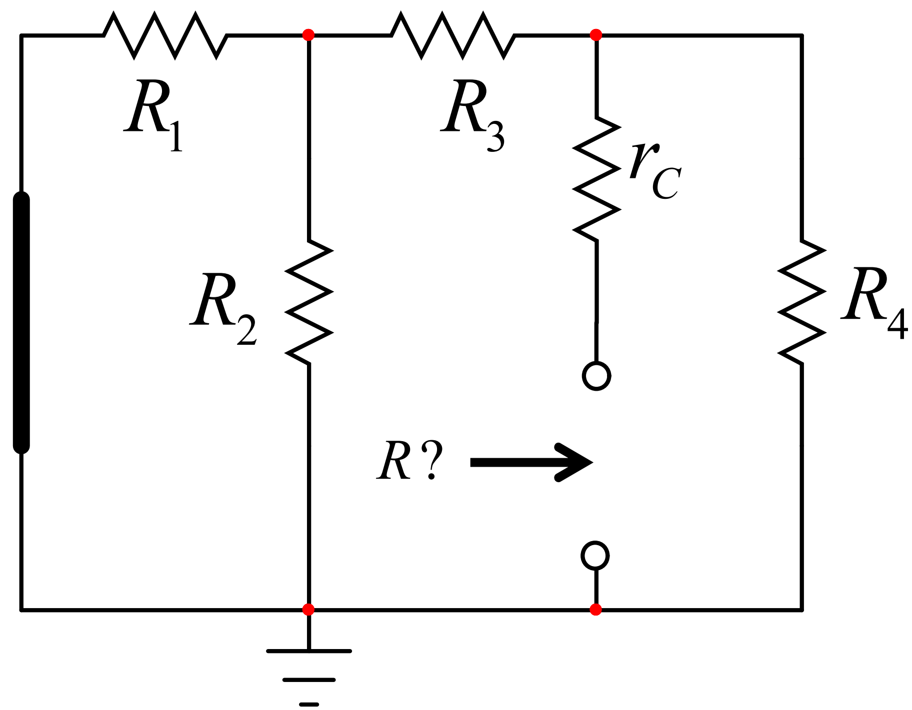
- The capacitor is open circuited, read the schematic!

$$H_0 = \frac{R_2}{R_1 + R_2} \frac{R_4}{R_1 \| R_2 + R_3 + R_4}$$

Determine the First Time Constant

- Look at the resistance driving the storage element

1. When the excitation is turned off, V_{in} = 0 V

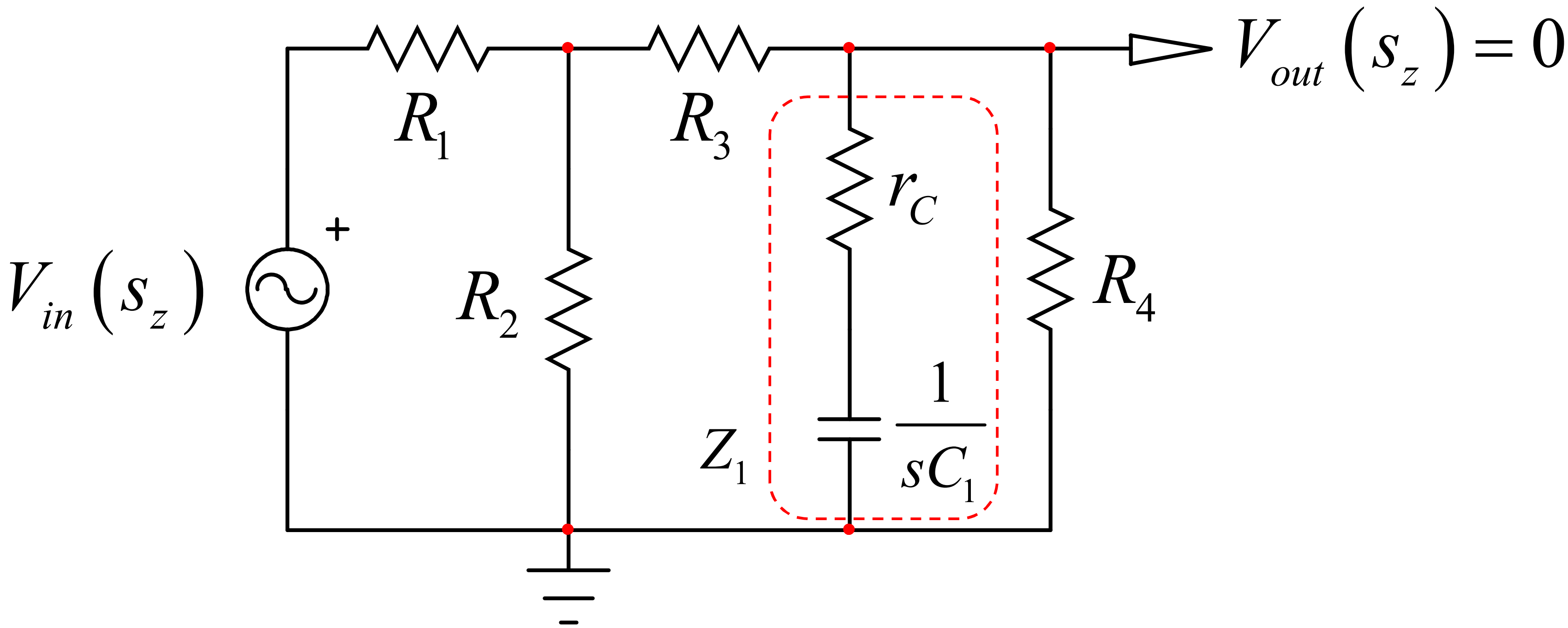


$$\tau_1 = \left[r_C + \left(R_1 \,||\, R_2 + R_3 \right) || \, R_4 \right] C_1$$

Denominator calculation

Determine the Second Time Constant

- Inspect the circuit and find the transformed short circuit



- If Z_1 is equal to 0, the output is nulled

$$r_C + \frac{1}{s_z C_1} = 0 \quad \Rightarrow \quad s_z = -\frac{1}{r_C C_1}$$

Numerator calculation

Assemble the Terms

- You immediately have a *low-entropy* form

$$H(s)=H_0\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$$

$$H_0=\frac{R_2}{R_1+R_2}\frac{R_4}{R_1 \| R_2+R_3+R_4}$$

$$\omega_p=\frac{1}{\left[r_C+\left(R_1 \| R_2+R_3\right) \| R_4\right]C_1}$$

$$\omega_z=\frac{1}{r_C C_1}$$

Way cool!

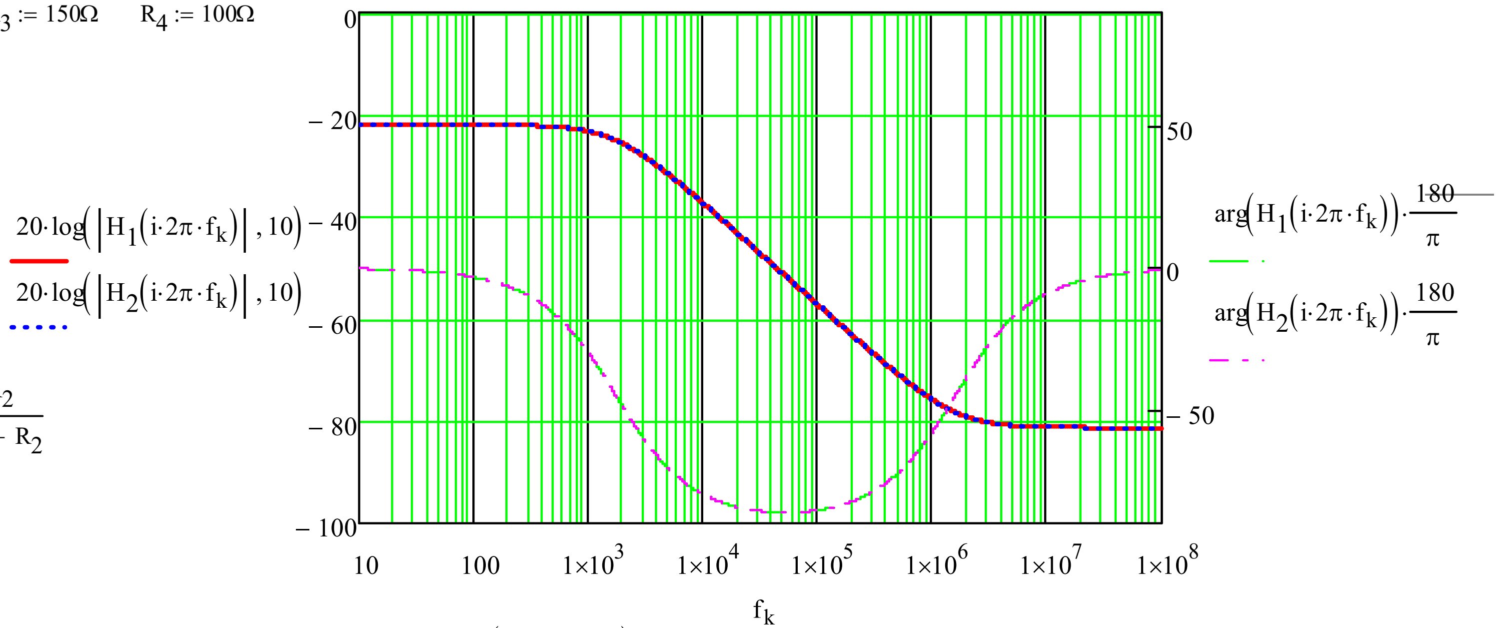
- We did not write a single line of algebra!

Use Mathcad® to Check Results

$R_1 := 1k\Omega \qquad R_2 := 22k\Omega \qquad r_C := 0.1\Omega \qquad R_3 := 150\Omega \qquad R_4 := 100\Omega$

$\|(x,y) := \dfrac{x \cdot y}{x+y} \qquad C_1 := 1\mu F$

$$H(s) = \frac{\dfrac{R_4 \cdot \left(r_C + \dfrac{1}{s \cdot C_1}\right)}{R_4 + r_C + \dfrac{1}{s \cdot C_1}}}{\dfrac{R_4 \cdot \left(r_C + \dfrac{1}{s \cdot C_1}\right)}{R_4 + r_C + \dfrac{1}{s \cdot C_1}} + \dfrac{R_1 \cdot R_2}{R_1 + R_2} + R_3} \cdot \frac{R_2}{R_1 + R_2}$$



$$H_2(s) := \frac{R_2 \cdot R_4 \cdot (C_1 \cdot r_C \cdot s + 1)}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4 + C_1 \cdot R_1 \cdot R_2 \cdot R_4 \cdot s + C_1 \cdot R_1 \cdot R_3 \cdot R_4 \cdot s + C_1 \cdot R_2 \cdot R_3 \cdot R_4 \cdot s + C_1 \cdot R_1 \cdot R_2 \cdot r_C \cdot s + C_1 \cdot R_1 \cdot R_3 \cdot r_C \cdot s + C_1 \cdot R_1 \cdot R_4 \cdot r_C \cdot s + C_1 \cdot R_2 \cdot R_3 \cdot r_C \cdot s + C_1 \cdot R_2 \cdot R_4 \cdot r_C \cdot s}$$

$\tau_2 := C_1 \cdot \left[r_C + \left(R_1 \| R_2 + R_3\right) \| R_4\right] = 91.812 \cdot \mu s$

$\tau_1 := C_1 \cdot r_C = 100 \cdot ns$

$H_0 := \dfrac{R_4}{R_4 + R_1 \| R_2 + R_3} \cdot \dfrac{R_2}{R_1 + R_2} = 0.079$

$H_1(s) := H_0 \cdot \dfrac{1 + s \cdot \tau_1}{1 + s \cdot \tau_2}$

Superimposing both transfer functions, matching should be perfect. If not, there is mistake.

Fractions and Dimensions

- A 1st-order system follows the form

$$H(s)=\frac{N(s)}{D(s)}=\frac{a_0+a_1 s}{b_0+b_1 s}\xrightarrow{\text{factoring}}H(s)=\frac{a_0}{b_0}\frac{1+\frac{a_1}{a_0}s}{1+\frac{b_1}{b_0}s}$$

- A leading term (if any) carries the unit

$$\underset{[\Omega]}{Z(s)}=\underset{[\Omega]}{R_0}\underbrace{\boxed{\frac{1+\frac{a_1}{a_0}s}{1+\frac{b_1}{b_0}s}}}_{\text{Unitless}}\Rightarrow\left.\begin{array}{c}1+\frac{a_1}{a_0}s\\ \Updownarrow\\ 1+\frac{b_1}{b_0}s\\ \text{Unitless}\end{array}\right\}\begin{array}{l}\frac{a_1}{a_0}\to\underset{\text{time}}{[\mathrm{s}]}\to\tau_N\\ \\ \frac{b_1}{b_0}\to\underset{\text{time}}{[\mathrm{s}]}\to\tau_D\end{array}$$

2nd-Order System

- A 2nd-order system follows the form

$$H(s)=\frac{\alpha_0+\alpha_1 s+\alpha_2 s^2}{\beta_0+\beta_1 s+\beta_2 s^2} \xrightarrow[\text{Factoring } \beta_0]{\text{Factoring } \alpha_0} H(s)=\underbrace{H_0}_{\text{Carries the unit}}\boxed{\frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2}} \quad \text{Unitless}$$

- The second fraction is unitless

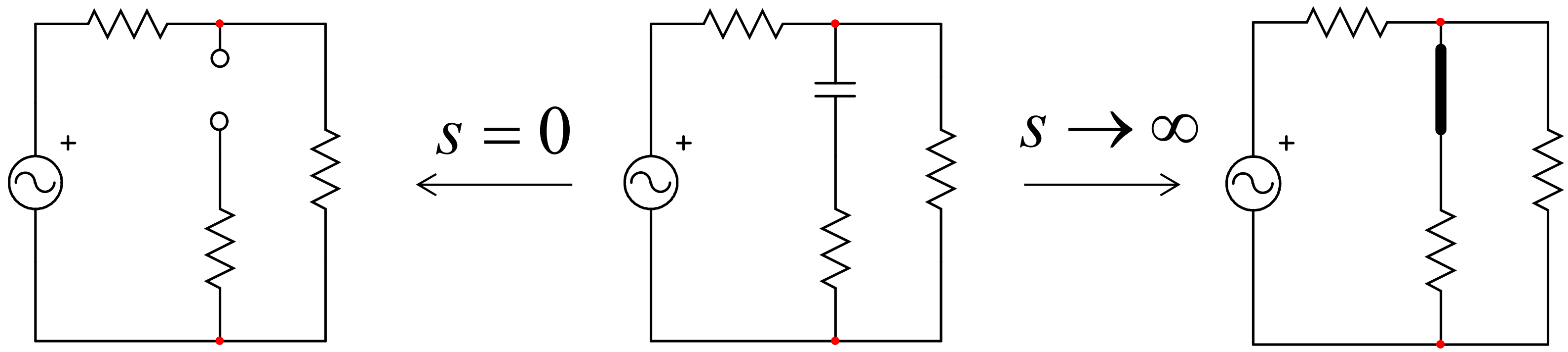
$$a_1=\frac{\alpha_1}{\alpha_0}\to[\mathrm{s}]\to\tau_{1N}+\tau_{2N} \qquad a_2=\frac{\alpha_2}{\alpha_0}\to\underbrace{\left[\mathrm{s}^2\right]}_{\text{time}^2}\to\tau_{1N}\tau_{2N}^1 \text{ or } \tau_{2N}\tau_{1N}^2$$

sum product

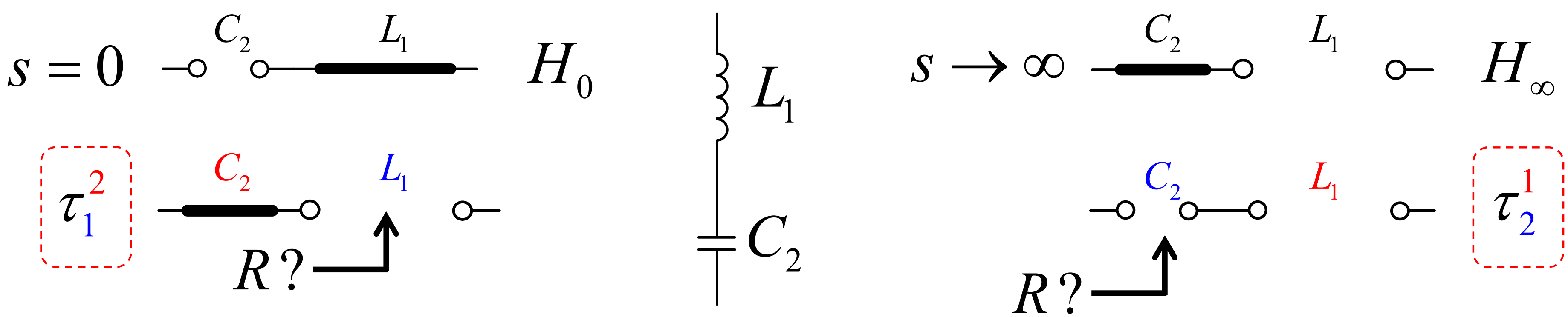
$$b_1=\frac{\beta_1}{\beta_0}\to[\mathrm{s}]\to\underbrace{\tau_{1D}}_{\text{reactance 1}}+\underbrace{\tau_{2D}}_{\text{reactance 2}} \qquad b_2=\frac{\beta_2}{\beta_0}\to\underbrace{\left[\mathrm{s}^2\right]}_{\text{time}^2}\to\tau_{1D}\tau_{2D}^1 \text{ or } \tau_{2D}\tau_{1D}^2$$

Alternating the Reactance States

- In a 1st-order circuit, there is one reactance
  - it is either in a high-frequency state or in a dc state



- In a 2nd-order circuit, there are two reactances
  - we can consider individual states



Introducing the Notation

- Set one reactance into its high-frequency state

$$\tau_2^1$$

- 1 → Reactance 1 is in its high-frequency state
- 2 → What resistance drives reactance 2?

$R?$

$$\tau_1^2$$

- 2 → Reactance 2 is in its high-frequency state
- 1 → What resistance drives reactance 1?

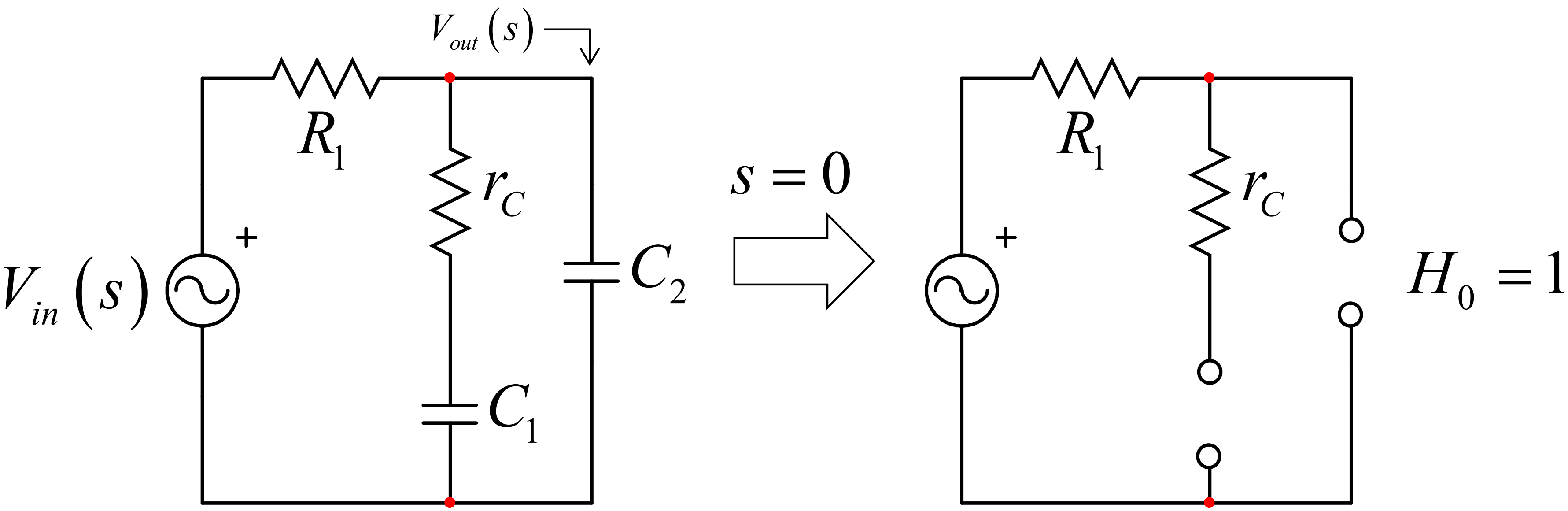
$R?$

- There is redundancy: pick the simplest result

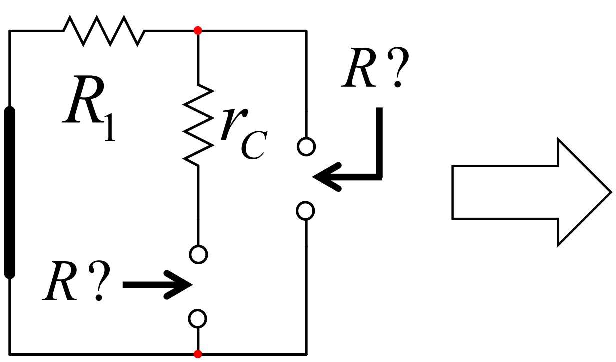
$$b_2 = \tau_1 \tau_2^1 \Longleftrightarrow b_2 = \tau_2 \tau_1^2$$

Example with Capacitors

- Assume the following 2-capacitor circuit



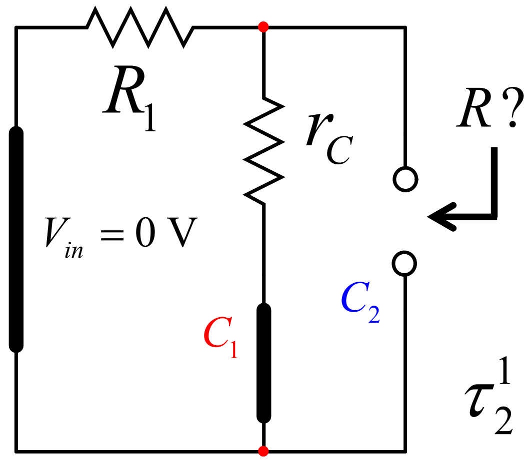
- Determine the two time constants while V_{in} is 0 V



$$\left.\begin{aligned}\tau_1 &= C_1\left(r_C + R_1\right)\\ \tau_2 &= C_2 R_1\end{aligned}\right\}\quad \begin{aligned} b_1 &= \tau_1 + \tau_2\\ b_1 &= C_1\left(r_C + R_1\right) + C_2 R_1\end{aligned}$$

Determining the Higher-Order Term

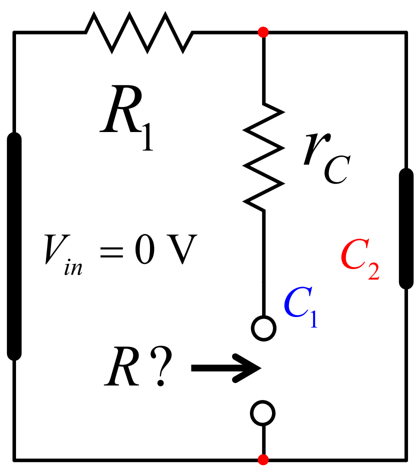
- Place C_1 in its high-frequency state and look into C_2



$$\tau_2^1 = \left(R_1 \,\|\, r_C\right) C_2$$

$$\Rightarrow \quad b_2 = \tau_1 \tau_2^1 = C_1\left(r_C + R_1\right) C_2 \left(R_1 \,\|\, r_C\right)$$

- Place C_2 in its high-frequency state and look into C_1



$$\tau_1^2 = r_C C_1$$

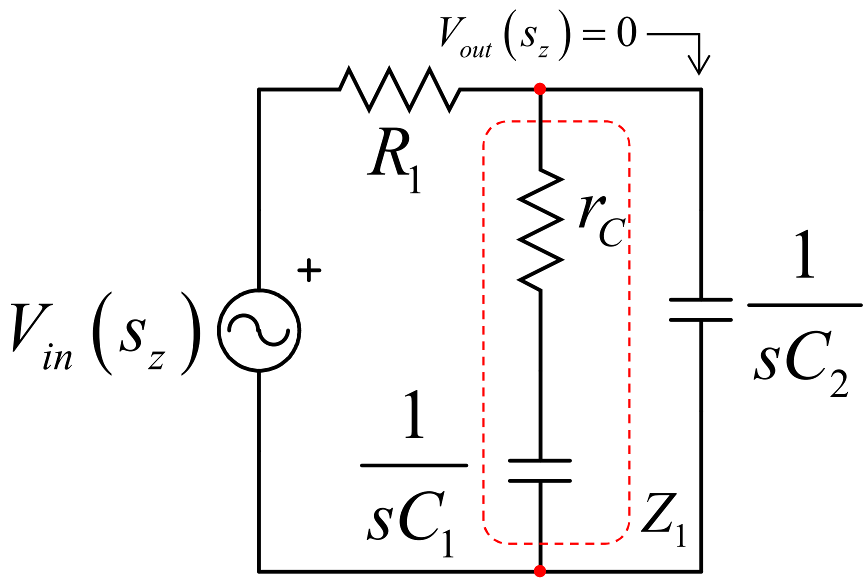
$$\Rightarrow \quad b_2 = \tau_2 \tau_1^2 = C_2 R_1 C_1 r_C$$

Denominator is Completed

- The denominator can be assembled

$$D(s) = 1 + b_1 s + b_2 s^2 = 1 + \left[C_1\left(r_C + R_1\right) + C_2 R_1\right]s + C_2 R_1 C_1 r_C s^2$$

- Is there a zero in this network?



$$r_C + \frac{1}{sC_1} = 0 \quad \left.\right\} \quad \begin{aligned} s_{z_1} &= -\frac{1}{r_C C_1} \\ \omega_{z_1} &= \frac{1}{r_C C_1} \end{aligned}$$

- If Z_1 becomes a transformed short, the response disappears

Final Expression and Conclusion

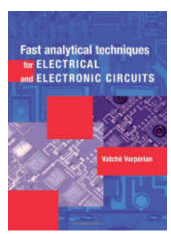
- Gather the pieces to form the transfer function

$$H(s)=\frac{1+sr_C C_1}{1+\left[C_1\left(r_C+R_1\right)+C_2R_1\right]s+C_2R_1C_1r_Cs^2}$$

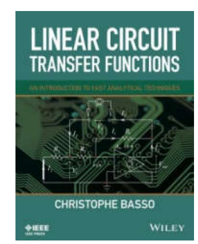
$$\Rightarrow \quad H(s)=H_0\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}$$

- This expression was determined in a flashing time — No algebra!
- We did not use KVL or KCL: inspection is easy

Fast Analytical Circuits Techniques for Electrical and Electronic Circuits – Vatché Vorpérian – Cambridge Press 2002



Linear Circuit Transfer Function – A Tutorial Introduction to Fast Analytical Techniques –Christophe Basso – Wiley & Sons 2016

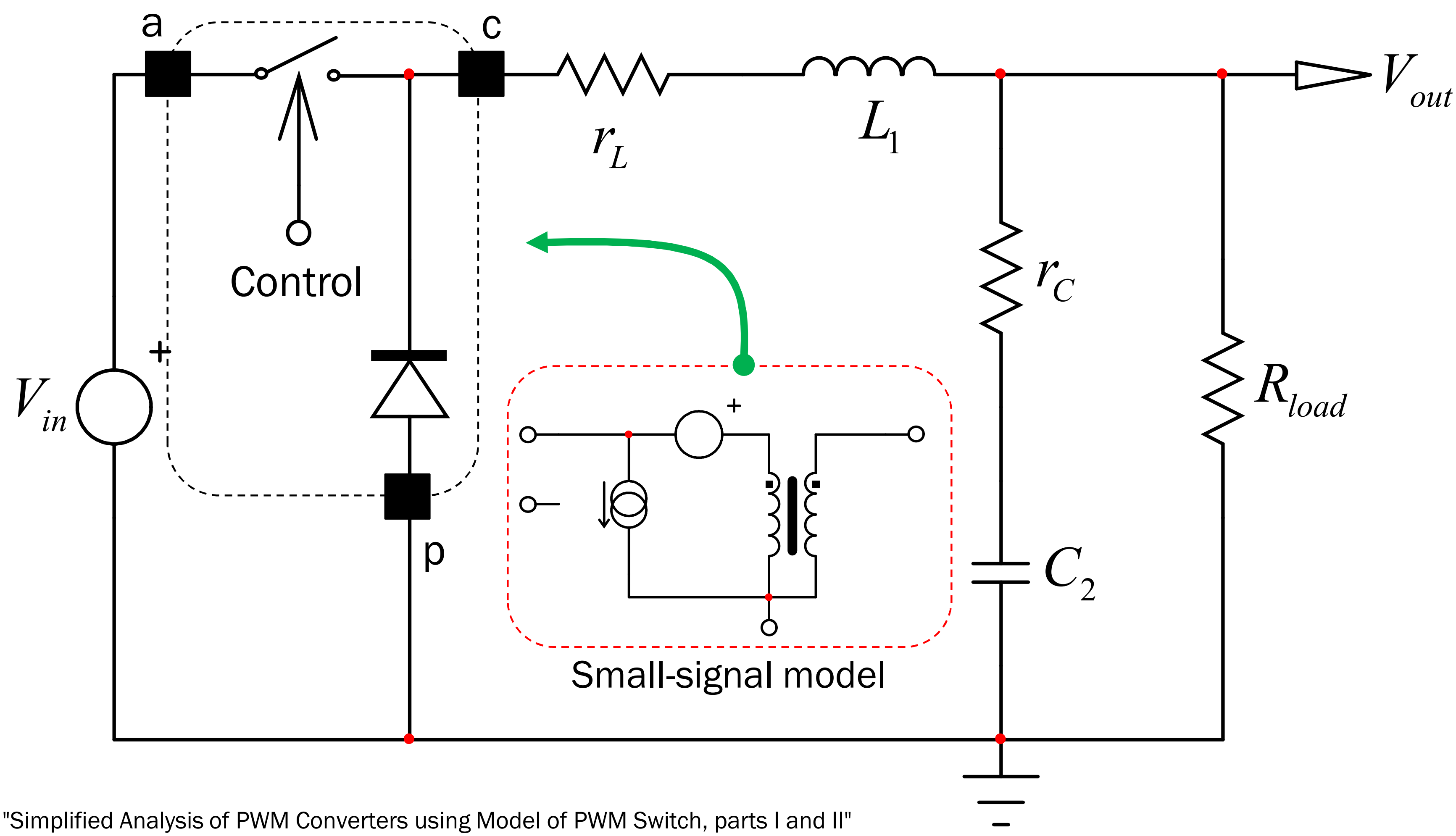


Course Agenda

- ❑ A Switching Regulator as a Load
- ❑ EMI Filter Impact
- ❑ An Introduction to FACTs
- ❑ Buck Converter Input/Output Impedances
- ❑ Filtering the Input Current
- ❑ Damping the Filter
- ❑ Optimum Component Selection
- ❑ A Practical Case Study
- ❑ Cascading Converters

A Buck Converter in Voltage Mode

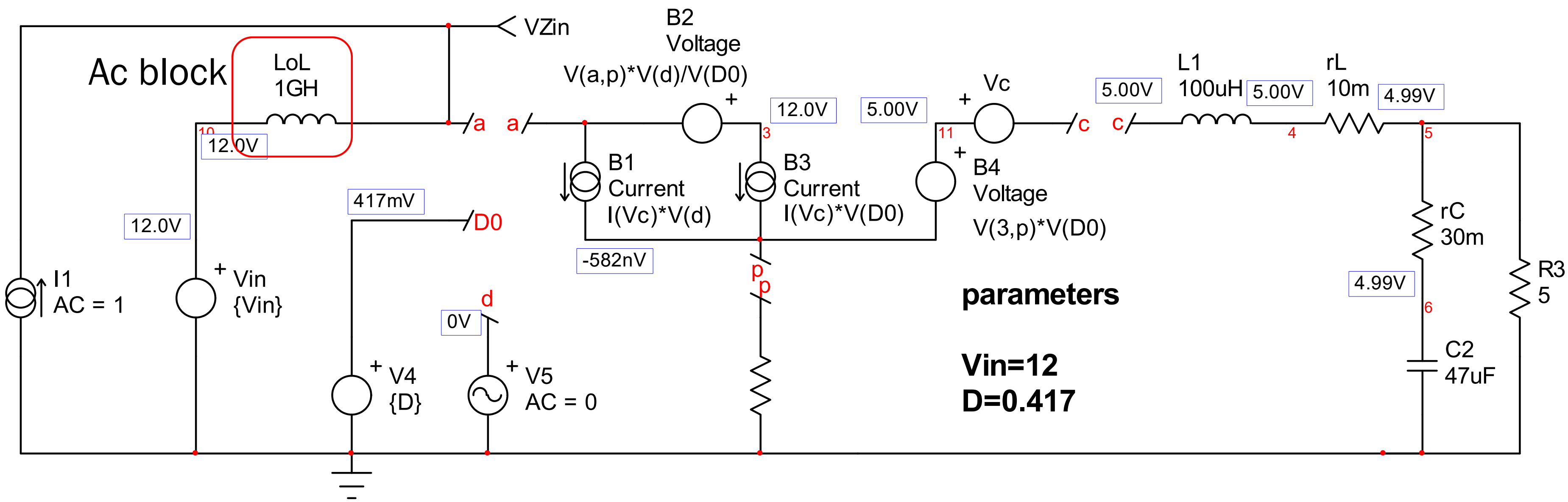
- Replace the diode and the switch by the PWM switch



V. Vorpérian, "Simplified Analysis of PWM Converters using Model of PWM Switch, parts I and II"
IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, NO. 3, 1990

Buck Input Impedance

- ❑ Inductance LoL lets you sweep the input to have Z_{in}



- ❑ In this mode, $\hat{d}$ is equal to zero

$$\left.\frac{V_{in}(s)}{I_{in}(s)}\right|_{\hat{d}=0}$$

Source B2 and B1 are zero

Node *p* is ground

$$V(a,p) = V_{in}$$

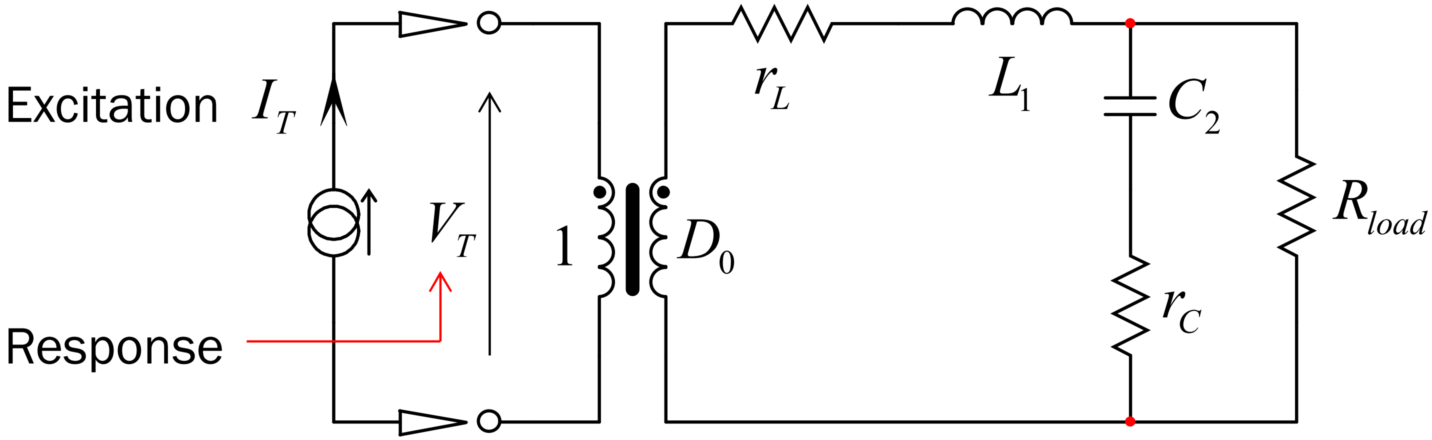
Simplify schematic

Check ac response

Input impedance

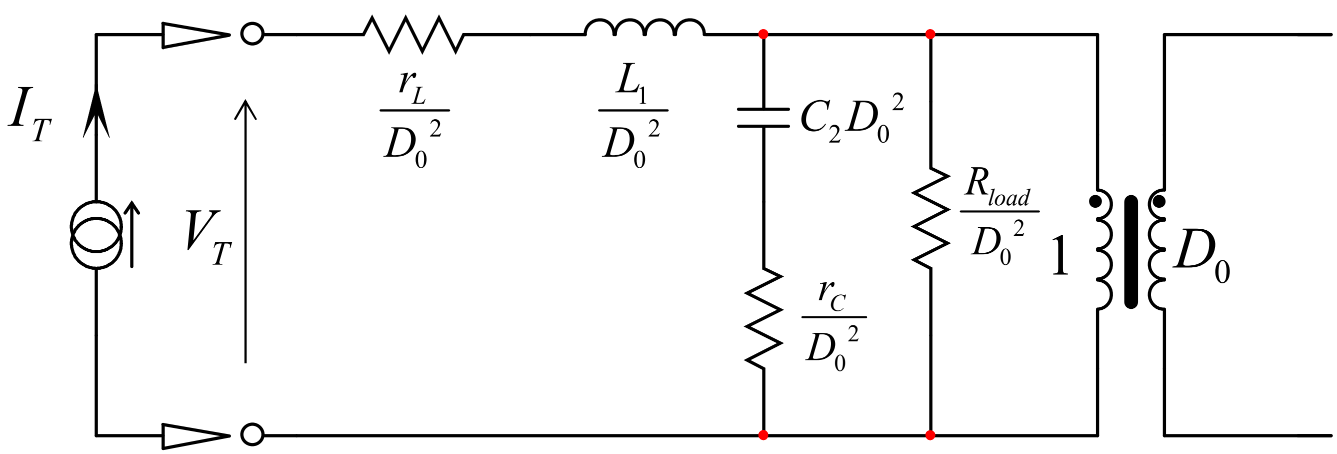
Simplifying and Rearranging is Key

- Install the dc transformer to obtain Z_{in}



$$Z_{in}(s) = \frac{V_T(s)}{I_{in}(s)}$$

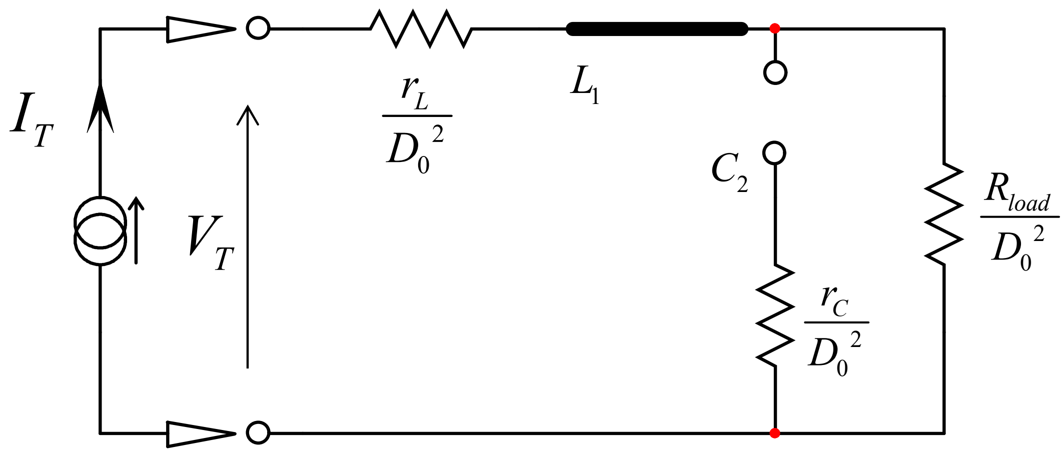
- Reflect elements to the primary side



Dc input resistance

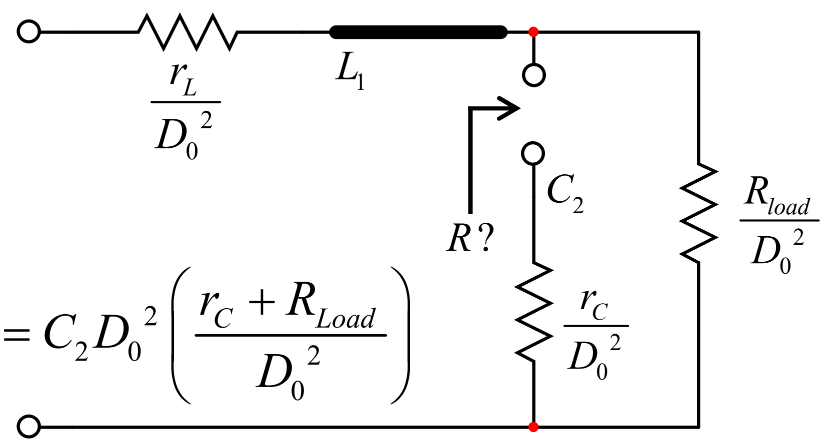
Start with $s = 0$ – Draw Circuit in dc

- Short the inductor, open the capacitor

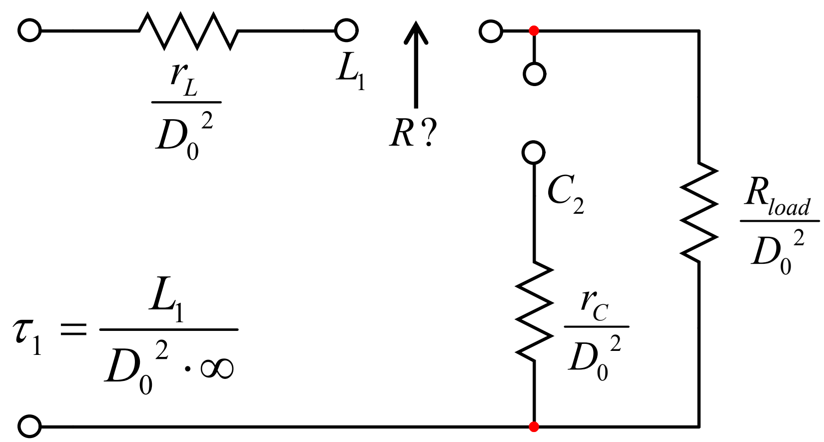


$$R_0 = \frac{r_L + R_{load}}{D_0^2}$$

- For the time constants, suppress the excitation, $I_T = 0$



$$\tau_2 = C_2 D_0^2 \left(\frac{r_C + R_{Load}}{D_0^2} \right)$$

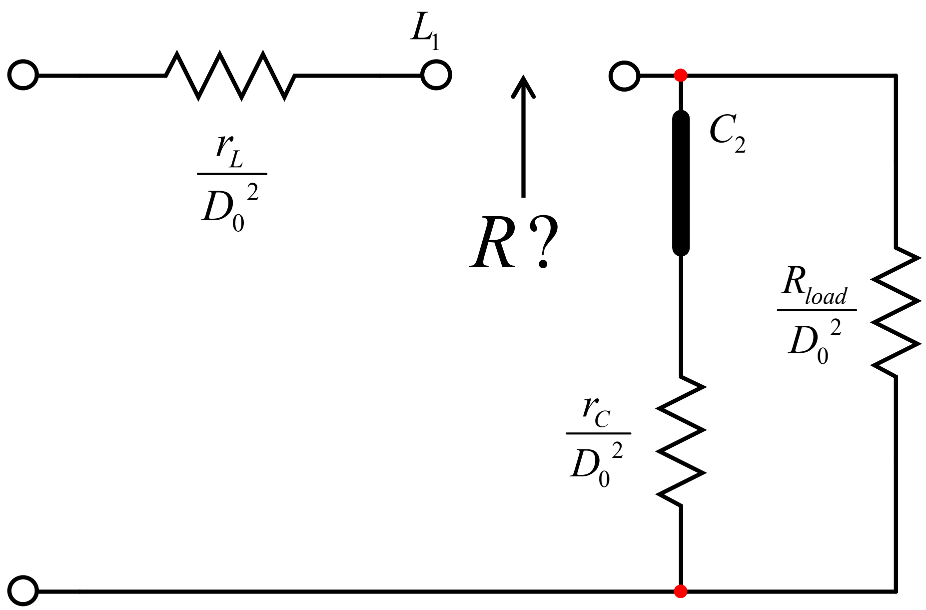


$$\tau_1 = \frac{L_1}{D_0^2 \cdot \infty}$$

Input impedance

Higher Order Coefficients

- Avoid indeterminacy with τ_1 : use τ_2 instead
- Determine τ_1^2



$$\tau_1^2 \xrightarrow{\quad} \text{High-frequency state}$$
$$\tau_1^2 \xleftarrow{\quad} R?$$

$$\tau_1^2 = \frac{L_1}{D_0^2 \cdot \infty}$$

$$b_2 = \tau_2 \tau_1^2 = C_2 D_0^2 \left(\frac{r_C + R_{Load}}{D_0^2} \right) \frac{L_1}{D_0^2 \cdot \infty} = 0$$

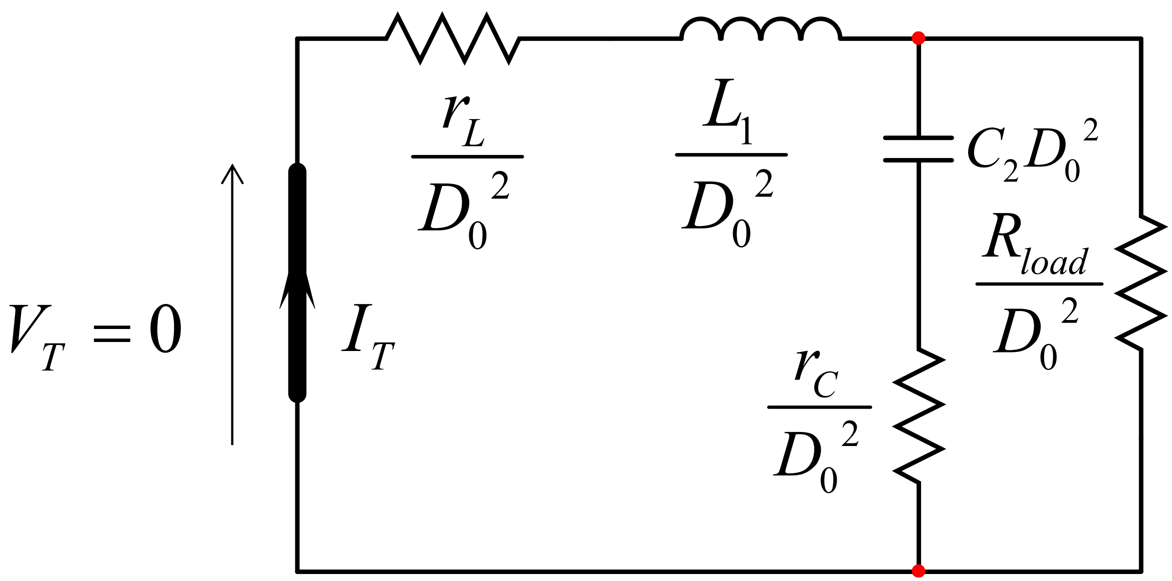
$$\Rightarrow D(s) = 1 + \left[C_2 D_0^2 \left(\frac{r_C + R_{Load}}{D_0^2} \right) + \frac{L_1}{D_0^2 \cdot \infty} \right] s = 1 + s C_2 \left(r_C + R_{Load} \right)$$

Input impedance

The Numerator is of 2nd-Order Type

- Null the response across the current source

⟶ Degenerate case, short the generator's terminals!



Use Fast Analytical Circuits Techniques!

$$N(s) = 1 + s\left(\frac{L_1}{r_L + R_{load}} + C_2\left[\left(r_L \| R_{load}\right) + r_C\right]\right) + s^2\left(L_1 C_2 \frac{r_C + R_{load}}{r_L + R_{load}}\right)$$

C. Basso, "Introduction to Fast Analytical Techniques, Application to Small-Signal Modeling", APEC 2016 Professional Seminar

Assemble the Pieces

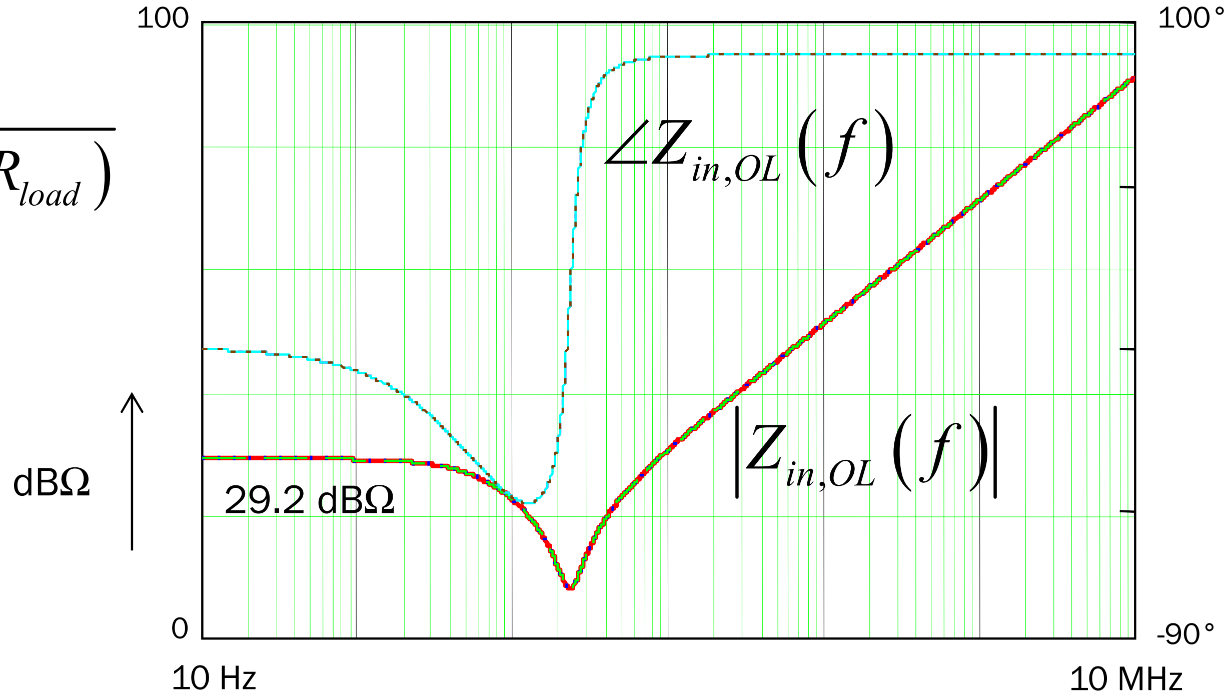
- The transfer function dimension is ohm

$$Z_{in,OL}(s) = R_0 \frac{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}{1 + \frac{s}{\omega_p}}$$

$$\omega_p = \frac{1}{(r_C + R_{Load})C_2} \qquad R_0 = \frac{r_L + R_{load}}{D_0^{\ 2}}$$

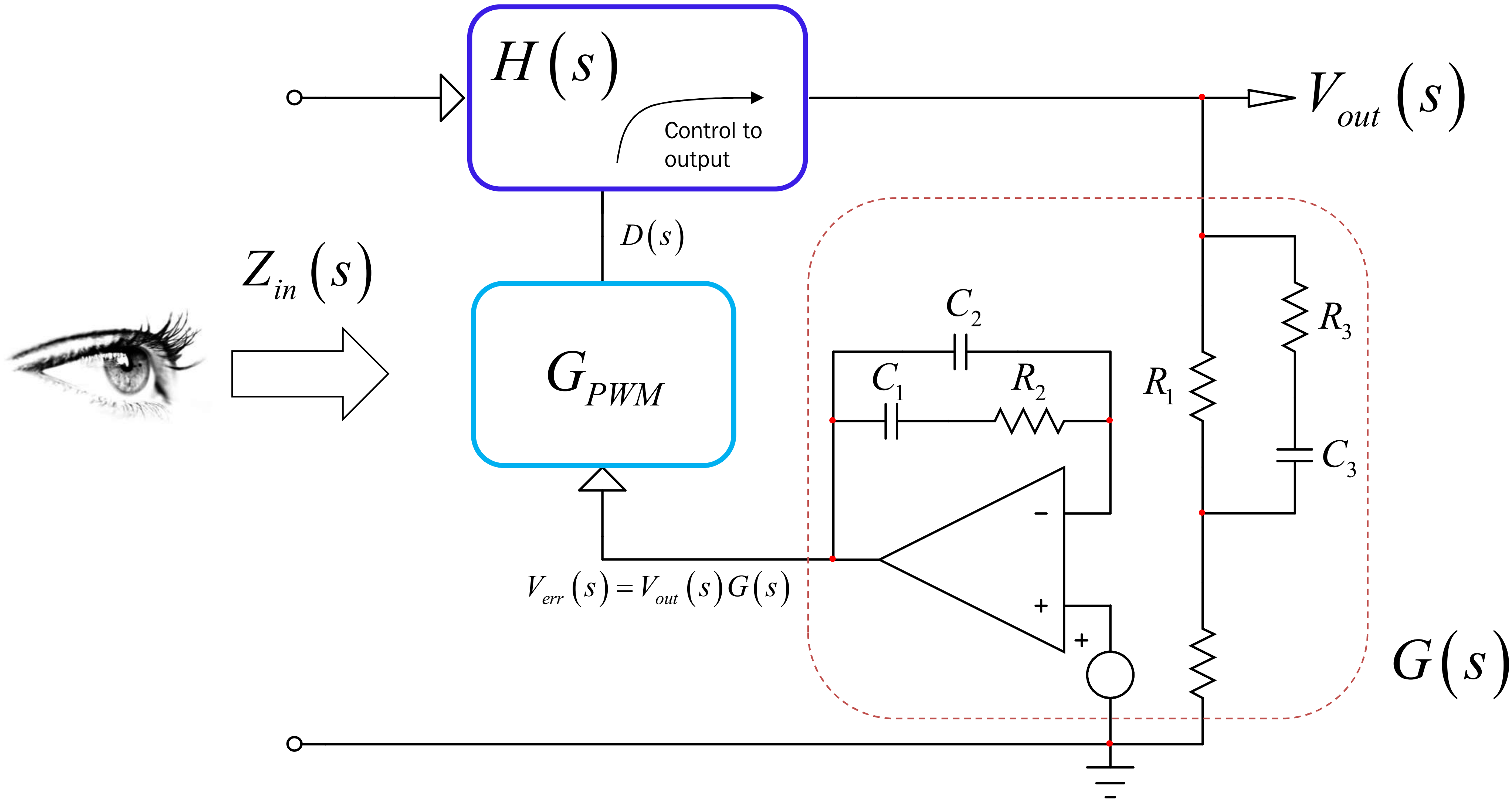
$$Q = \frac{L_1 C_2 \omega_0 (r_C + R_{load})}{L_1 + C_2(r_L r_C + r_L R_{load} + r_C R_{load})}$$

$$\omega_0 = \frac{1}{\sqrt{L_1 C_2}} \sqrt{\frac{r_L + R_{load}}{r_C + R_{load}}}$$



Closed-Loop Input Impedance

- We want the input impedance once the loop is closed



Stabilize the Buck with a Type 3

- A 10-kHz crossover is selected for the compensation

parameters

Vout=5
Pout=50
Rload=Vout^2/Pout
Rupper=10k
AOL=1T
Vp=2
fc=10k
Vin=20
Gfc=-22
PM=70
PS=-119
boost=PM-PS-90

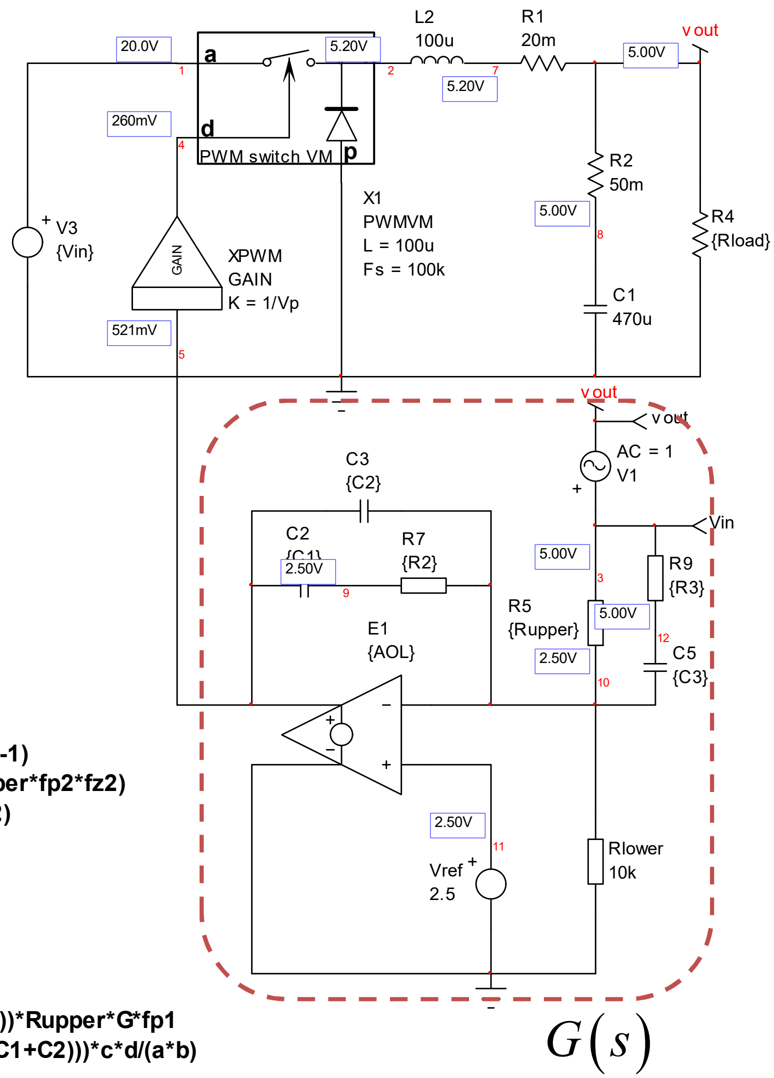
G=10^(-Gfc/20)
pi=3.14159

fz1=540
fz2=230
fp1=6.8k
fp2=fp12

C1=1/(2*pi*fz1*R2)
C2=C1/(C1*R2*2*pi*fp1-1)
C3=(fp2-fz2)/(2*pi*Rupper*fp2*fz2)
R3=Rupper*fz2/(fp2-fz2)

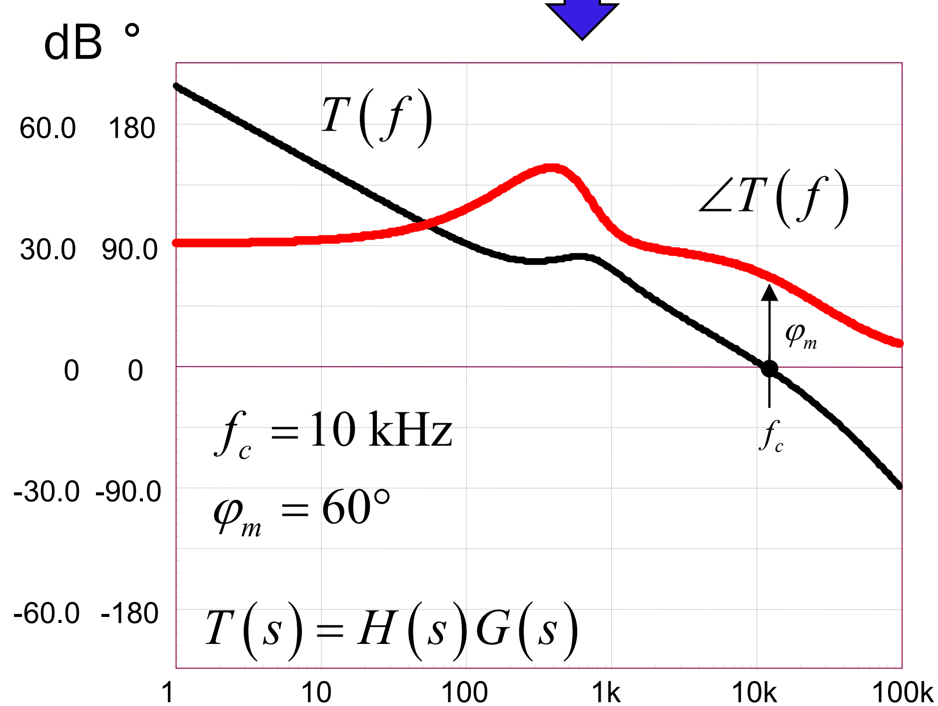
a=sqrt((fc^2/fp1^2)+1)
b=sqrt((fc^2/fp2^2)+1)
c=sqrt((fz1^2/fc^2)+1)
d=sqrt((fc^2/fz2^2)+1)

R2=((a*b/(c*d))/(fp1-fz1))*Rupper*G*fp1
G0=((R2*C1)/(Rupper*(C1+C2)))*c*d/(a*b)



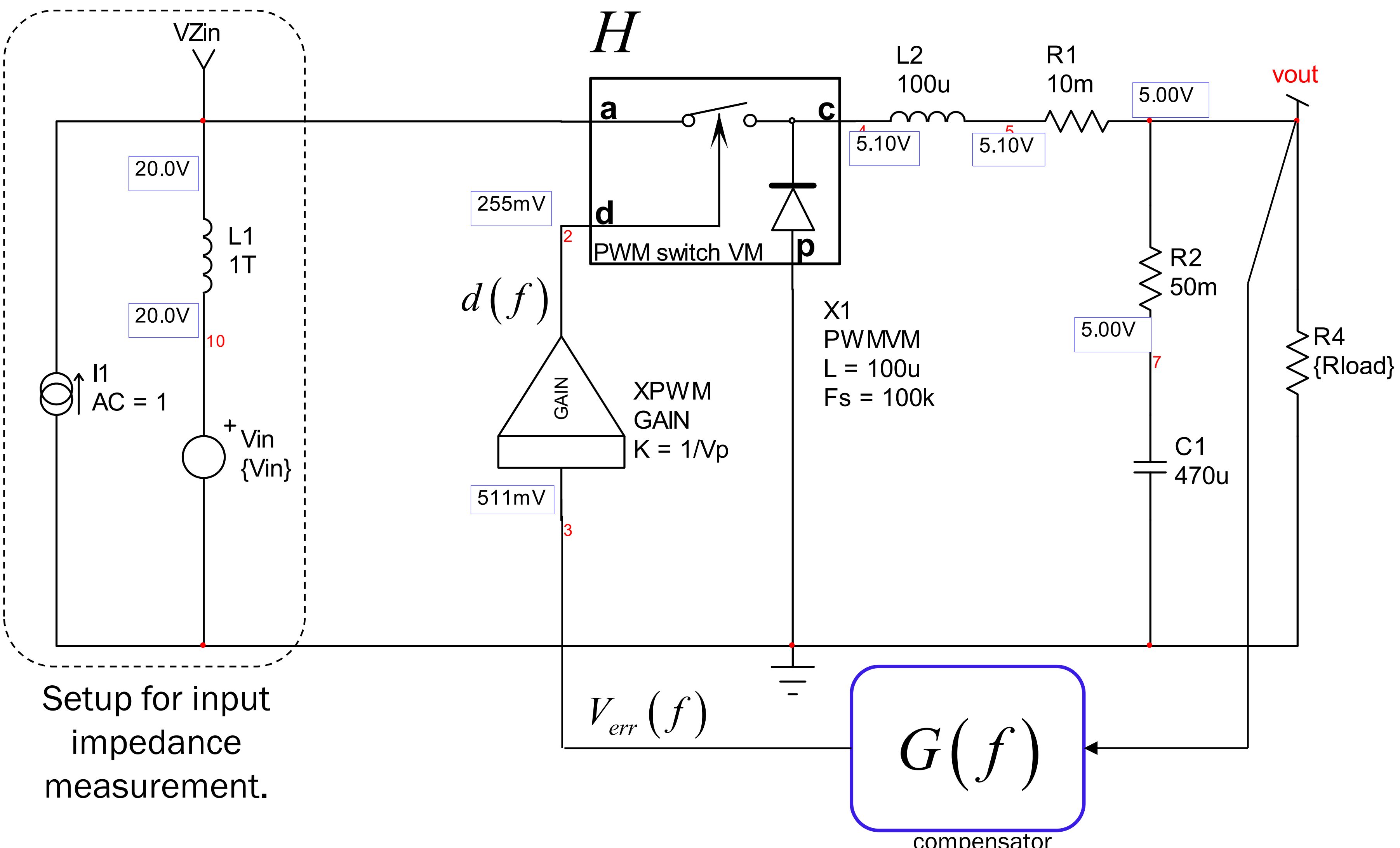
$$G(s) = -G_0 \frac{\left(1+\frac{\omega_{z_1}}{s}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

Neg. sign



Use Large-Signal Model for Reference

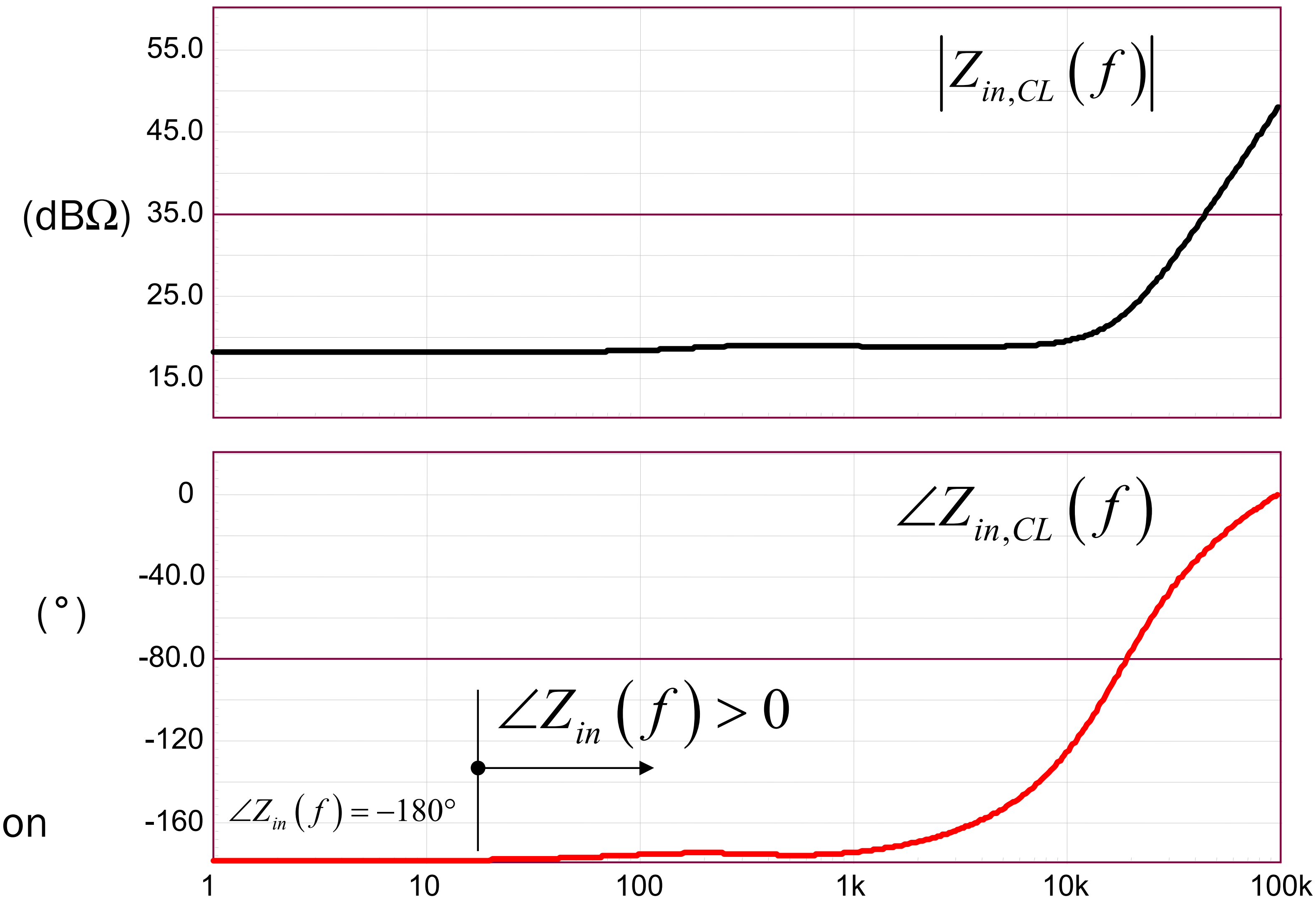
- Use the PWM switch to check the response



Setup for input impedance measurement.

Loop Gain Decreases as f Increases

- Input impedance is negative at low frequencies only



Simulation results

Why Does Z_{in} Become Positive?

- The below expression only holds if P_{out} is truly constant

$$R_{in} = \boxed{-}\frac{V_{in}^{\ 2}}{P_{out}}$$

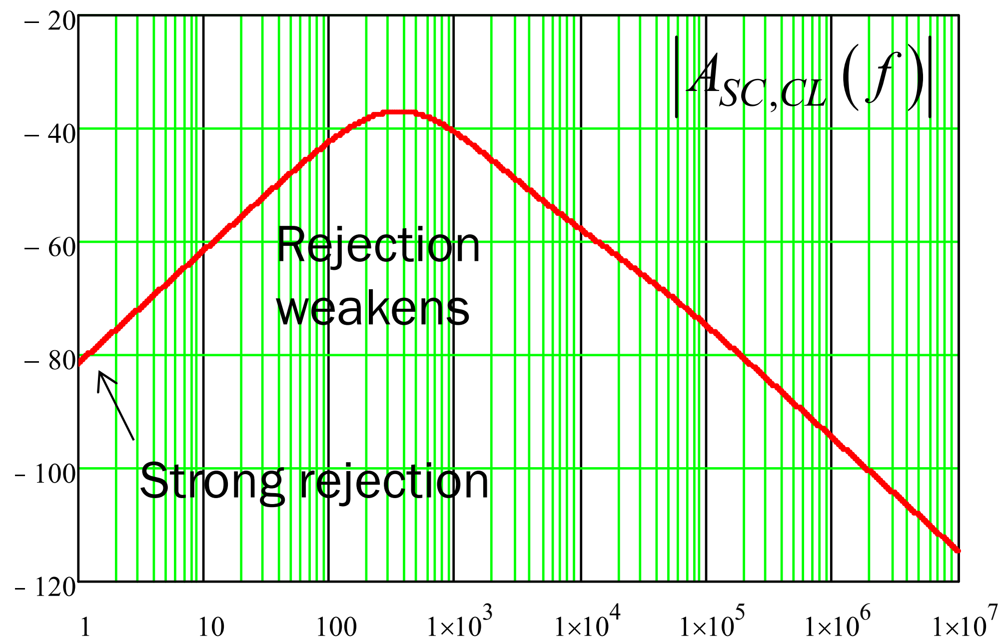
Infinite input voltage rejection

- It is true at dc, where loop gain is very high

$$A_{SC,OL}(s) = D\frac{R_{load}}{R_{load}+r_L}\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}$$

Audio susceptibility

$$A_{SC,CL}(s) = \frac{A_{SC,OL}(s)}{1+T(s)}$$

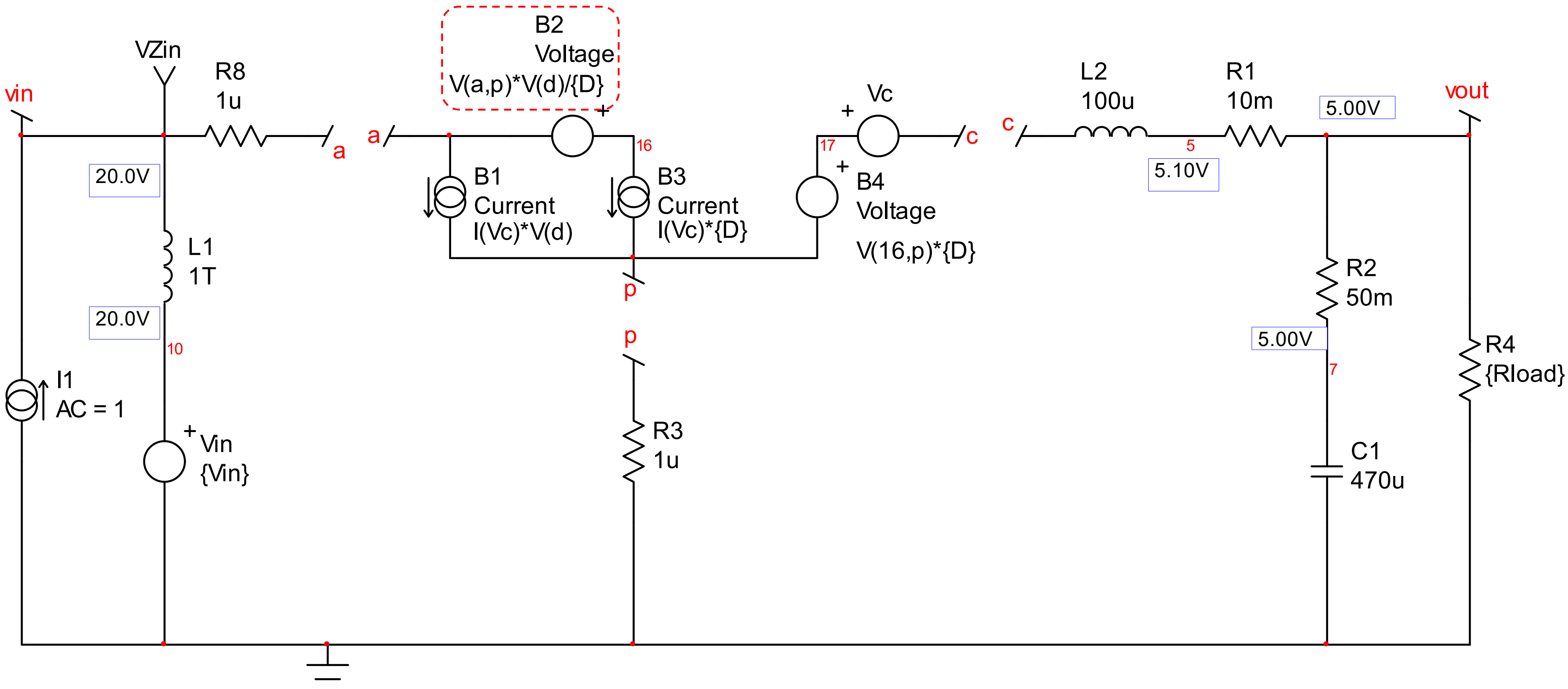


- Loop gain quickly drops as f increases

A_{SC}, audio susceptibility

Determining the Closed-Loop Impedance

- Update the large-signal model with linear sources



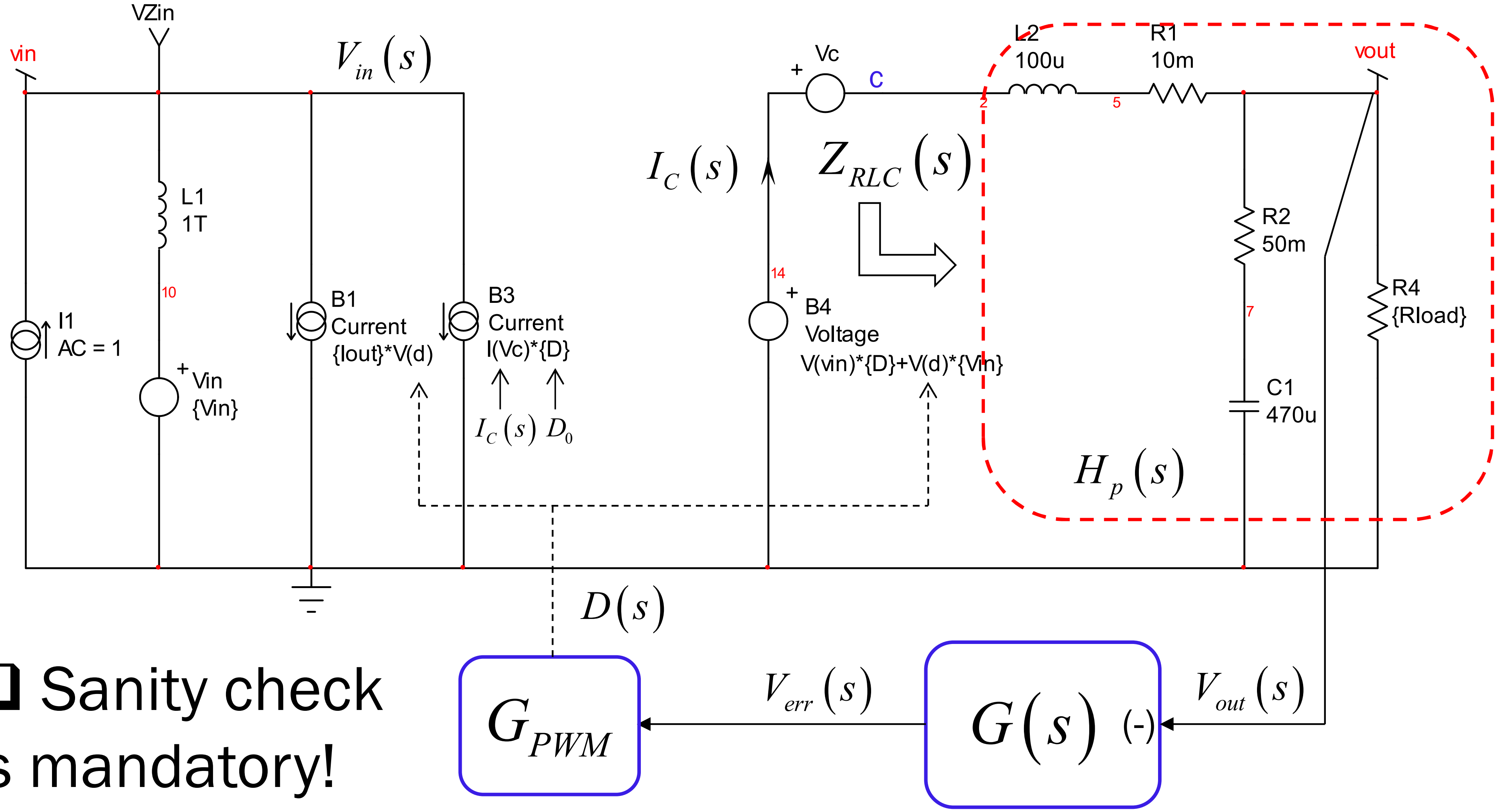
- B_2 needs to be linearized as d and V_{in} now include ac

$$\Rightarrow \quad \left(V_{in}+\hat{v}_{in}\right)\left(D_0+\hat{d}\right)\approx \hat{d}V_{in}+\hat{v}_{in}D_0$$

Analytical analysis

Simplified Circuit Helps Analysis

- Final closed-loop schematic to perform analysis



- Sanity check is mandatory!

$Z_{RLC}(s)$ is the filter input impedance

Derive Equations for Individual Variables

- The duty ratio is linked to V_{out} by the error amplifier G

$$D(s) = G(s) G_{PWM} V_{out}(s)$$

- From the previous slide, we have

$$V_{out}(s) = \left[D_0 V_{in}(s) + D(s) V_{in} \right] \frac{H(s)}{V_{in}}$$

Control-to-output function of the buck

Divide by V_{in} to get the transmittance H_p

$$\Rightarrow \quad D(s) = \frac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)}$$

- Current in terminal c is B_4 applied to the RLC network

$$\Rightarrow \quad I_C(s) = \frac{V_{in}(s) D_0 + V_{in} \dfrac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)}}{Z_{RLC}(s)}$$

Substitute Expressions and Rearrange

- The input current depends on the two input sources:

$$I_{in}(s) = I_{out} D(s) + I_C(s) D_0$$

Buck dc output current

Static duty ratio

- Substitute $D(s)$ and $I_C(s)$ previously obtained

$$I_{in}(s) = I_{out} \frac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)} + D_0 \frac{D_0 + V_{in} \dfrac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)}}{Z_{RLC}(s)}$$

Factor

Rearrange

$$\frac{1}{Z_{in}(s)} = \frac{I_{in}(s)}{V_{in}(s)} = \frac{D_0 I_{out}}{V_{in}} \frac{\overbrace{\boxed{G_{PWM} G(s) H(s)}}^{T(s)}}{1 - \boxed{G_{PWM} G(s) H(s)}} + \frac{D_0^{\,2}}{Z_{RLC}(s)} \left(1 + \frac{\boxed{G_{PWM} G(s) H(s)}}{1 - \boxed{G_{PWM} G(s) H(s)}} \right)$$

$$V_{in} = \frac{V_{out}}{D_0}$$

Final Expression for Z_{in} in Closed Loop

- Final expression involves two contributors

$$\frac{1}{Z_{in}(s)} = \frac{D_0^{\ 2}}{R_{load}} \frac{T(s)}{1-T(s)} + \frac{D_0^{\ 2}}{Z_{RLC}(s)} \left(\frac{1}{1-T(s)} \right)$$

- In dc or low frequency, $T(s)$ is >> 1

$$\frac{1}{Z_{in}(s)} \approx \frac{D_0^{\ 2}}{R_{load}} \frac{T(s)}{1-T(s)} \approx -\frac{D_0^{\ 2}}{R_{load}} \longrightarrow \boxed{\underset{s \to 0}{Z_{in}(s)} \approx -\frac{R_{load}}{D_0^{\ 2}}}$$

- As s exceeds f_c and increases, $T(s)$ is << 1

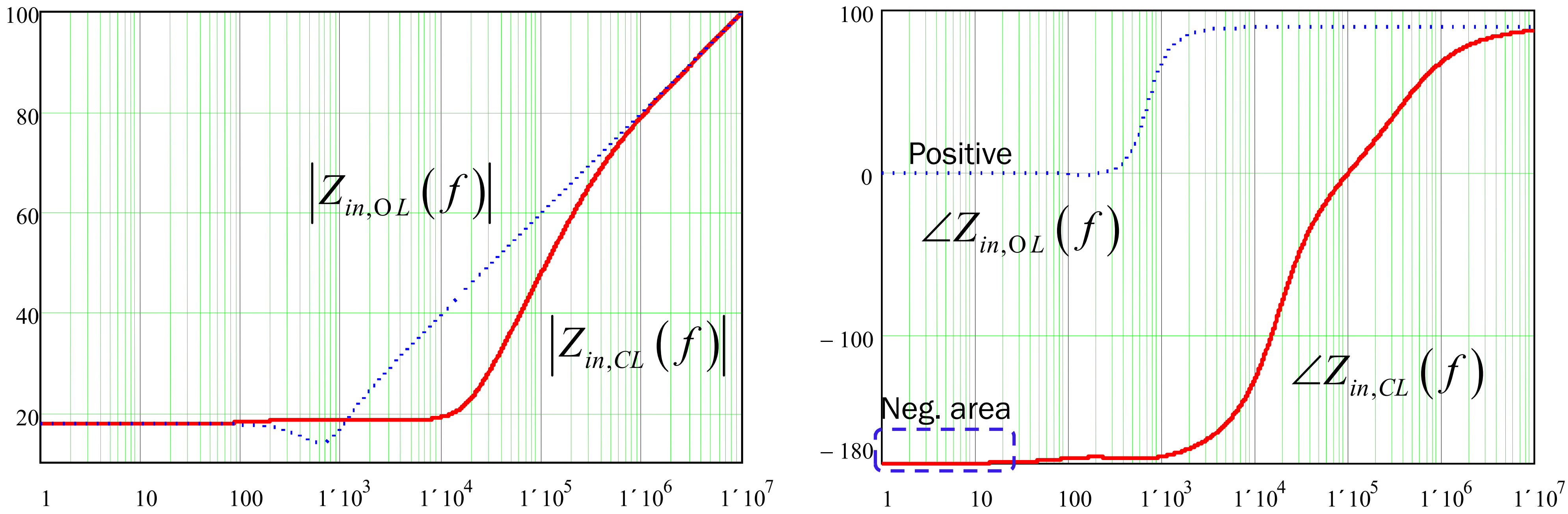
$$\frac{1}{Z_{in}(s)} \approx \frac{D_0^{\ 2}}{Z_{RLC}(s)} \left(\frac{1}{\underbrace{1-\cancel{T(s)}}_{\approx 1}} \right) \approx \frac{D_0^{\ 2}}{Z_{RLC}(s)} \longrightarrow \boxed{Z_{in}(s) \approx \frac{Z_{RLC}(s)}{D_0^{\ 2}}}$$

No gain means open-loop operation
Same as $D(s) = 0$

$$Z_{RLC}(s) = (r_L + R_{load}) \frac{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0} \right)^2}{1 + s(r_C + R_{load}) C_1}$$

Plot the Closed-Loop Input Impedance

❑ Magnitude becomes open-loop plot in high-frequency

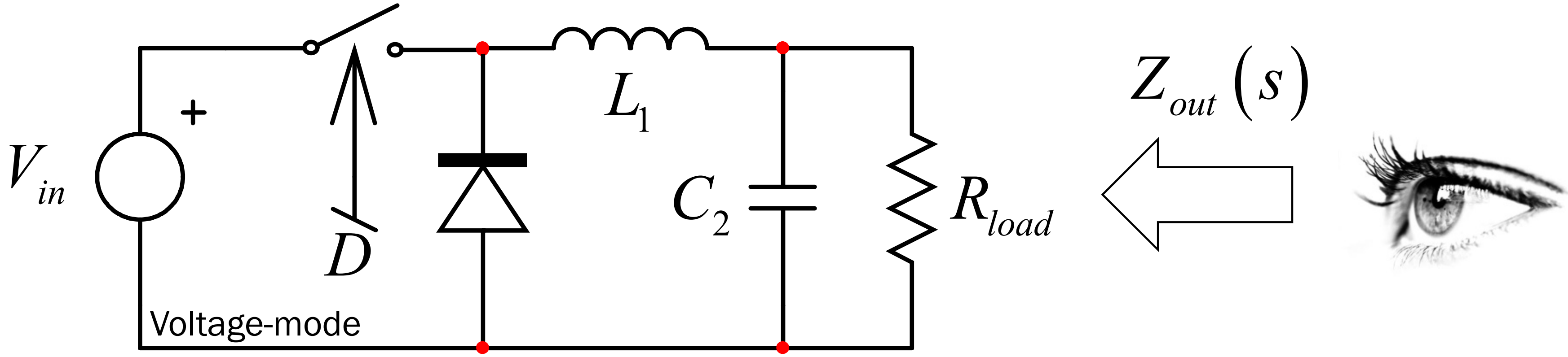


❑ Argument meets open-loop plot in high-frequency

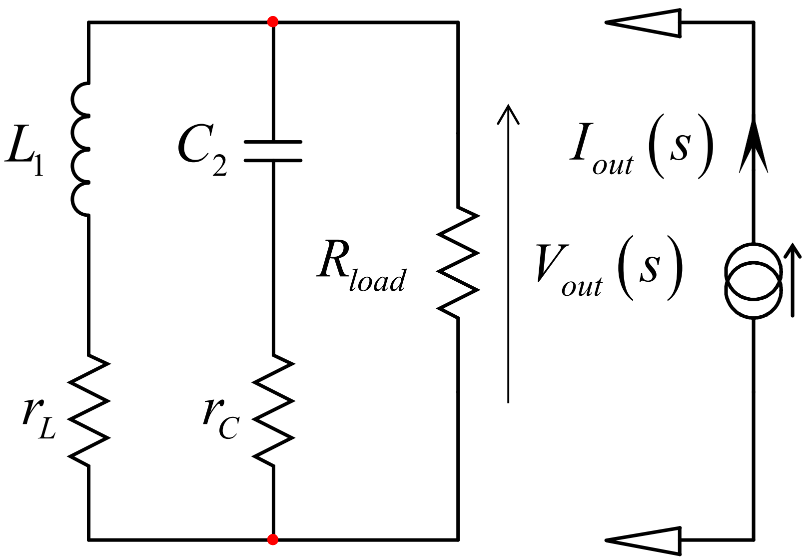
➢ Negative argument occurs only at low-frequency

Open-Loop Output Impedance

- What is the buck converter output impedance?



- Consider parasitic elements for L and C



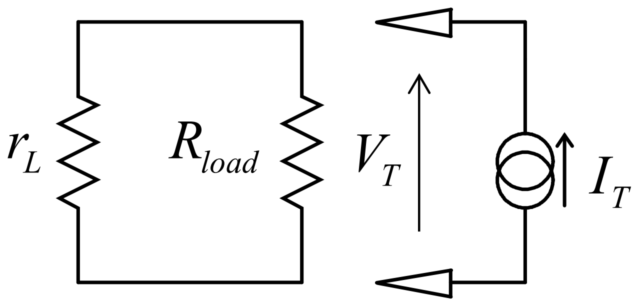
$$Z_{out}(s) = \frac{V_{out}(s)}{I_{out}(s)}$$

response

excitation

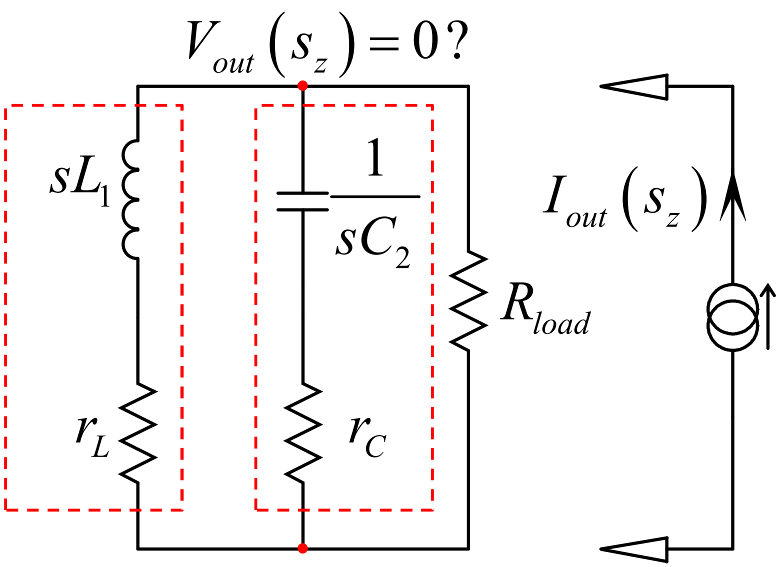
Buck Output Impedance

- Let's find the term R_0 in dc: open caps, short inductors



$$R_0 = \frac{V_T}{I_T} = r_L \,\|\, R_{load}$$

- The zeros cancel the response



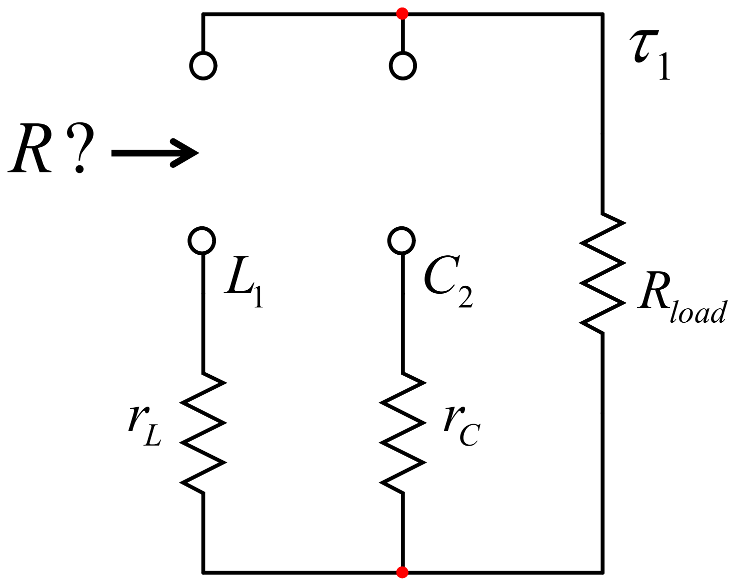
$$sL_1 + r_L = 0 \qquad r_L\left(s\frac{L_1}{r_L} + 1\right) = 0 \qquad \omega_{z_1} = \frac{r_L}{L_1}$$

$$\frac{1}{sC_2} + r_C = 0 \qquad 1 + sr_C C_2 = 0 \qquad \omega_{z_2} = \frac{1}{r_C C_2}$$

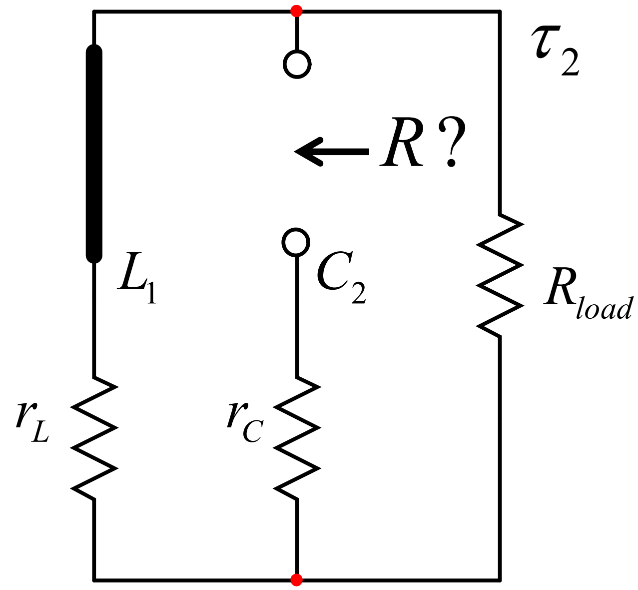
$$N(s) = \left(1 + s\frac{L_1}{r_L}\right)(1 + sr_C C_2)$$

Low-Frequency Time Constants

- All elements are in their dc state
  - Look at R driving L then R driving C



$$R = r_L + R_{load}$$

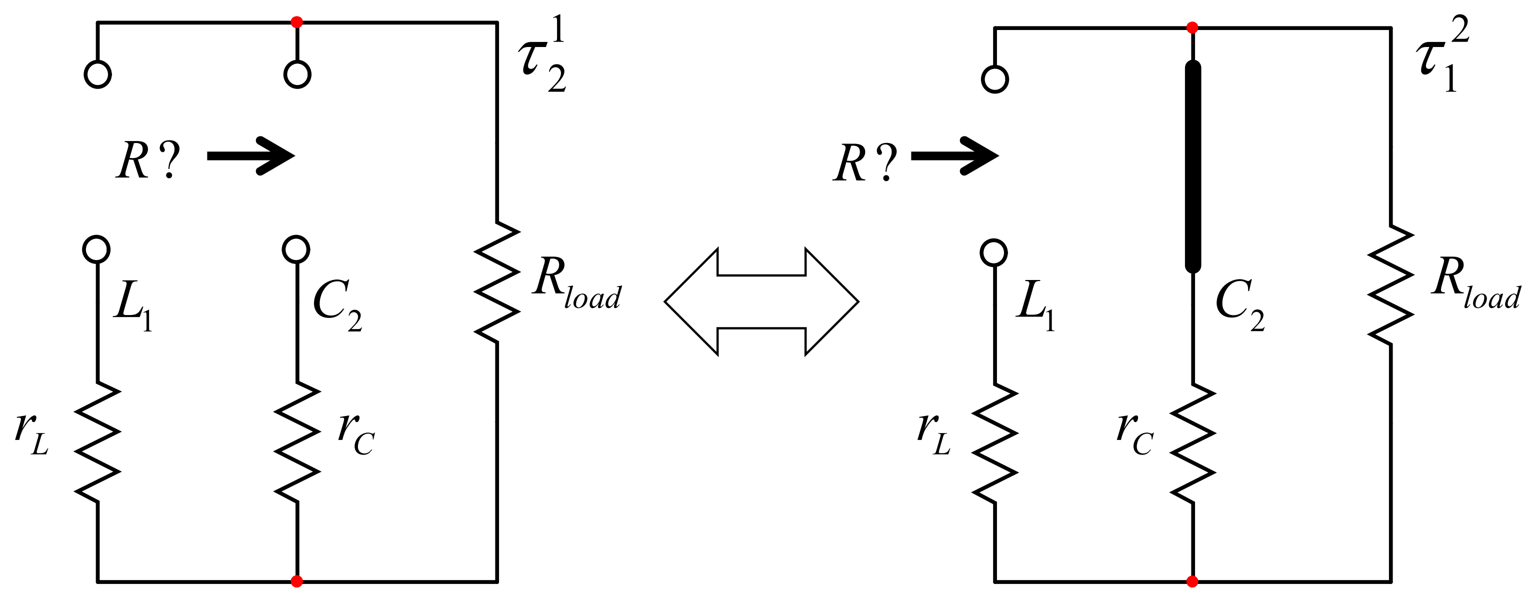


$$R = \left(r_L \parallel R_{load}\right) + r_C$$

$$b_1 = \frac{L_1}{r_L + R_{load}} + C_2\left[\left(r_L \parallel R_{load}\right) + r_C\right]$$

High-Frequency Time Constants

- Set L_1 in high frequency state and look at R driving C_2



$$\tau_2^1 = C_2\left(r_c + R_{load}\right) \qquad \tau_1^2 = \frac{L_1}{r_L + R_{load} \parallel r_C}$$

$$b_2 = \frac{L_1}{r_L + R_{load}} C_2\left(r_c + R_{load}\right) = C_2\left[\left(r_L \parallel R_{load}\right) + r_C\right]\frac{L_1}{r_L + R_{load} \parallel r_C}$$

$b_2 = \tau_1\tau_2^1 \quad \Leftarrow$ redundancy $\Rightarrow \quad b_2 = \tau_2\tau_1^2$

Final Expression for Z_{OUT}

- We have our denominator!

$$D(s) = 1 + s\left(\frac{L_1}{r_L + R_{load}} + C_2\left[\left(r_L \parallel R_{load}\right) + r_C\right]\right) + s^2\left(L_1 C_2 \frac{r_C + R_{load}}{r_L + R_{load}}\right)$$

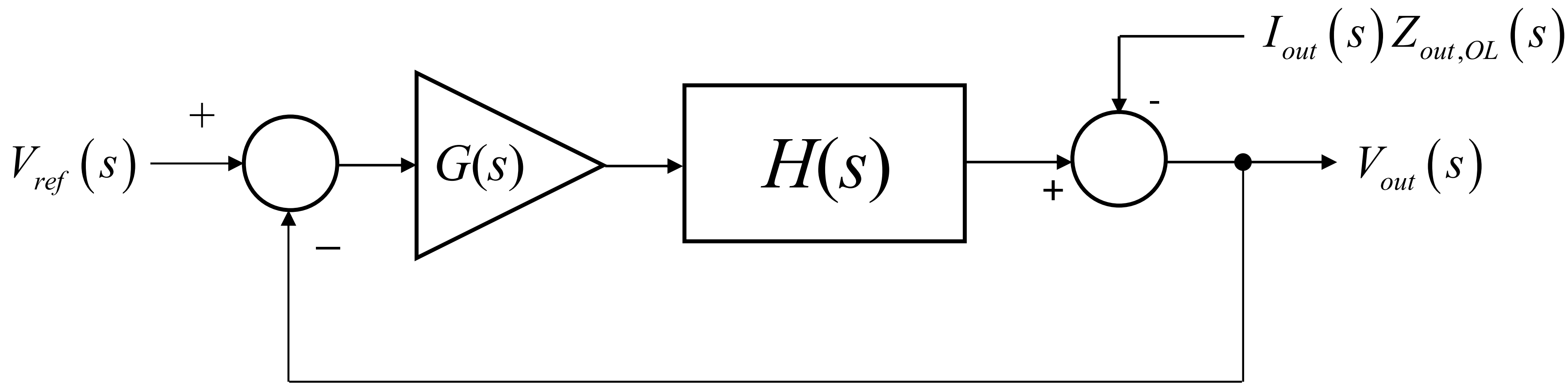
- The complete transfer function is now:

$$\underset{\substack{\uparrow \\ \text{Open loop}}}{Z_{out,OL}}(s) = \left(r_L \parallel R_{load}\right)\frac{\left(1 + s\frac{L_1}{r_L}\right)\left(1 + s r_C C_2\right)}{1 + s\left(\frac{L_1}{r_L + R_{load}} + C_2\left[\left(r_L \parallel R_{load}\right) + r_C\right]\right) + s^2\left(L_1 C_2 \frac{r_C + R_{load}}{r_L + R_{load}}\right)}$$

- This is the open-loop output impedance

A Closed-Loop System

- The converter fights output perturbations



- Because the system is linear, superposition applies

$$V_{out1}(s) = V_{ref}(s)\frac{T_{OL}(s)}{1+T_{OL}(s)} \qquad V_{out2}(s) = I_{out}(s)Z_{out,OL}(s) - V_{out}(s)T_{OL}(s)$$

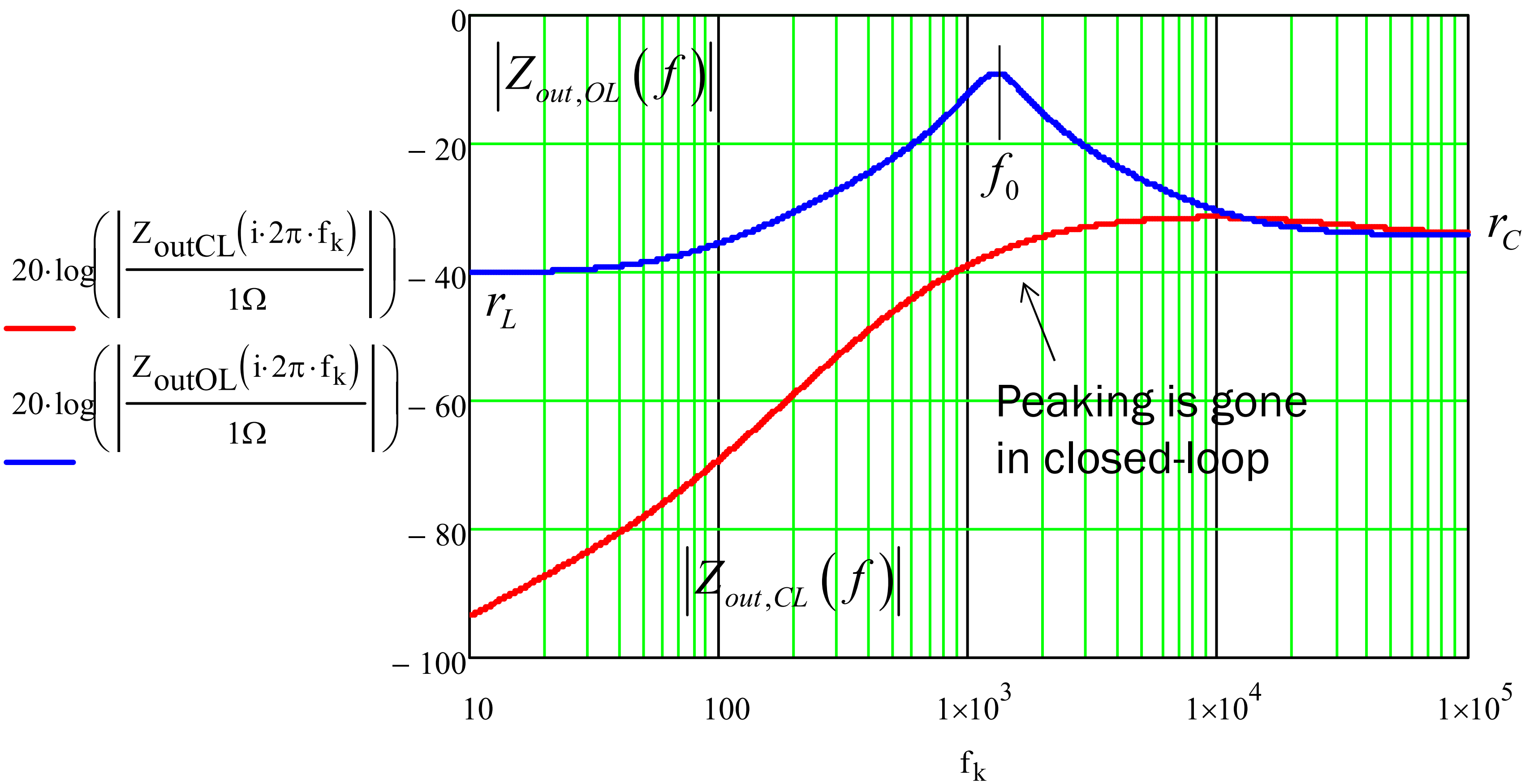
$$V_{out}(s) = V_{out1}(s) + V_{out2}(s) = V_{ref}(s)\frac{T_{OL}(s)}{1+T_{OL}(s)} - I_{out}(s)\underbrace{\boxed{\frac{Z_{out,OL}(s)}{1+T_{OL}(s)}}}_{Z_{out,CL}}$$

- The loop gain affects the final expression

Open loop

Loop Gain Impact on the Impedance

- Plots superimpose as loop gain approaches 0



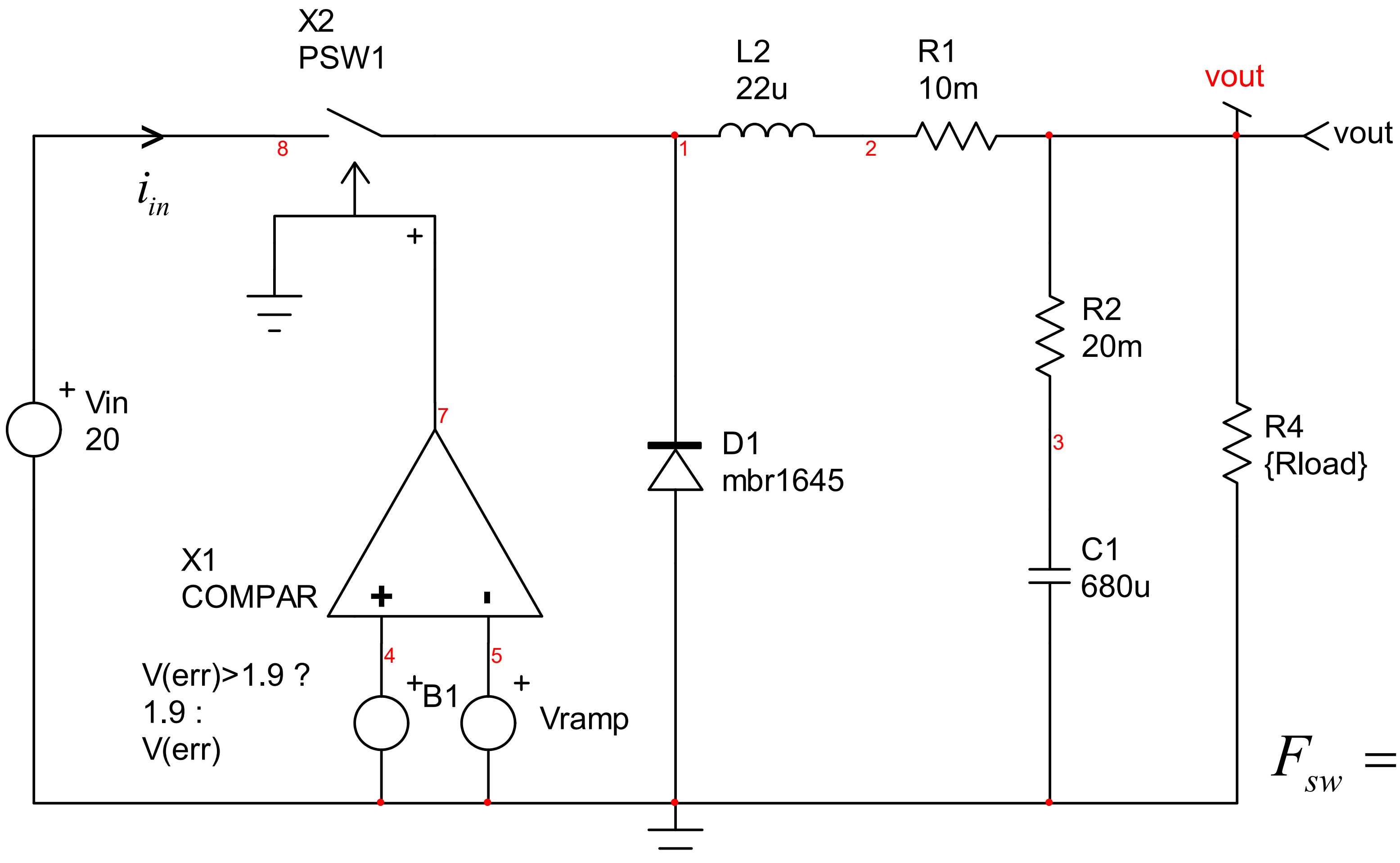
- Having gain at f_0 is important to damp the filter

Course Agenda

- ❑ A Switching Regulator as a Load
- ❑ EMI Filter Impact
- ❑ An Introduction to FACTs
- ❑ Buck Converter Input/Output Impedances
- ❑ **Filtering the Input Current**
- ❑ Damping the Filter
- ❑ Optimum Component Selection
- ❑ A Practical Case Study
- ❑ Cascading Converters

A Design Example with a Buck

- Assume the following 5-V/50-W buck converter

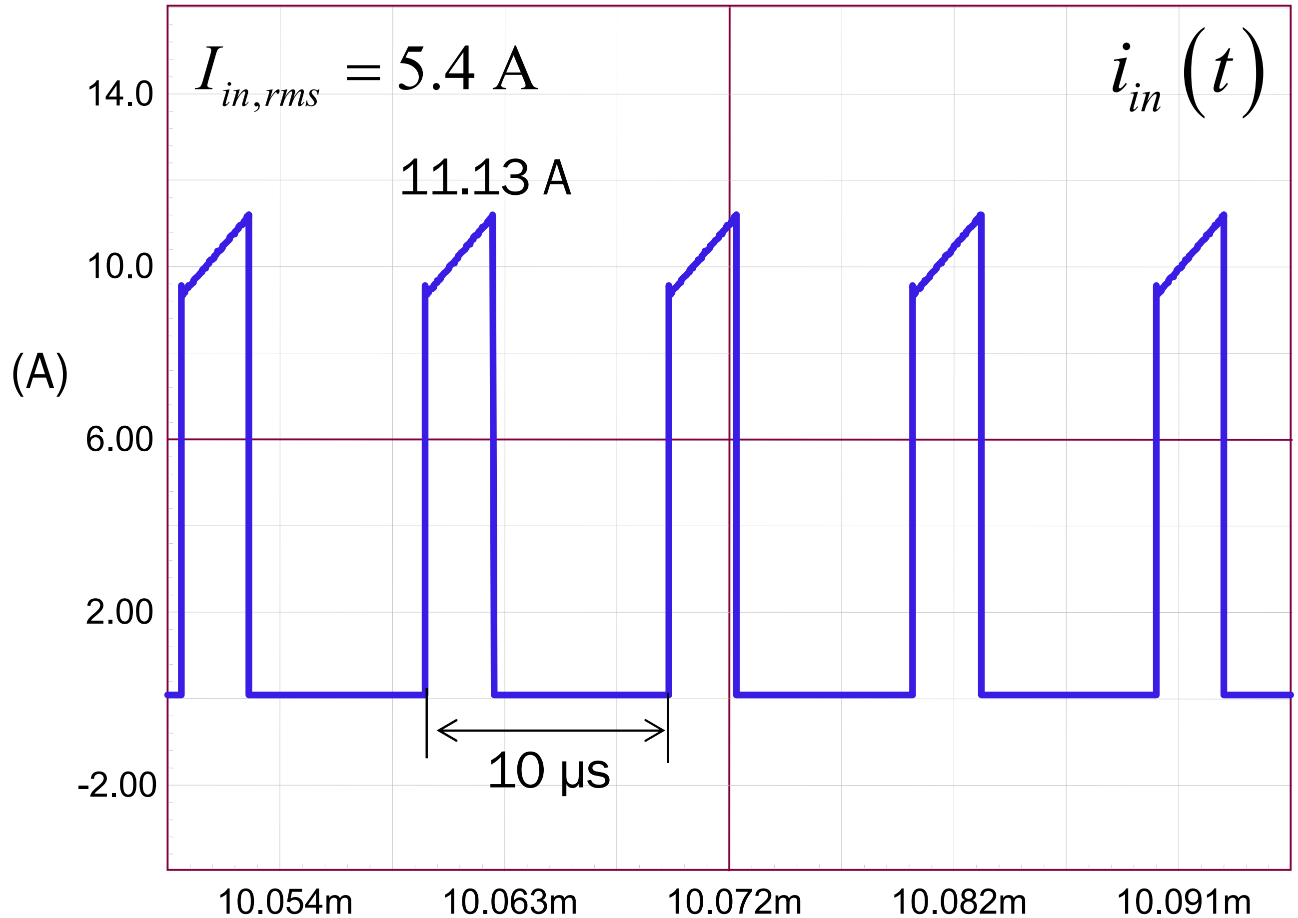


$$F_{sw} = 100 \text{ kHz}$$

- Specifications are less than 15 mA peak of input ripple

What is the Necessary Attenuation?

- Use SPICE to analyze the input current signature



.FOUR 100kHz I(V4)

$$I_1 = 4.94 \text{ A peak}$$

$$I_{in} < 15 \text{ mA peak}$$

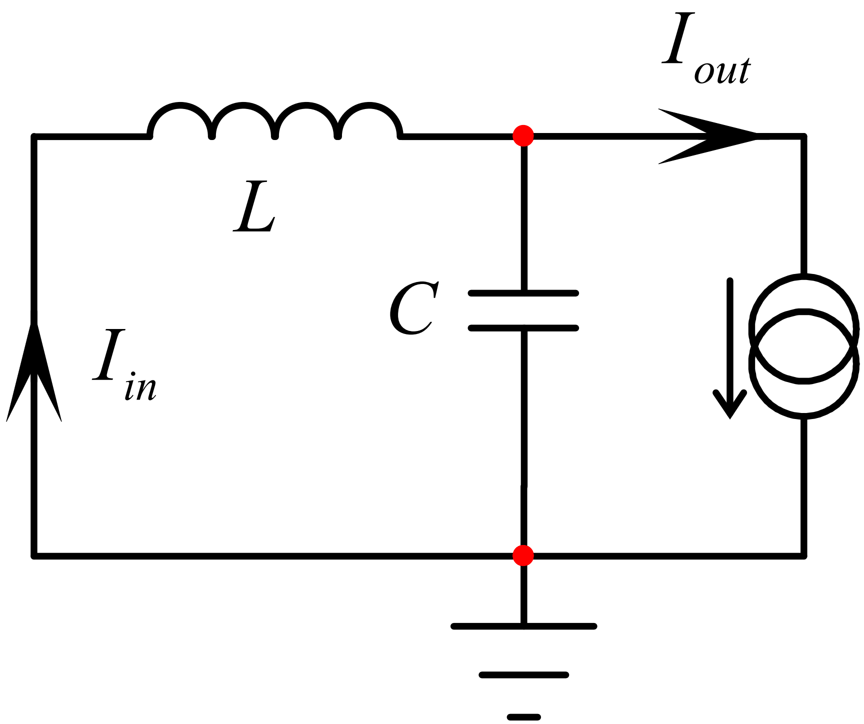
Attenuate by 3m
or 50 dB

$$I_{ac} = \sqrt{I_{in,rms}{}^2 - I_{dc}{}^2} = \sqrt{5.4^2 - 2.85^2} \approx 4.6 \text{ A}$$

Current in the capacitor

Where do you Place the Double Pole?

- Insert a LC filter to attenuate the pulsating current



At high frequency

$$\left\| \frac{I_{in}}{I_{out}} \right\| \approx \left(\frac{\omega_0}{\omega} \right)^2$$

- Position f_0 to provide a 50-dB attenuation at 100 kHz

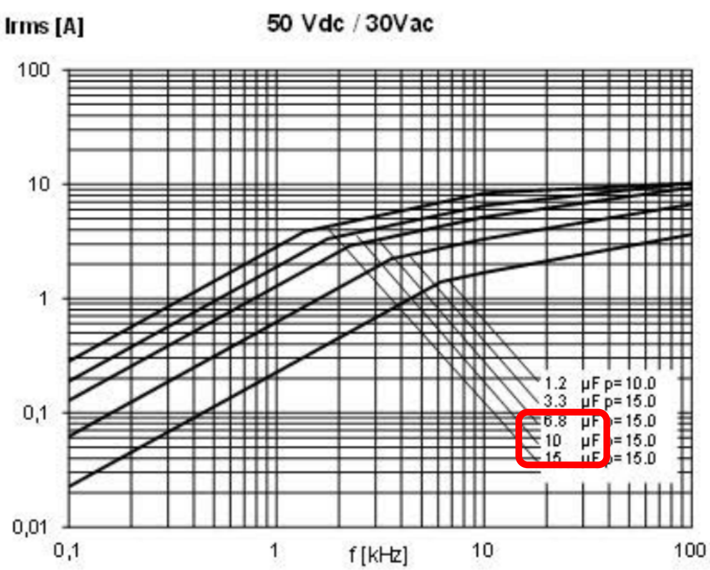
$$f_0 = \sqrt{A_{filter}} \cdot f_{SW} = \sqrt{3m} \times 100k \approx 17 \text{ kHz}$$

Select the Filter Elements

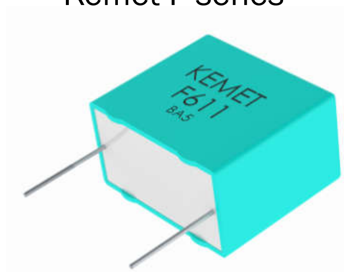
- ❑ The resonant frequency lets us chose L and C

$$LC = \frac{1}{4\pi^2 f_0^{\,2}} = 8.34\times 10^{-10}\,\mathrm{s}^2$$

- ❑ Component selection depends on volume, cost etc.
  - ✓ the capacitor sees the buck ac input current
  - ✓ the inductor ripple current is small, consider dc only



Kemet F series



4 x 10 µF

$I_{tot,rms}$ > 40 A

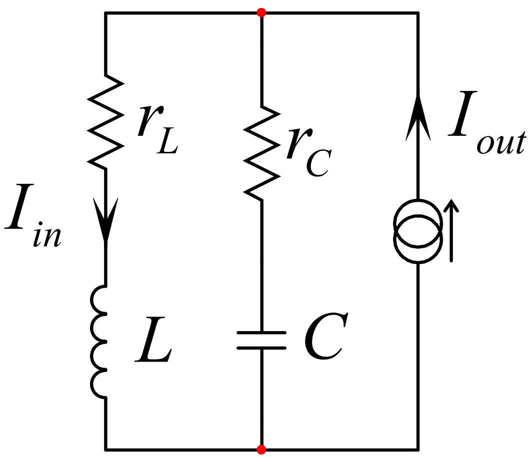
$$L = 22\ \mu\mathrm{H}$$

$$I_{in} < 3\ \mathrm{A}$$

7447714220 Würth

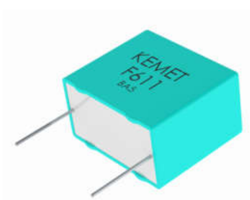
Always Consider Component ESR

- We can show that the complete attenuation is



$$\left|\frac{I_{in}}{I_{out}}\right| = \sqrt{\frac{r_C{}^2 + \dfrac{1}{(C_1\omega)^2}}{(r_C + r_L)^2 + \dfrac{1}{(\omega C_1)^2} - \dfrac{2L_1}{C_1} + (L_1\omega)^2}}$$

- Check if attenuation is ok with selected components



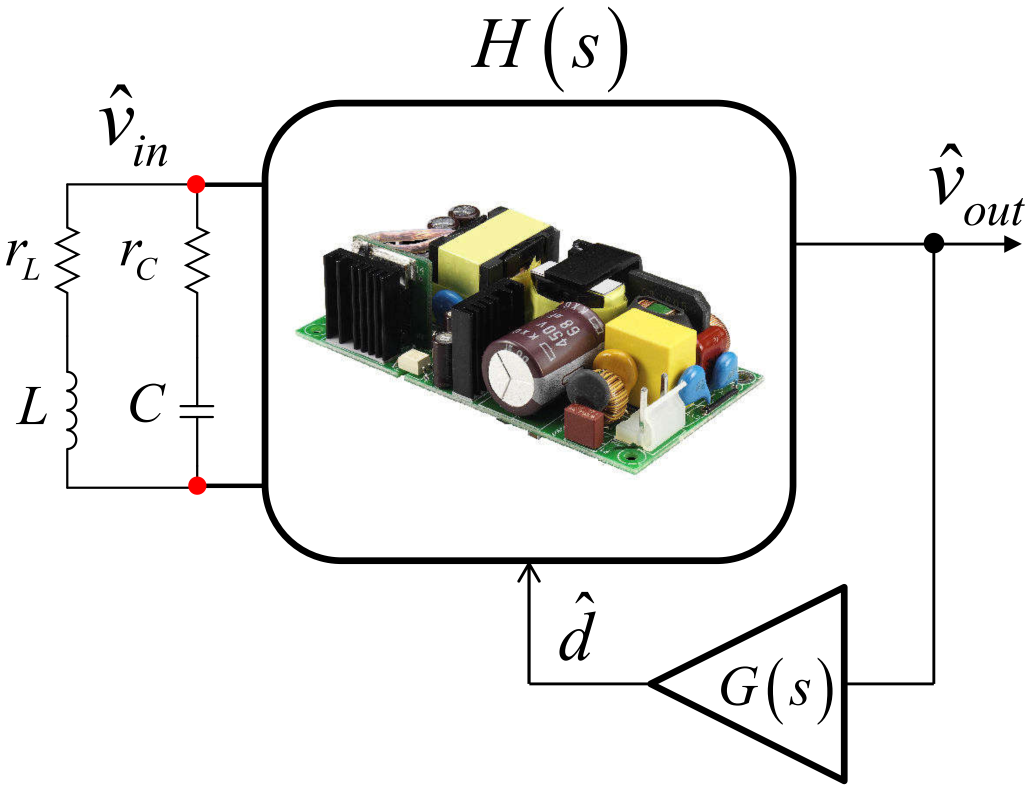
X 4 $r_C = \dfrac{5\ \text{m}\Omega}{4} \approx 1.3\ \text{m}\Omega$

$r_L = 50\ \text{m}\Omega$

$$\left|\frac{I_{in}}{I_{out}}\right| = -50.8\ \text{dB at } 100\ \text{kHz}$$

Control-to-Output Transfer Function

- The addition of the filter affects the converters



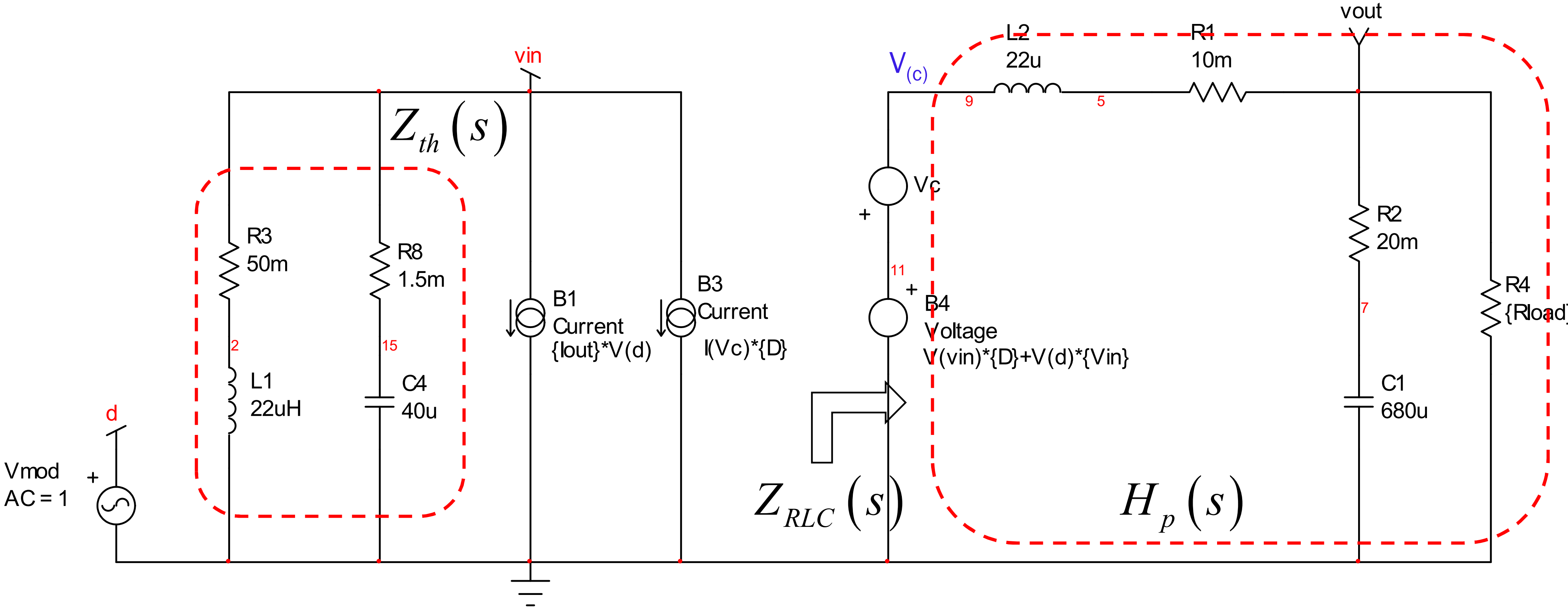
$$H(s) \neq V_{in}\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}$$

Buck control-to-output TF without filter.

- The small-signal model needs an update

A More Complex Architecture

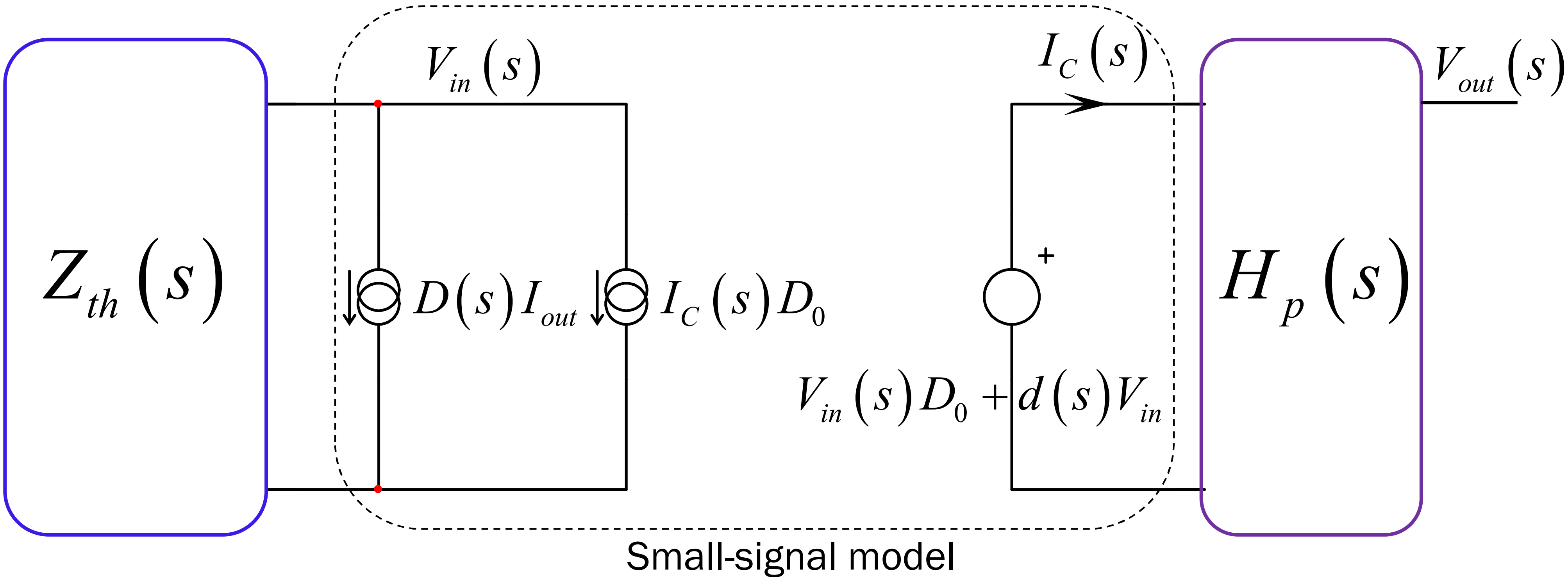
- ❑ The converter becomes a 4th-order system



- ❑ Simplify the circuit before solving the function

A Simplified Circuit

- You can reuse previously-determined transfer functions



- Apply KVL and KCL to obtain the new expression

Determine and Substitute Variables

- Determine the input voltage expression $V_{in}(s)$

$$V_{in}(s) = -Z_{th}(s)\left[I_{out}D(s) + I_C(s)D_0\right]$$

- Terminal c current is the voltage $V_{(c)}$ divided by $Z_{RLC}(s)$

$$I_C(s) = \frac{V_{in}(s)D_0 + D(s)V_{in}}{Z_{RLC}(s)} \qquad V_{in}(s) = -Z_{th}(s)\left[I_{out}D(s) + \frac{V_{in}(s)D_0 + D(s)V_{in}}{Z_{RLC}(s)}D_0\right]$$

$$\Rightarrow \boxed{V_{in}(s)} = \frac{D(s)Z_{RLC}(s)Z_{th}(s)I_{out} + D_0V_{in}D(s)Z_{th}(s)}{Z_{th}(s)D_0^{\,2} + Z_{RLC}(s)}$$

- The output voltage involves the RLC transmittance H_p

$$V_{out}(s) = \left[V_{in}(s)D_0 + D(s)V_{in}\right]H_p(s)$$

Final Expression is Complicated

- Rearrange the final transfer function

$$\frac{V_{out}(s)}{D(s)} = \frac{R_{load}V_{in}}{R_{load}+r_L}\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}\;\frac{1-\frac{D_0^{\,2}}{R_{load}}Z_{th}(s)}{1+\frac{D_0^{\,2}}{R_{load}+r_L}Z_{th}(s)\frac{1+sC_1\left(r_L+R_{load}\right)}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}}$$

Classical buck, no filter

Filter effect

$$Z_{th}(s) = r_{Lf}\frac{\left(1+sr_{Cf}C_f\right)\left(1+s\frac{L_f}{r_{Lf}}\right)}{1+sC_f\left(r_{Cf}+r_{Lf}\right)+s^2L_fC_f}$$

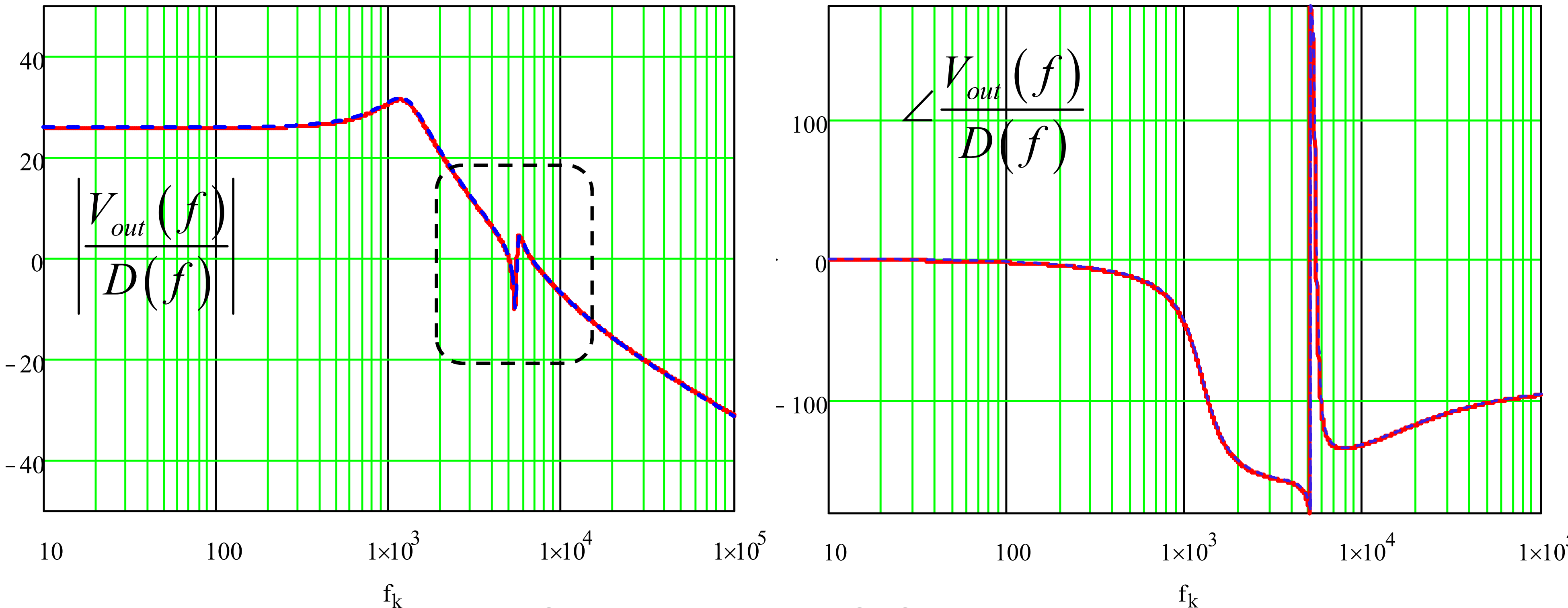
$$Q_f = \sqrt{\frac{L_f}{C_f}}\frac{1}{r_{Cf}+r_{Lf}} \qquad \omega_{0f} = \frac{1}{\sqrt{L_fC_f}}$$

$$\omega_{z_1} = \frac{1}{r_{C_f}C_f} \qquad \omega_{z_2} = \frac{r_{Lf}}{L_f}$$

- Filter output impedance impacts the transfer function

Plots Show a Distorted Transfer Function

- Mathcad and SPICE match each other quite well

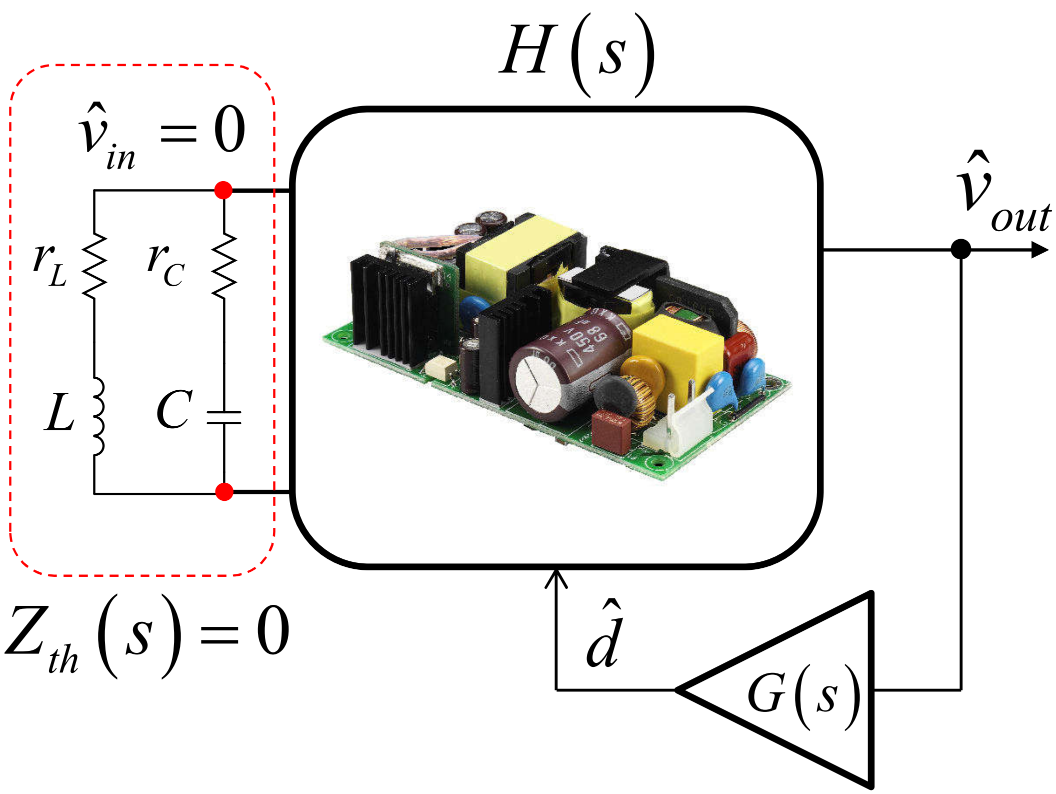


Control-to-output transfer function

- The notch occurs because V_{in} drops at resonance

The Extra Element Theorem at Work

- What is the EET principle?
  - Identify an element whose presence complicates the analysis
  - Calculate the transfer function with that element removed
  - Apply a correction factor k to the transfer function: voilà!



$$H(s)\big|_{Z_{th}=0} = V_{in}\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}$$

$$\Downarrow$$

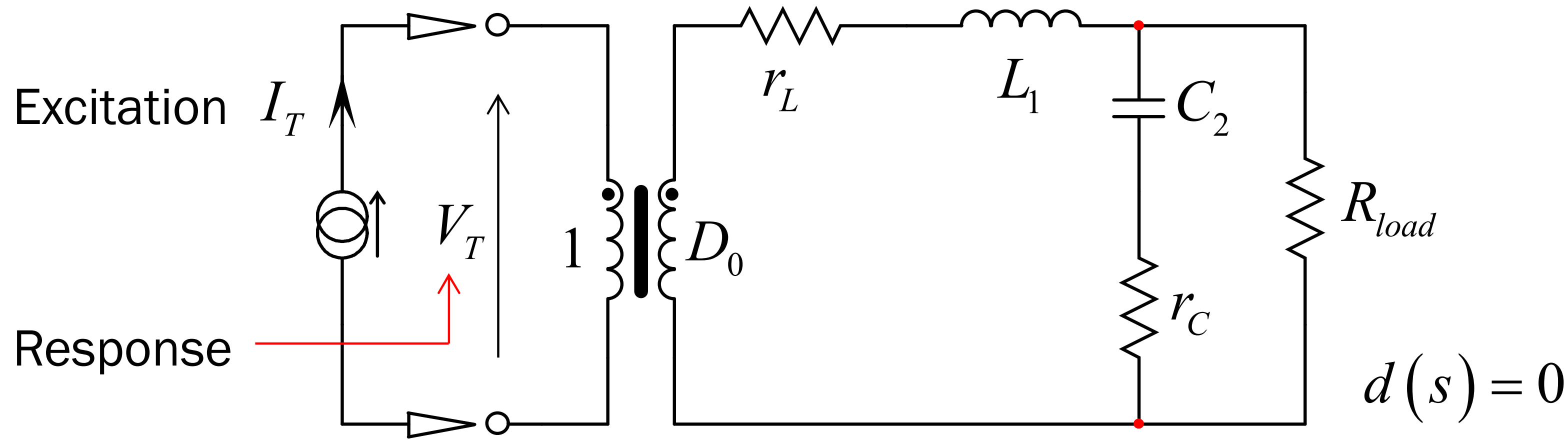
$$H(s) = H(s)\big|_{Z_{th}=0} \cdot k$$

Correction factor

Determining the Correction Factor

- The correction factor requires two terms:

1. The converter input impedance obtained in open-loop
2. The converter input impedance obtained for $V_{out}(s)=0$

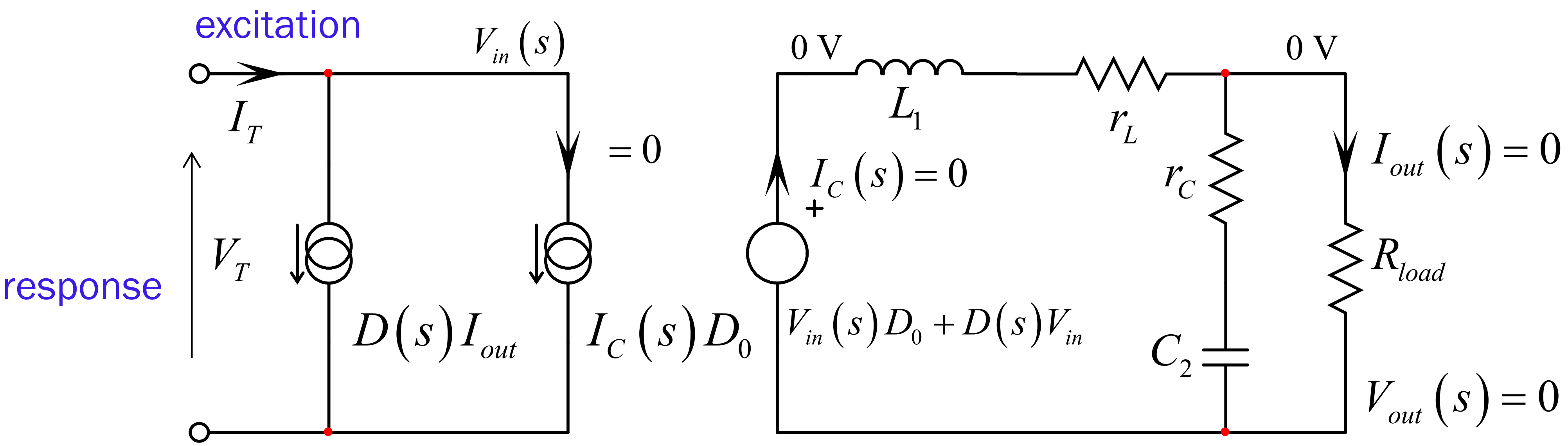


- The open-loop impedance has already been derived

$$Z_D(s)=\frac{R_{load}+r_L}{D_0^{\ 2}}\frac{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}{1+\frac{s}{\omega_p}}$$

The Null Propagates in the Circuit

- Despite excitation with I_T, there is no ac response
  - This is the principle behind nulling the response



- 0 A in the load and 0 V at r_L-r_C implies 0 V across L_1

$$\Rightarrow \quad V_{in}(s)D_0 + D(s)V_{in} = 0 \longrightarrow D(s) = -D_0\frac{V_{in}(s)}{V_{in}}$$

Expression for Z_N Comes Easily

- Substitute/rearrange test and voltage expressions

$$I_T(s) = I_{out} D(s) + I_C(s) D_0$$

$$D(s) = -D_0 \frac{V_{in}(s)}{V_{in}}$$

$$\frac{I_T(s)}{V_T(s)} = -\frac{I_{out} D_0}{V_{in}} \qquad I_{out} \rightarrow \frac{V_{out}}{R_{load}} \qquad V_{in} \rightarrow \frac{V_{out}}{D_0}$$

- The input impedance Z_N for a nulled response is:

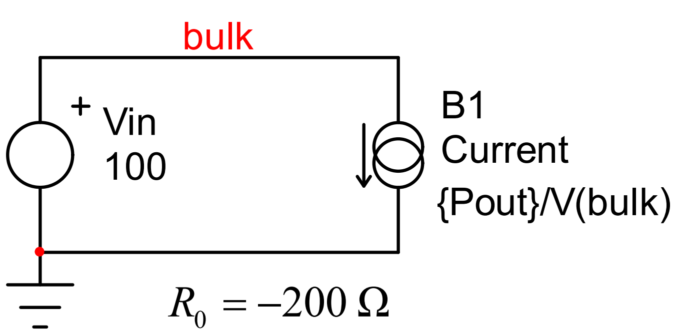
$$Z_N(s) = -\frac{R_{load}}{{D_0}^2} \rightarrow$$

Nulled response implies <u>infinite</u> input rejection
It is the incremental input resistance seen before

- We already had this with the SPICE simulation

parameters

Pout=50W



For a buck

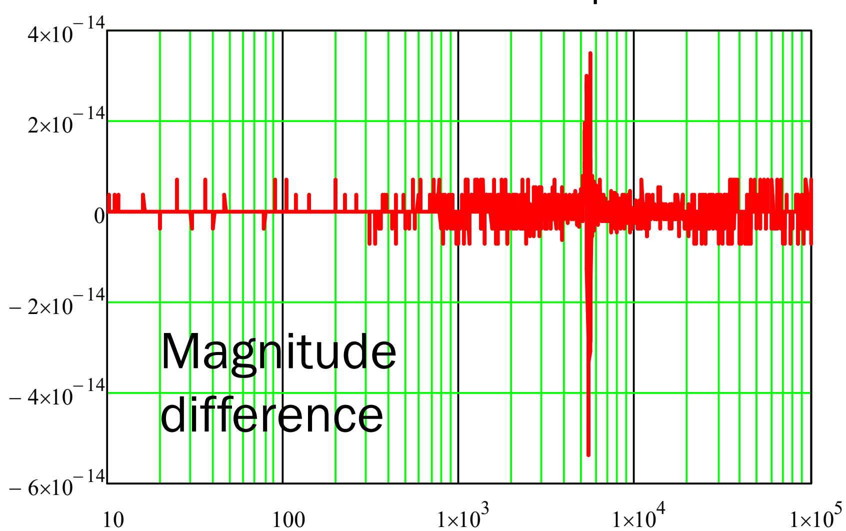
$$R_0 = -\frac{{V_{in}}^2}{P_{out}} = -\frac{\left(\frac{V_{out}}{D_0}\right)^2}{\frac{{V_{out}}^2}{R_{load}}} = -\frac{R_{load}}{{D_0}^2}$$

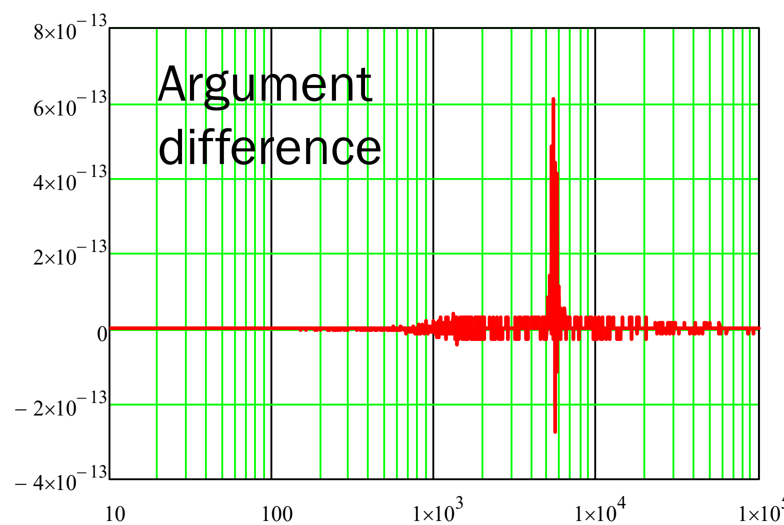
Final Expression with EET

- The final expression includes the filter impact

$$\frac{V_{out}(s)}{D(s)} = V_{in}\frac{R_{load}}{R_{load}+r_L}\frac{1+\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}\underbrace{\frac{1-\dfrac{\dfrac{Z_{th}(s)}{R_{load}}}{D_0^{\,2}}}{1+\dfrac{Z_{th}(s)}{\dfrac{R_{load}+r_L}{D_0^{\,2}}\dfrac{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}{1+\dfrac{s}{\omega_p}}}}}_{k}$$

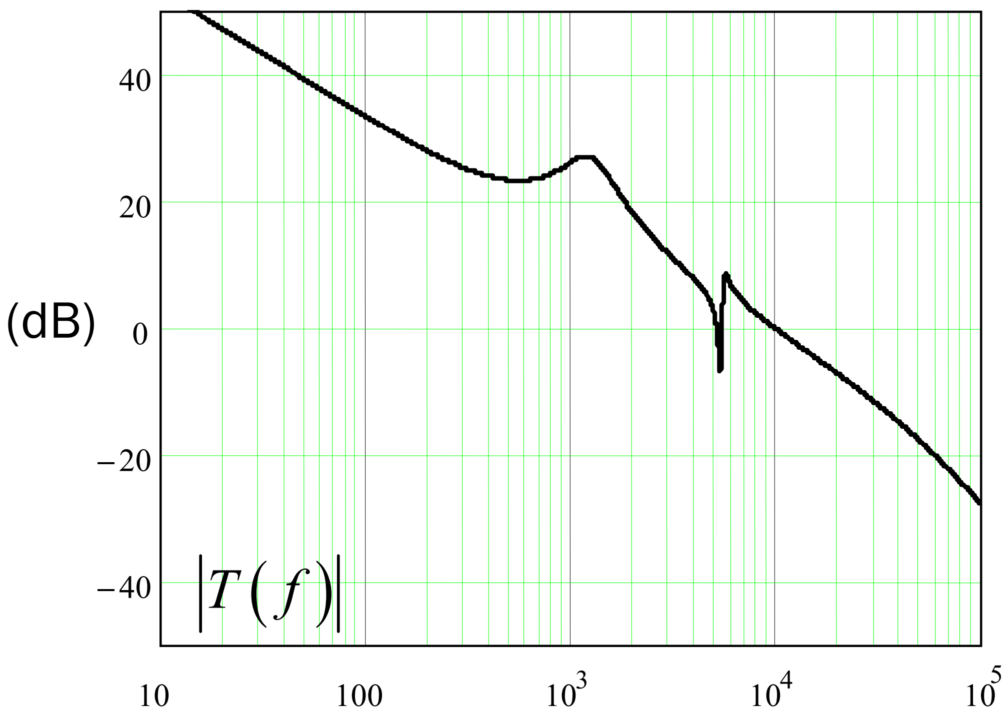
Comparison between EET and full formula

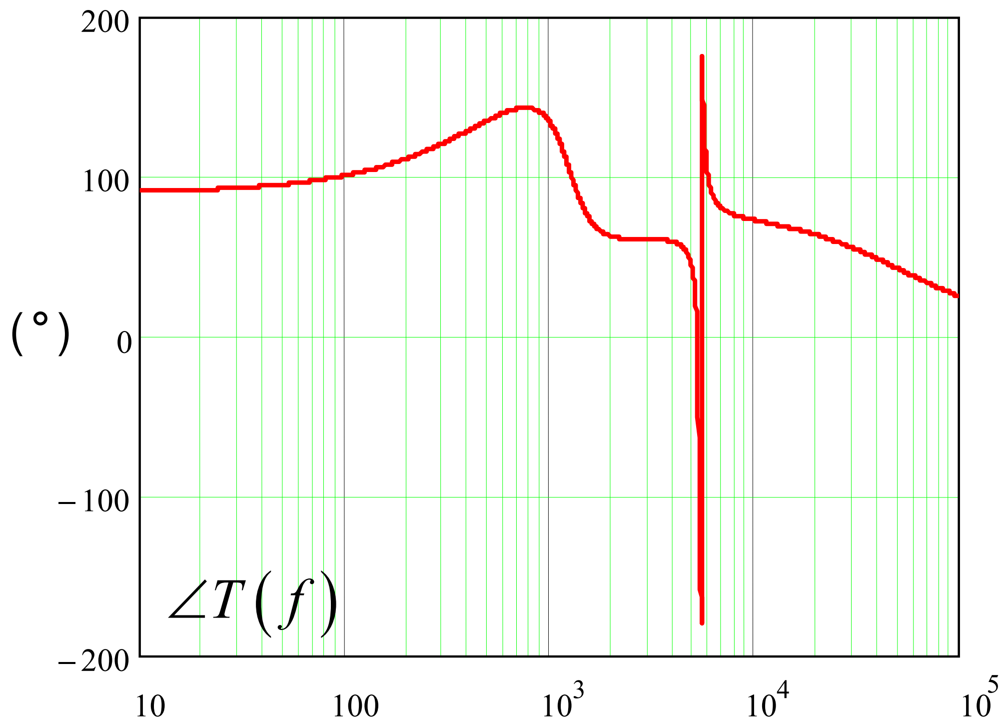




What is the Impact on Stability?

- When adding the filter, the loop gain is awfully ugly!





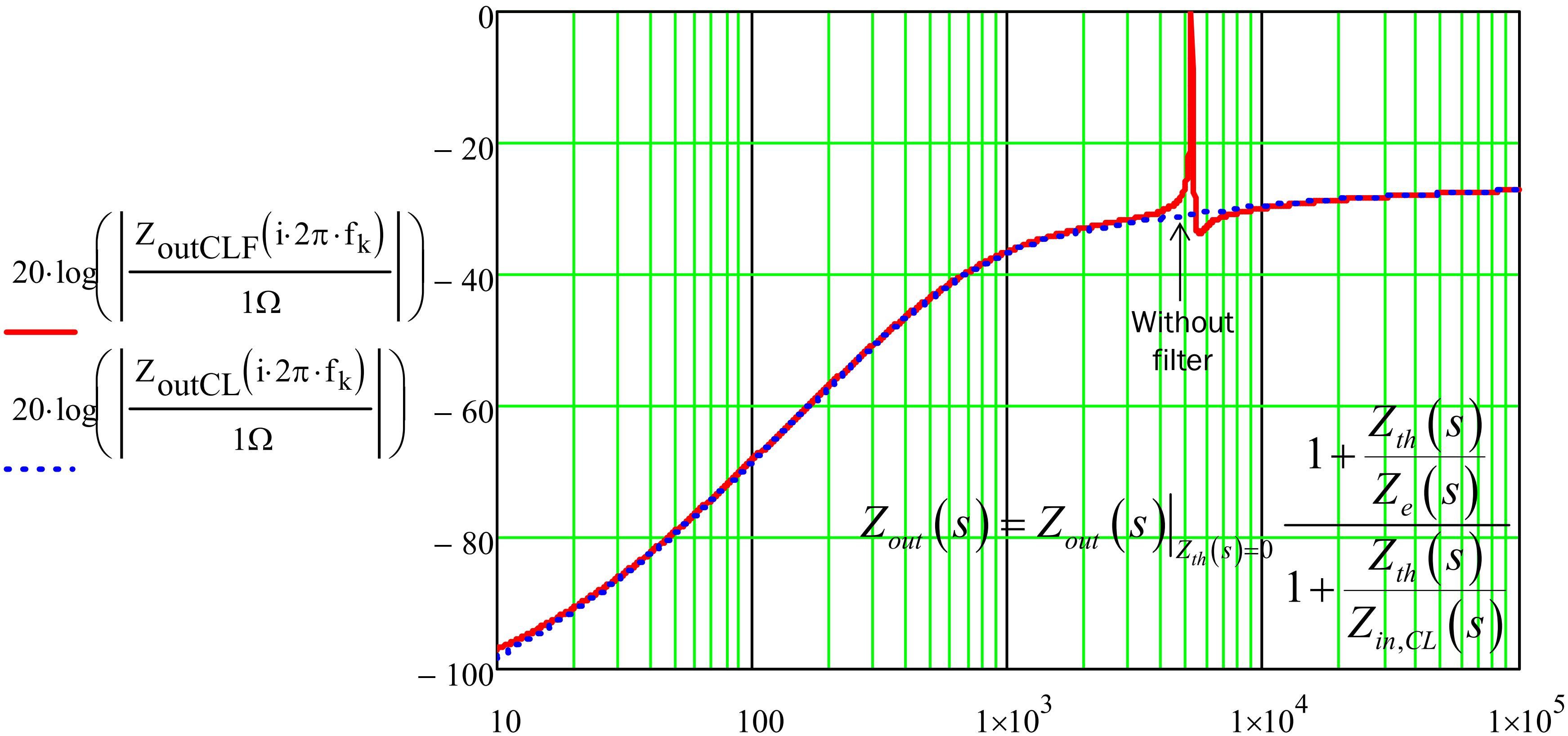
Filter damping is necessary:

$$|Z_{th}(s)| << |Z_N(s)|$$

$$|Z_{th}(s)| << |Z_D(s)|$$

The Output Impedance is Affected

- ❑ Closed-loop output impedance peaks as filter resonates



$$Z_{out}(s) = Z_{out}(s)\Big|_{Z_{th}(s)=0} \frac{1+\frac{Z_{th}(s)}{Z_e(s)}}{1+\frac{Z_{th}(s)}{Z_{in,CL}(s)}}$$

Filter damping is necessary: $|Z_{th}(s)| << |Z_e(s)|$

Course Agenda

- ❑ A Switching Regulator as a Load
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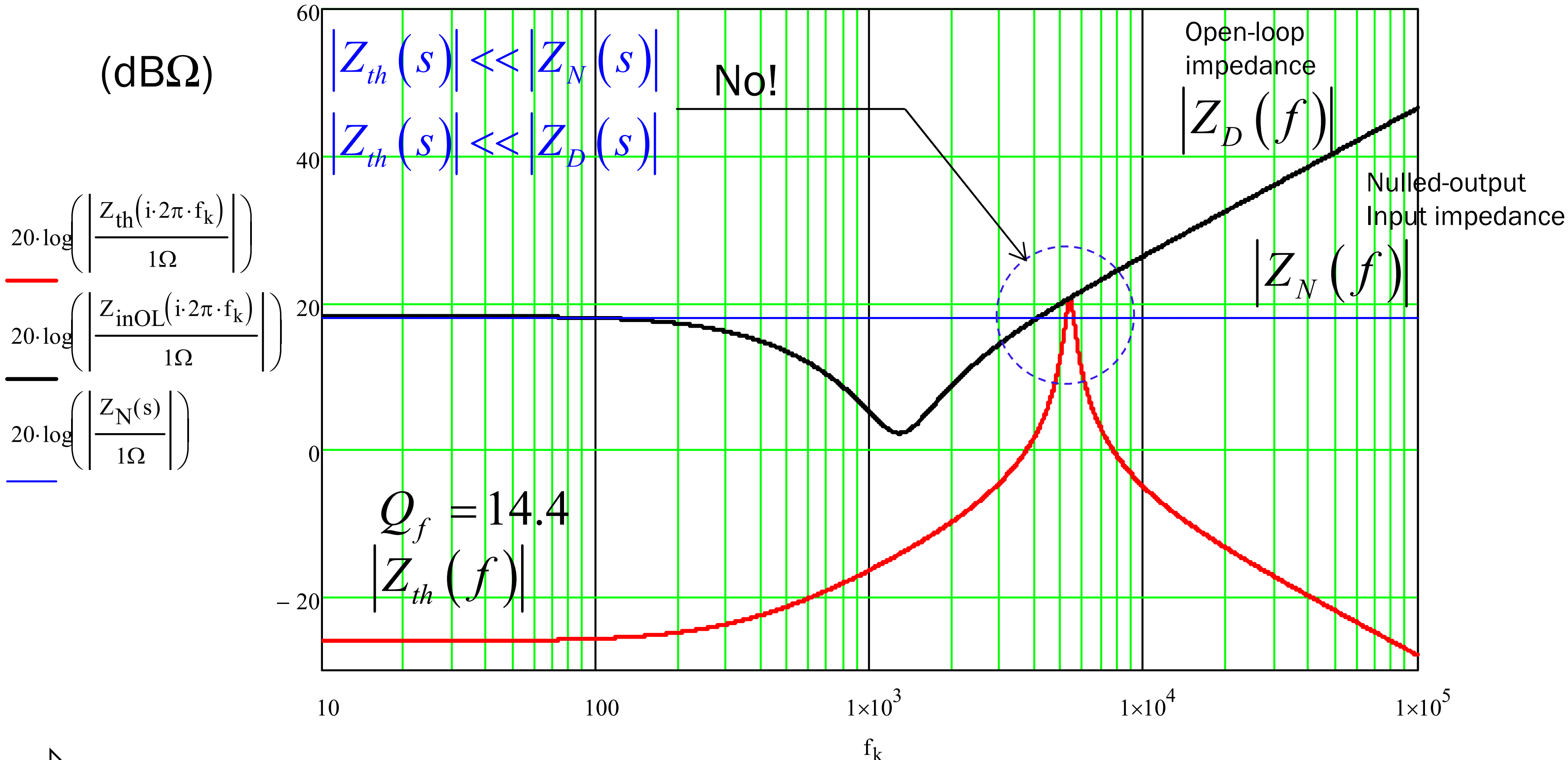
Check Filter Output Impedance

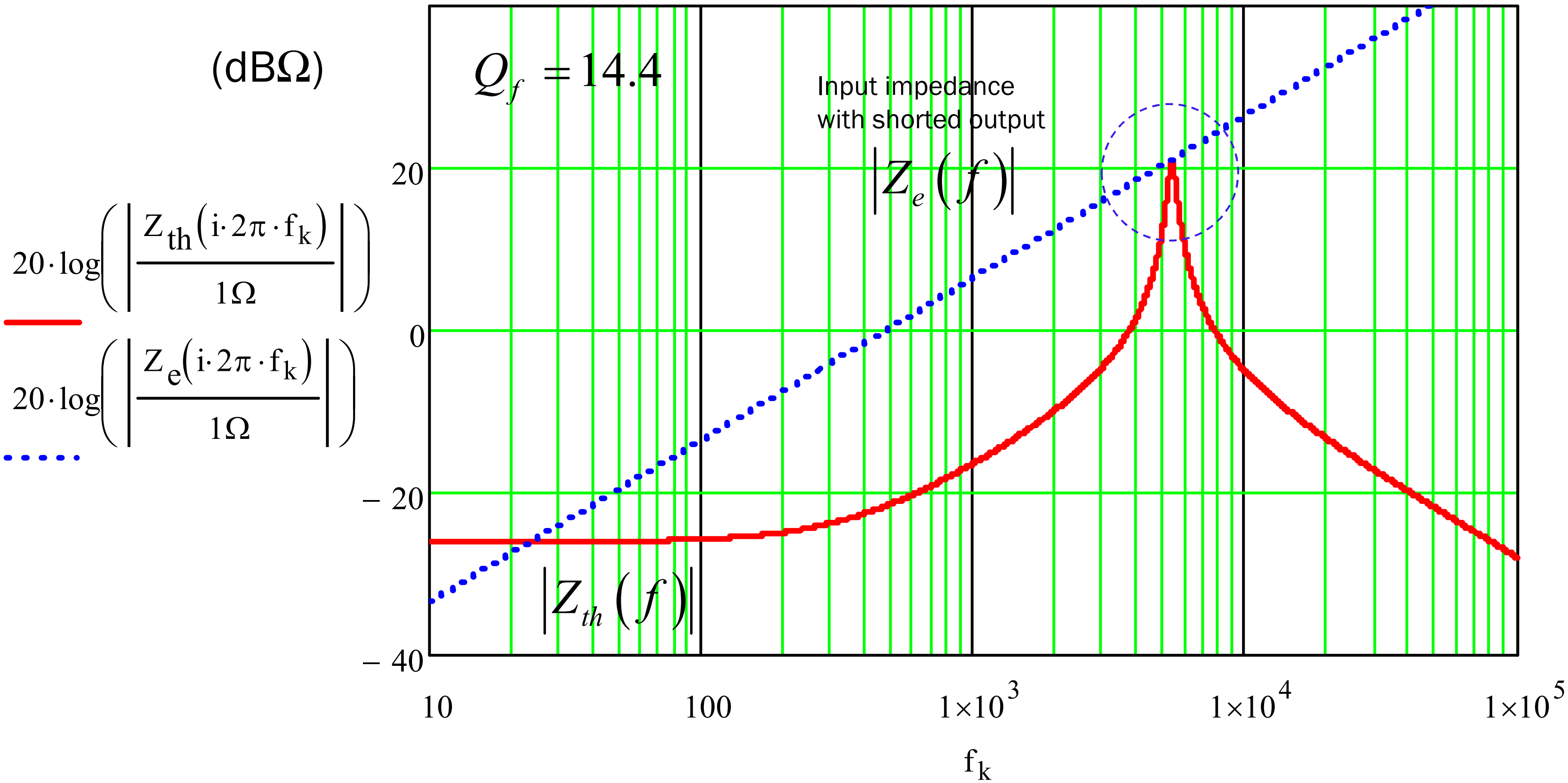
- Plot shows that design inequalities are not respected



Gain and phase distortion of k must be minimized

Filter Peaking Also Affects Z_{out}

- Inequality for the closed-loop Z_{out} is not respected



- Filter damping must ensure $|Z_{th}(s)| << |Z_e(s)|$

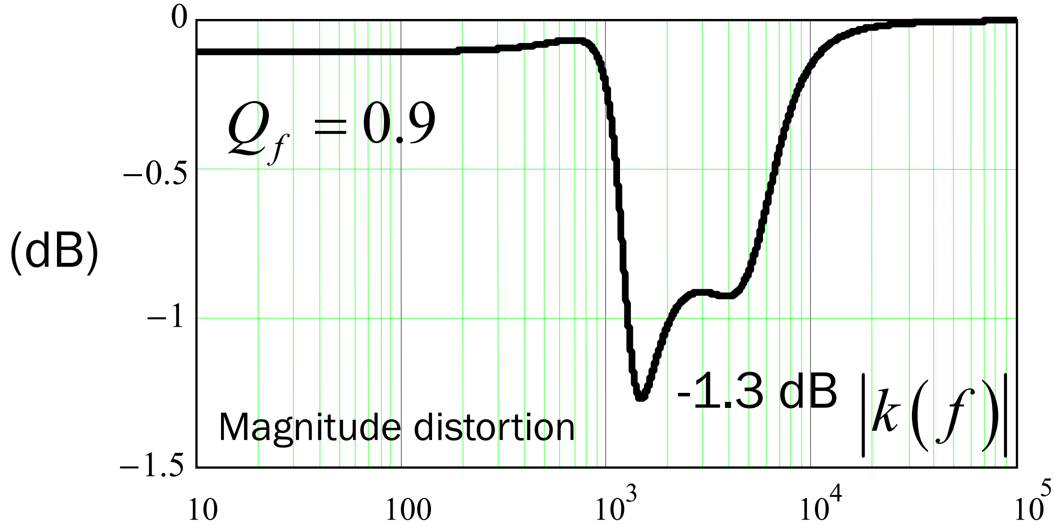
Look at the Correction Factor k

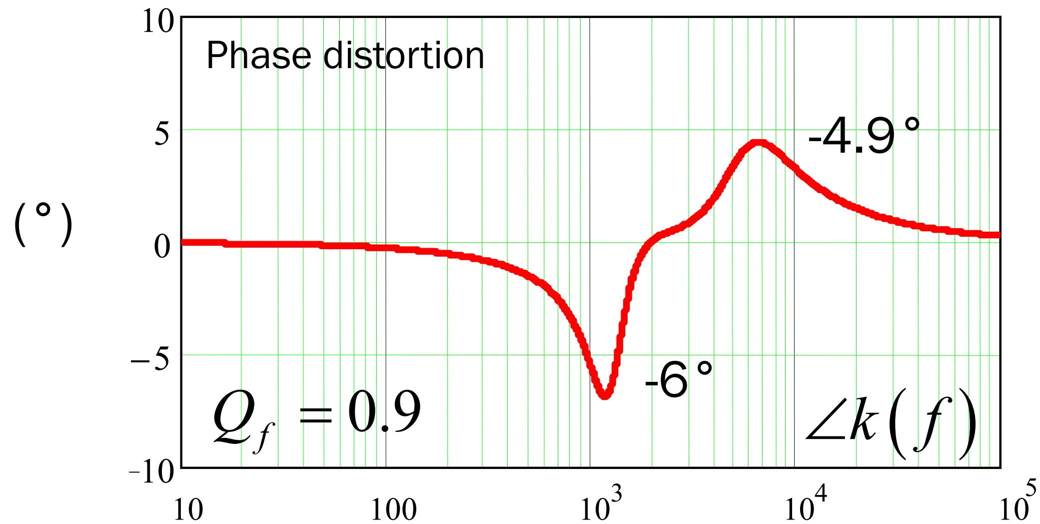
- Sweep Q_f for the least phase and gain deviation

$$k(s)=\frac{1-\dfrac{\dfrac{Z_{th}(s)}{R_{load}}}{D_0{}^2}}{1+\dfrac{Z_{th}(s)}{\dfrac{R_{load}+r_L}{D_0{}^2}\dfrac{1+\dfrac{s}{\omega_0 Q}+\left(\dfrac{s}{\omega_0}\right)^2}{1+\dfrac{s}{\omega_p}}}}$$

$$Z_{th}(s)=R_0\frac{\left(1+\dfrac{s}{\omega_{z_1}}\right)\left(1+\dfrac{s}{\omega_{z_2}}\right)}{1+\dfrac{s}{\omega_0 Q_f}+\left(\dfrac{s}{\omega_0}\right)^2}$$

$$\Rightarrow \quad Q_f=\sqrt{\frac{L_f}{C_f}}\frac{1}{r_{Cf}+r_{Lf}}$$

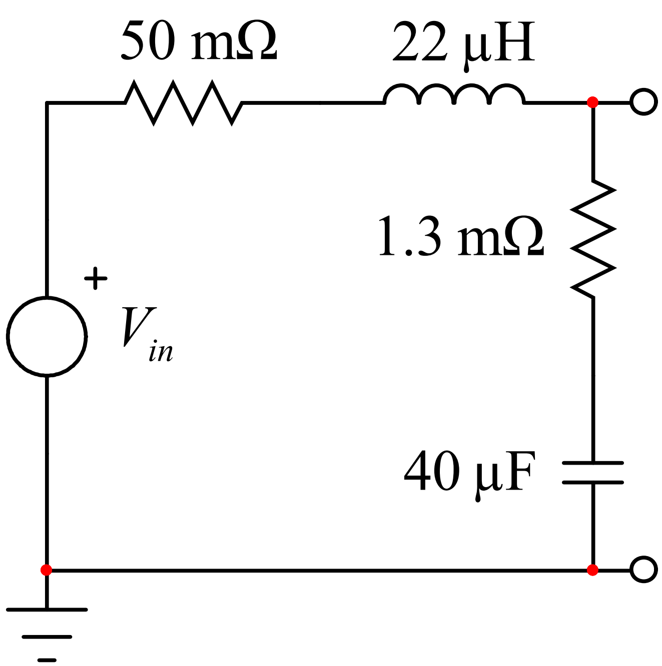




Determine Maximum Filter Peaking

- Determine maximum peaking with selected Q_f

$$\left|Z_{th}\right|_{MAX} = \frac{R_0 Q_f}{\omega_{z_1}\omega_{z_2}}\sqrt{\left(\omega_0^{\,2}+\omega_{z_1}^{\,2}\right)\left(\omega_0^{\,2}+\omega_{z_2}^{\,2}\right)}$$



$$Q_f = 14.4$$

$$\left|Z_{th}\right|_{MAX} \approx 10.7\ \Omega$$

Damp

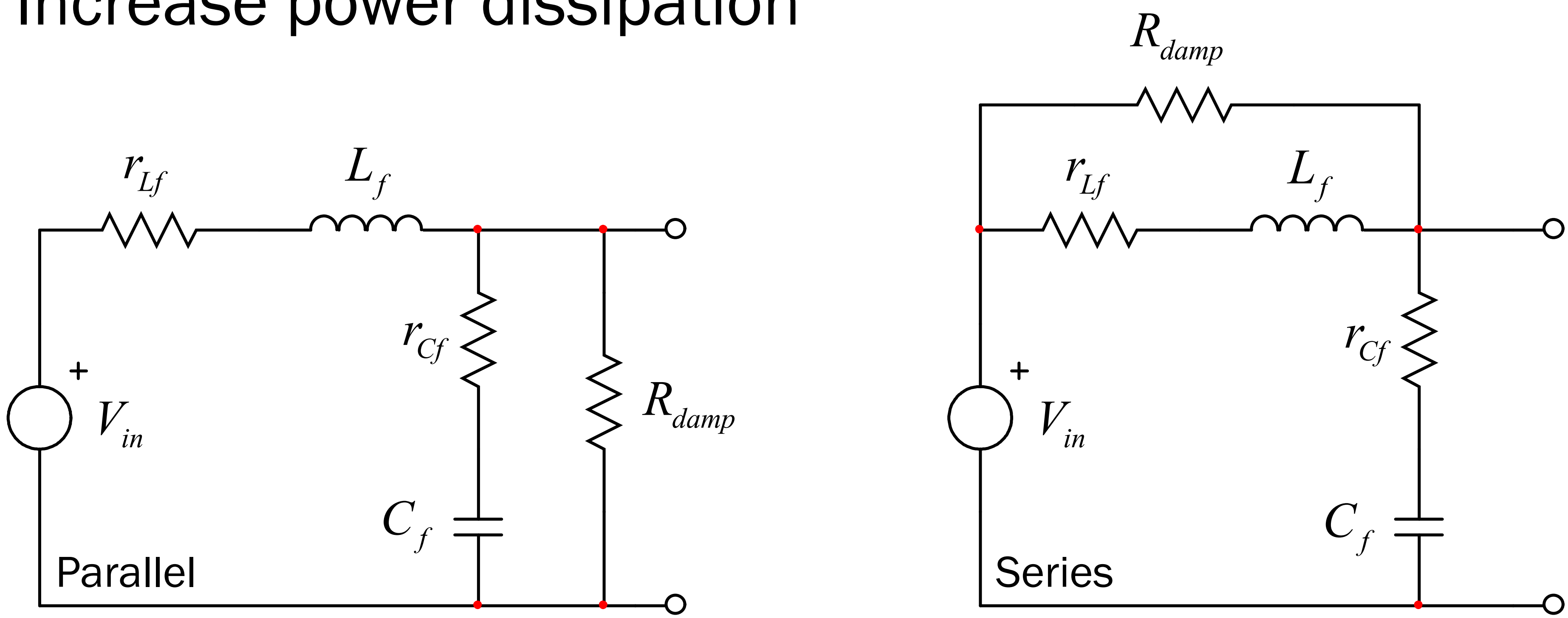
$$Q_f = 0.9$$

$$\left|Z_{th}\right|_{MAX} \approx 0.7\ \Omega \longleftarrow \text{The target}$$

- What damping elements will reduce $\left|Z_{th}\right|_{MAX}$ to 0.7 Ω?

Available Damping Techniques

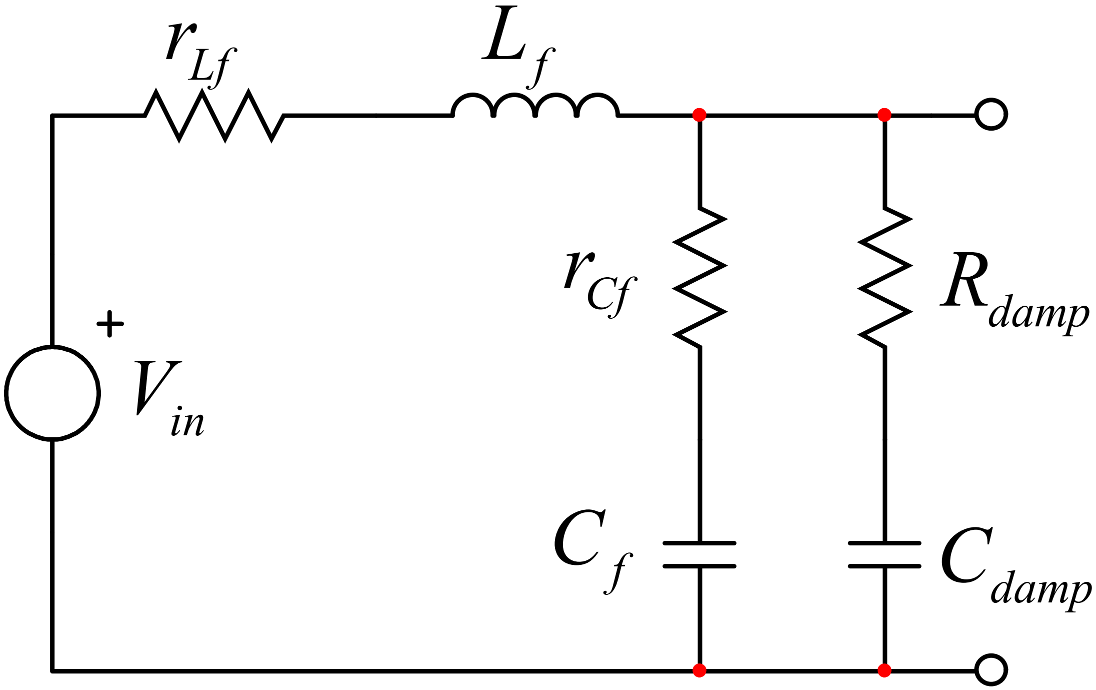
- ❑ Damping means increasing the loss per cycle
  - ➢ Increase power dissipation

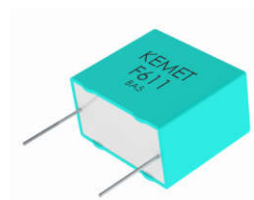


- ❑ r_C-C_f parallel damping: R_{damp} dissipates power
- ❑ r_L-L_f parallel damping: R_{damp} adds a zero and alters filter

Adding a Blocking Capacitor

- ❑ The series capacitor blocks the dc component
- ❑ Literature recommends $C_{damp} = 10 \text{ x } C_f$





$$C_f = 40\ \mu\text{F}$$

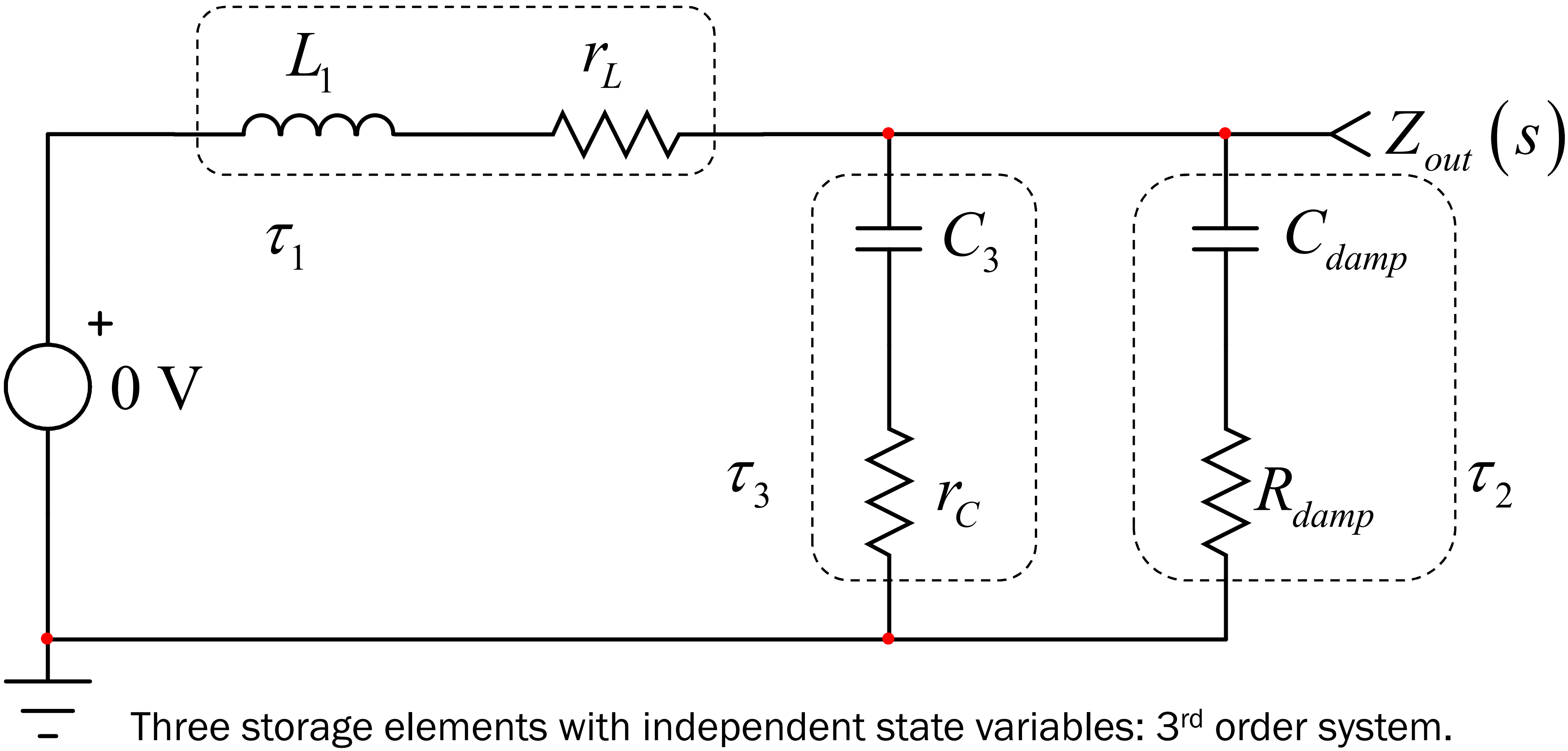
x10

$$C_{damp} = 400\ \mu\text{F}$$

- ❖ C_{damp} can be extremely bulky
- ❖ In ac R_{damp} dissipation can be an issue

Optimum combination?

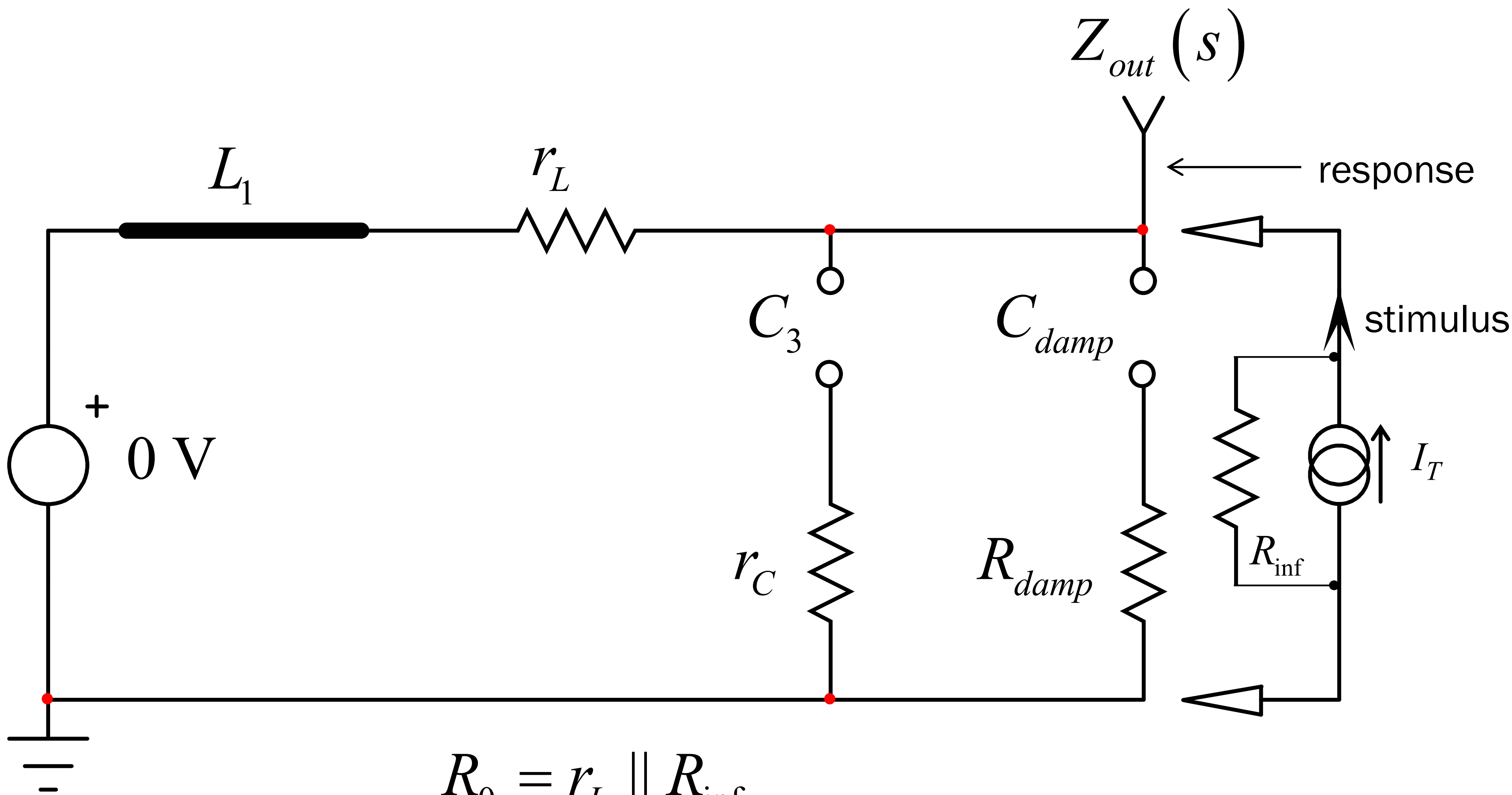
Analyzing the Filter Output Impedance



Three storage elements with independent state variables: 3rd order system.

$$D(s) = 1 + b_1 s + a_2 s^2 + a_3 s^3$$

Start with Dc Analysis $s = 0$

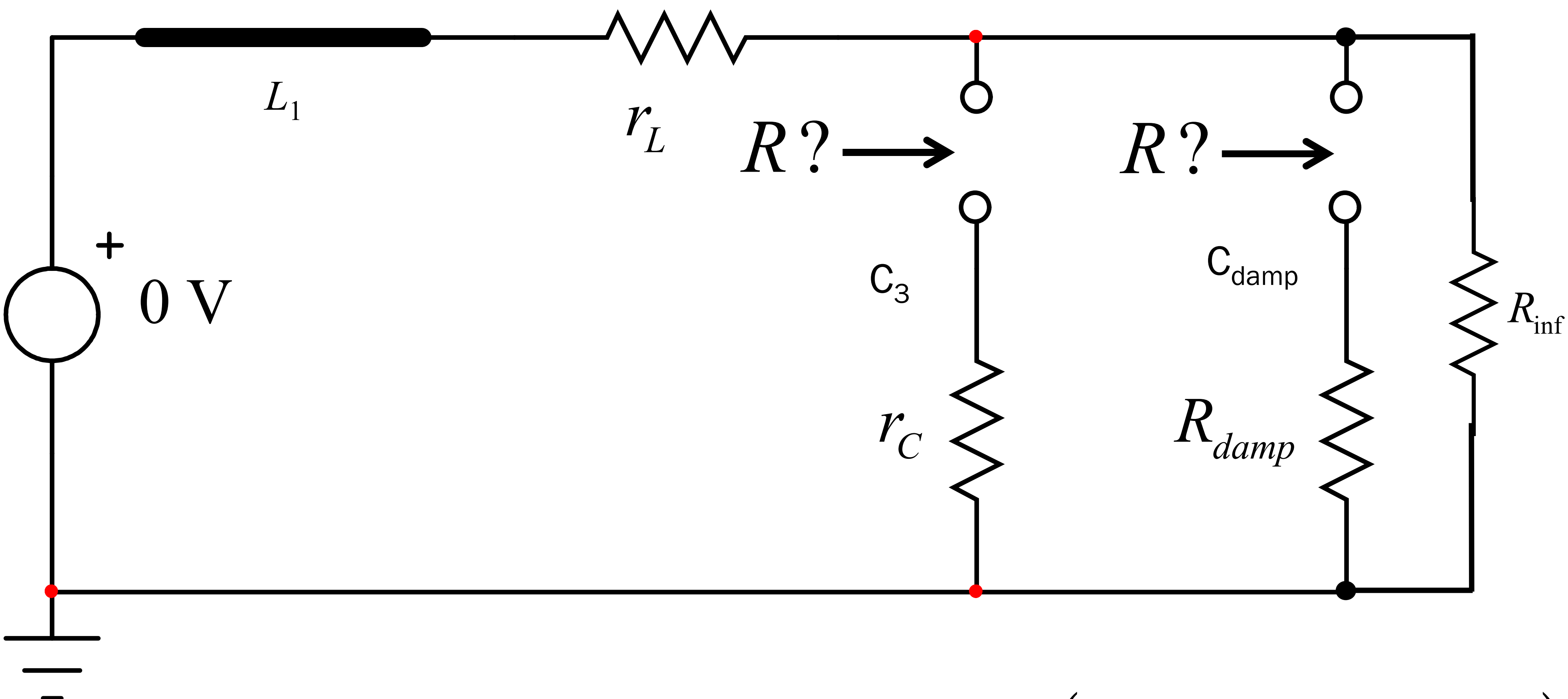


$$R_0 = r_L \parallel R_{inf}$$

R_{inf} is the current source output resistance and ensures a dc path when $I_T = 0$

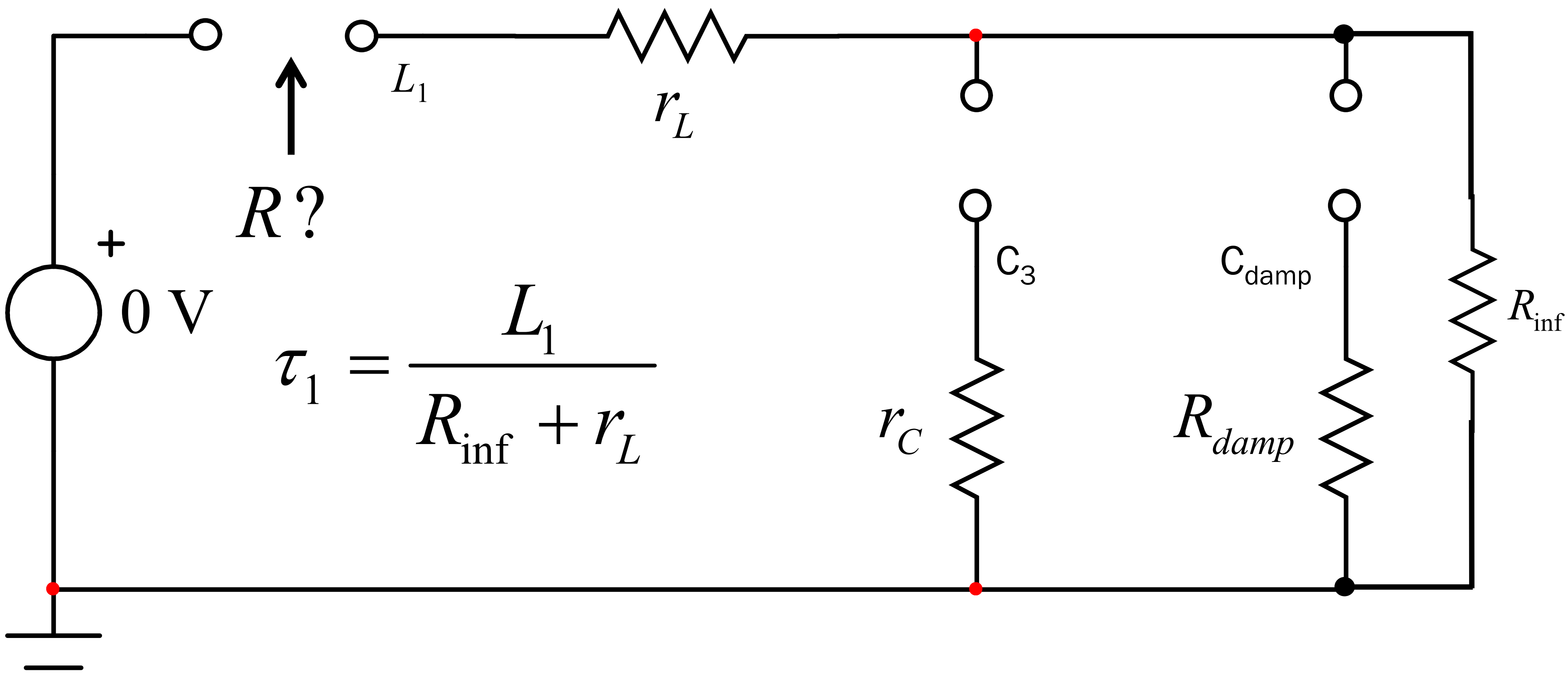
Determine Time Constants

- Turn excitation off, current source is set to 0 A



$$\tau_3 = C_3\left(r_C + r_L \parallel R_{\mathrm{inf}}\right) \qquad \tau_2 = C_{damp}\left(R_{damp} + r_L \parallel R_{\mathrm{inf}}\right)$$

Assemble Time Constants for b_1



$$\tau_1 = \frac{L_1}{R_{inf} + r_L}$$

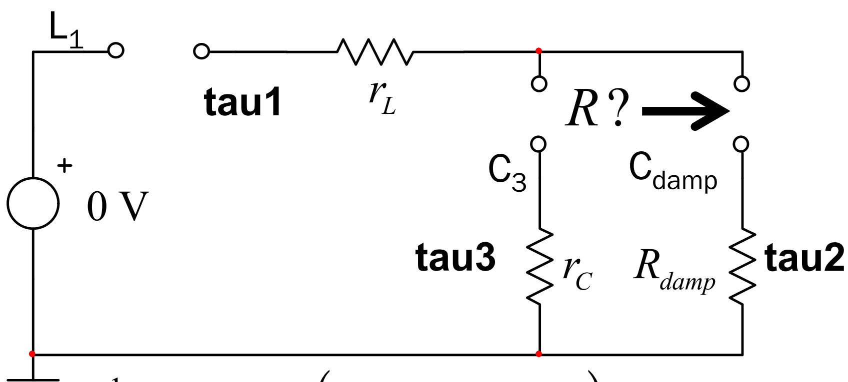
$$D(s) = 1 + \boxed{\left(\tau_1 + \tau_2 + \tau_3\right)} s + \left(\tau_1 \tau_2^1 + \tau_1 \tau_3^1 + \tau_2 \tau_3^2\right) s^2 + \left(\tau_1 \tau_2^1 \tau_3^{12}\right) s^3$$

$$b_1 = \tau_1 + \tau_2 + \tau_3 = \cancel{\frac{L_1}{R_{inf} + r_L}} + C_{damp}\left(R_{damp} + r_L \parallel R_{inf}\right) + C_3\left(r_C + r_L \parallel R_{inf}\right)$$

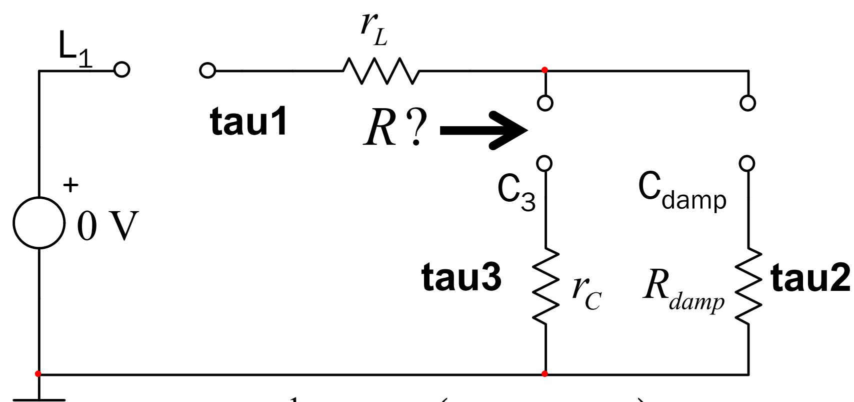
$R_{inf} \to \infty$

Second-Order Time Constants

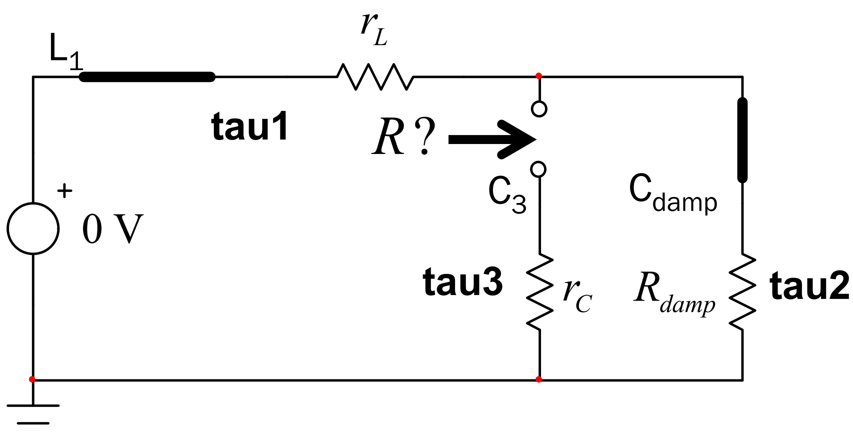
$$D(s) = 1 + (\tau_1 + \tau_2 + \tau_3)s + \boxed{\left(\tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2\right)}s^2 + \left(\tau_1\tau_2^1\tau_3^{12}\right)s^3$$



$$\tau_2^1 = C_{damp}\left(R_{damp} + R_{\inf}\right)$$



$$\tau_3^1 = C_3\left(r_C + R_{\inf}\right)$$



$$\tau_3^2 = C_3\left(r_C + R_{damp} \,||\, r_L \,||\, R_{\inf}\right)$$

τ_2^1 — $1 \rightarrow$ Reactance 1 is in its high-frequency state

$2 \rightarrow$ What resistance drives reactance 2?

$R?$

R_{inf} is not added for clarity purposes

Assemble Time Constants for b_2

$$b_2 = \frac{L_1}{R_{\inf} + r_L} C_{damp}\left(R_{damp} + R_{\inf}\right) + \frac{L_1}{R_{\inf} + r_L} C_3\left(r_C + R_{\inf}\right) + C_{damp}\left(R_{damp} + r_L \,||\, R_{\inf}\right) C_3\left(r_C + R_{damp} \,||\, R_{\inf} \,||\, r_L\right)$$

$$b_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2$$

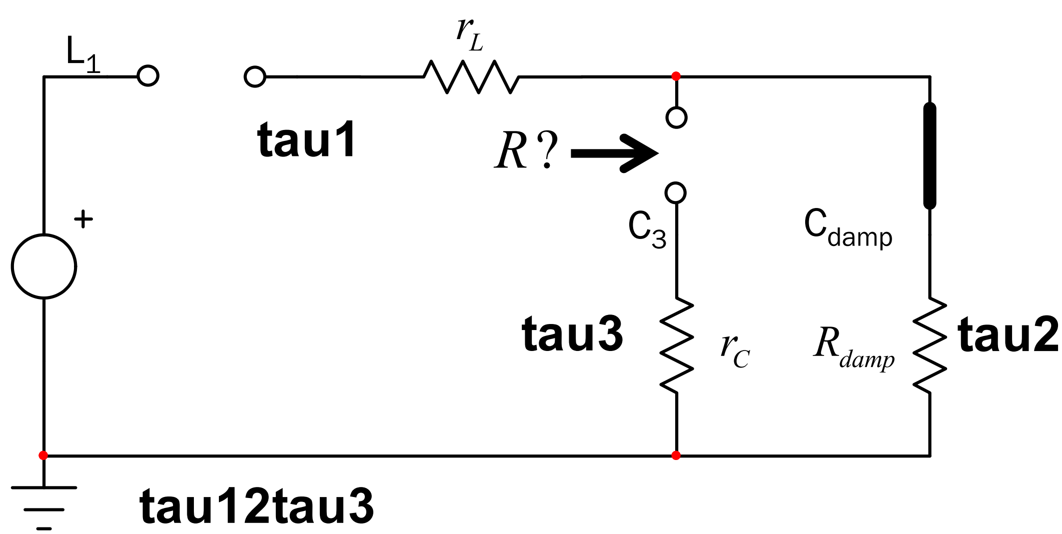
$$R_{\inf} \to \infty \longrightarrow b_2 = L_1\left(C_{damp} + C_3\right) + C_{damp}\left(R_{damp} + r_L\right) C_3\left(r_C + r_L \,||\, R_{damp}\right)$$

$$b_3 = \tau_1\tau_2^1\tau_3^{12}$$

HF state

$$\tau_3^{12} = C_3\left(r_C + R_{damp} \,||\, R_{\inf}\right)$$

What resistance drives reactance 3?



Build the 3rd-Order Denominator

- Gather time constants and rearrange to form $D(s)$

$$D(s)=1+b_1s+b_2s^2+b_3s^3$$

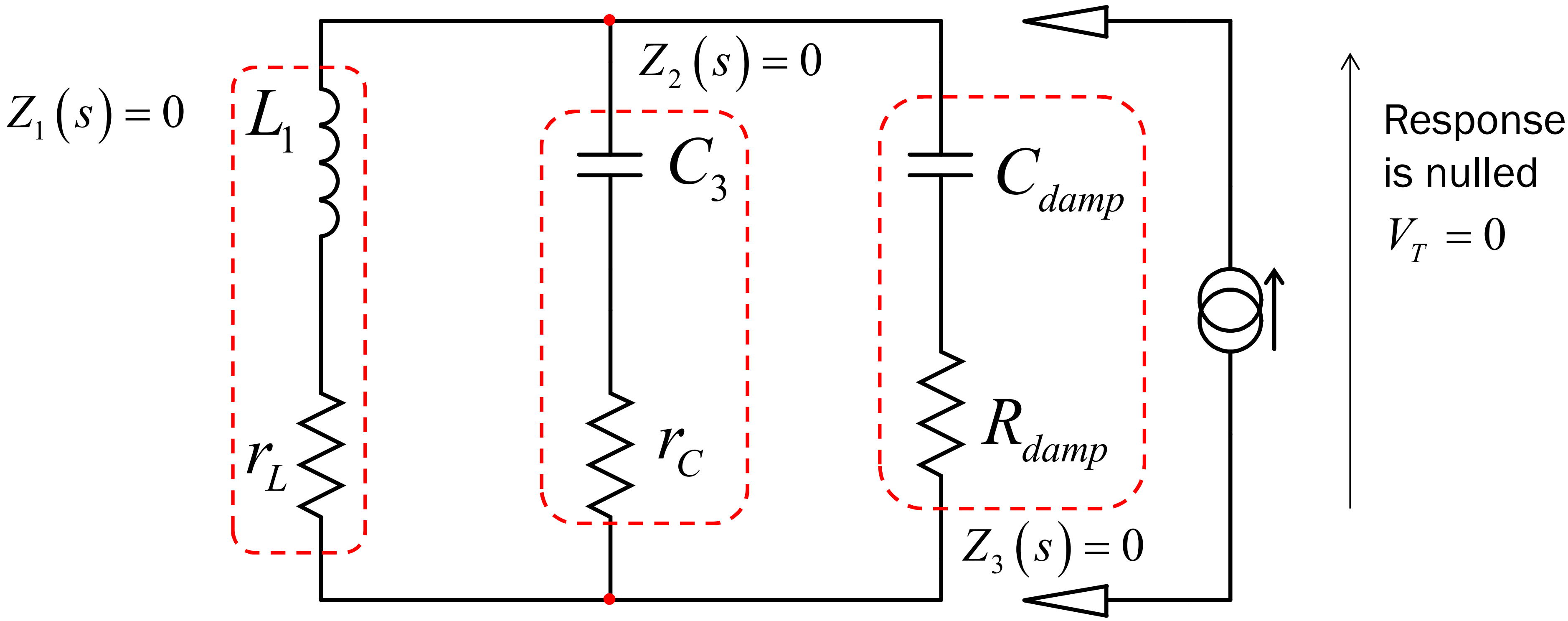
$$b_1=C_{damp}\left(R_{damp}+r_L\right)+C_3\left(r_C+r_L\right)$$

$$b_2=L_1\left(C_{damp}+C_3\right)+C_{damp}\left(R_{damp}+r_L\right)C_3\left(r_C+r_L\,||\,R_{damp}\right)$$

$$b_3=\frac{L_1}{R_{\inf}}C_{damp}\left(R_{damp}+R_{\inf}\right)C_3\left(r_C+R_{damp}\right)=L_1C_{damp}C_3\left(r_C+R_{damp}\right)$$

$$\begin{aligned}D(s)=1&+s\left[C_{damp}\left(R_{damp}+r_L\right)+C_3\left(r_C+r_L\right)\right]\\&+s^2\left[L_1\left(C_{damp}+C_3\right)+C_{damp}\left(R_{damp}+r_L\right)C_3\left(r_C+r_L\,||\,R_{damp}\right)\right]\\&+s^3\left[L_1C_{damp}C_3\left(r_C+R_{damp}\right)\right]\end{aligned}$$

Determine $N(s)$ Swiftly with Inspection



Three zeros when $Z_1(s_{z_1})=0 \quad Z_2(s_{z_2})=0 \quad Z_3(s_{z_3})=0$

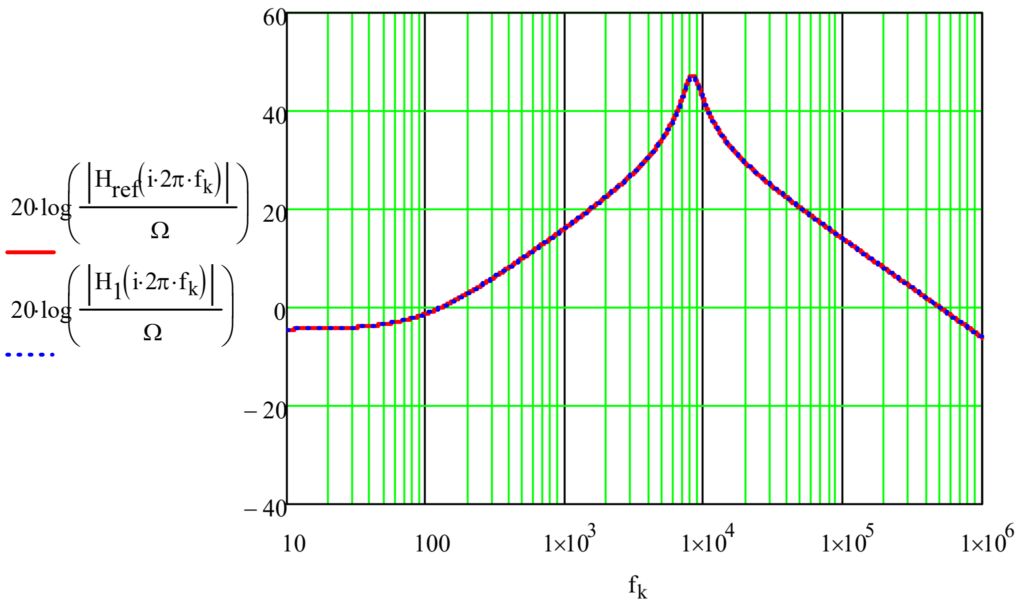
$$N(s)=\left(1+s\frac{L_1}{r_L}\right)\left(1+sr_C C_3\right)\left(1+sR_{damp}C_{damp}\right)$$

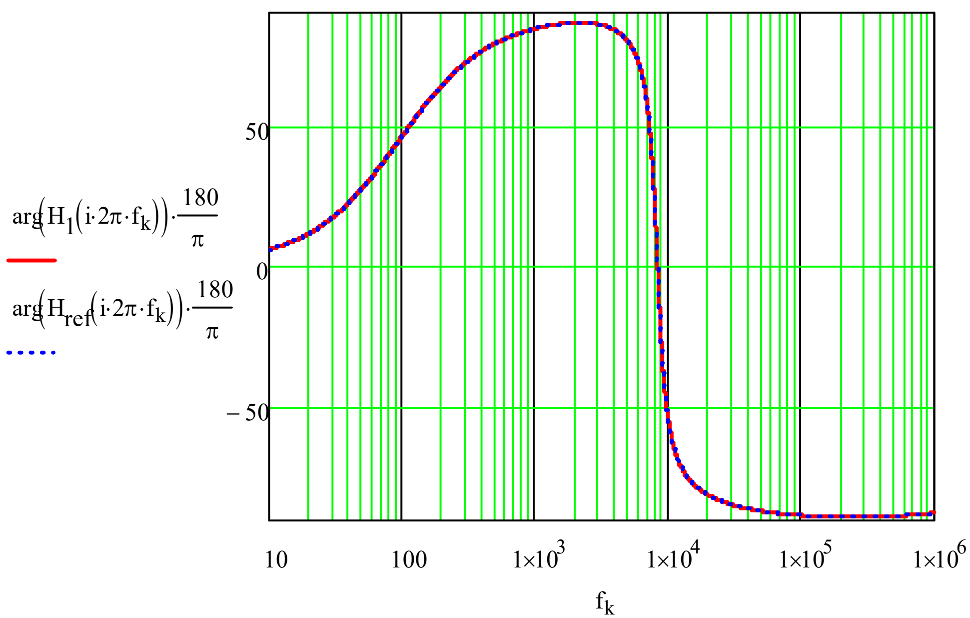
Run a Sanity Check to Verify Results

$$Z_{out}(s) = R_0 \frac{N(s)}{D(s)}$$ The complete *low-entropy* transfer function is thus

$$Z_{out}(s) = R_0 \frac{(1+sr_C C_3)(1+sR_{damp}C_{damp})\left(1+s\frac{L_1}{r_L}\right)}{1+s\left[C_{damp}(R_{damp}+r_L)+C_3(r_C+r_L)\right]+s^2\left[L_1(C_{damp}+C_3)+C_{damp}(R_{damp}+r_L)C_3(r_C+r_L \| R_{damp})\right]+s^3\left[L_1 C_{damp} C_3 (r_C+R_{damp})\right]}$$

The raw transfer function is $Z_{out}(s) = Z_1(s) \| Z_2(s) \| Z_3(s)$





Raw and full TF plots are superimposed: good to go!

Trying to Rewrite the Transfer Function

In this 3rd-order system, it is difficult to find a canonical form such as

$$Z_{out}(s) = R_0 \frac{N(s)}{\left(1+\frac{s}{\omega_p}\right)\left(1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2\right)}$$

If r_L is larger than r_C and $C_{damp}R_{damp}$ larger than $C_3 r_L$

$$D(s) \approx 1 + sC_{damp}R_{damp} + s^2 L_1\left(C_{damp} + C_3\right) + s^3 L_1 C_{damp} C_3 R_{damp}$$

Unfortunately, $1+sC_{damp}R_{damp}$ does not dominate at low freq.

Cannot factor the 3rd-order denominator

Neglect Parasitic Terms r_L and r_C

- Simplify the transfer function considering 0 r_L and r_C

$$Z_{out}(s) = r_L \frac{(1+sr_C C_3)(1+sR_{damp}C_{damp})\left(1+s\frac{L_1}{r_L}\right)}{1+s\left[C_{damp}(R_{damp}+r_L)+C_3(r_C+r_L)\right]+s^2\left[L_1(C_{damp}+C_3)+C_{damp}(R_{damp}+r_L)C_3(r_C+r_L \parallel R_{damp})\right]+s^3\left[L_1 C_{damp} C_3 (r_C + R_{damp})\right]}$$

- Factor the L_1/r_L term

$$Z_{out}(s) = r_L s\frac{L_1}{r_L} \frac{(1+sR_{damp}C_{damp})\left(\frac{r_L}{sL_1}+1\right)}{1+s\left[C_{damp}(R_{damp}+r_L)+C_3 r_L\right]+s^2\left[L_1(C_{damp}+C_3)+C_{damp}(R_{damp}+r_L)C_3(r_L \parallel R_{damp})\right]+s^3\left[L_1 C_{damp} C_3 R_{damp}\right]}$$

- Have r_L go to zero

$$Z_{out}(s) = sL_1 \frac{1+sR_{damp}C_{damp}}{1+sC_{damp}R_{damp}+s^2 L_1(C_{damp}+C_3)+s^3\left[L_1 C_{damp} C_3 R_{damp}\right]}$$

- Consider $C_{damp} = nC_3$

$$Z_{out}(s) = sL_1 \frac{1+sR_{damp}nC_3}{1+sR_{damp}nC_3+s^2 L_1 C_3(1+n)+s^3\left[L_1 n C_3^{\,2} R_{damp}\right]}$$

Rearrange Expressions

- Determine the magnitude of this transfer function

$$Z_{out}(j\omega) = \frac{L_1\omega^2 nR_{damp}C_3 - jL_1\omega}{\left(C_3L_1\omega^2 + C_3L_1\omega^2 n - 1\right) - j\left(C_3R_{damp}\omega n\right)\left(1 - C_3L_1\omega^2\right)}$$

$$\left|Z_{out}(\omega)\right| = \frac{\sqrt{\left(L_1\omega^2 nR_{damp}C_3\right)^2 + \left(L_1\omega\right)^2}}{\sqrt{\left(C_3L_1\omega^2 + C_3L_1\omega^2 n - 1\right)^2 + \left(C_3R_{damp}\omega n\right)^2\left(1 - C_3L_1\omega^2\right)^2}} \longrightarrow \text{mag}_1$$

- Check with Middlebrook's definitions

$$R_0 = \sqrt{\frac{L_1}{C_3}} \qquad Q = \frac{R_{damp}}{R_0} \qquad p(s) = \frac{s}{\omega_0} \qquad \omega_0 = \frac{1}{\sqrt{L_1C_3}}$$

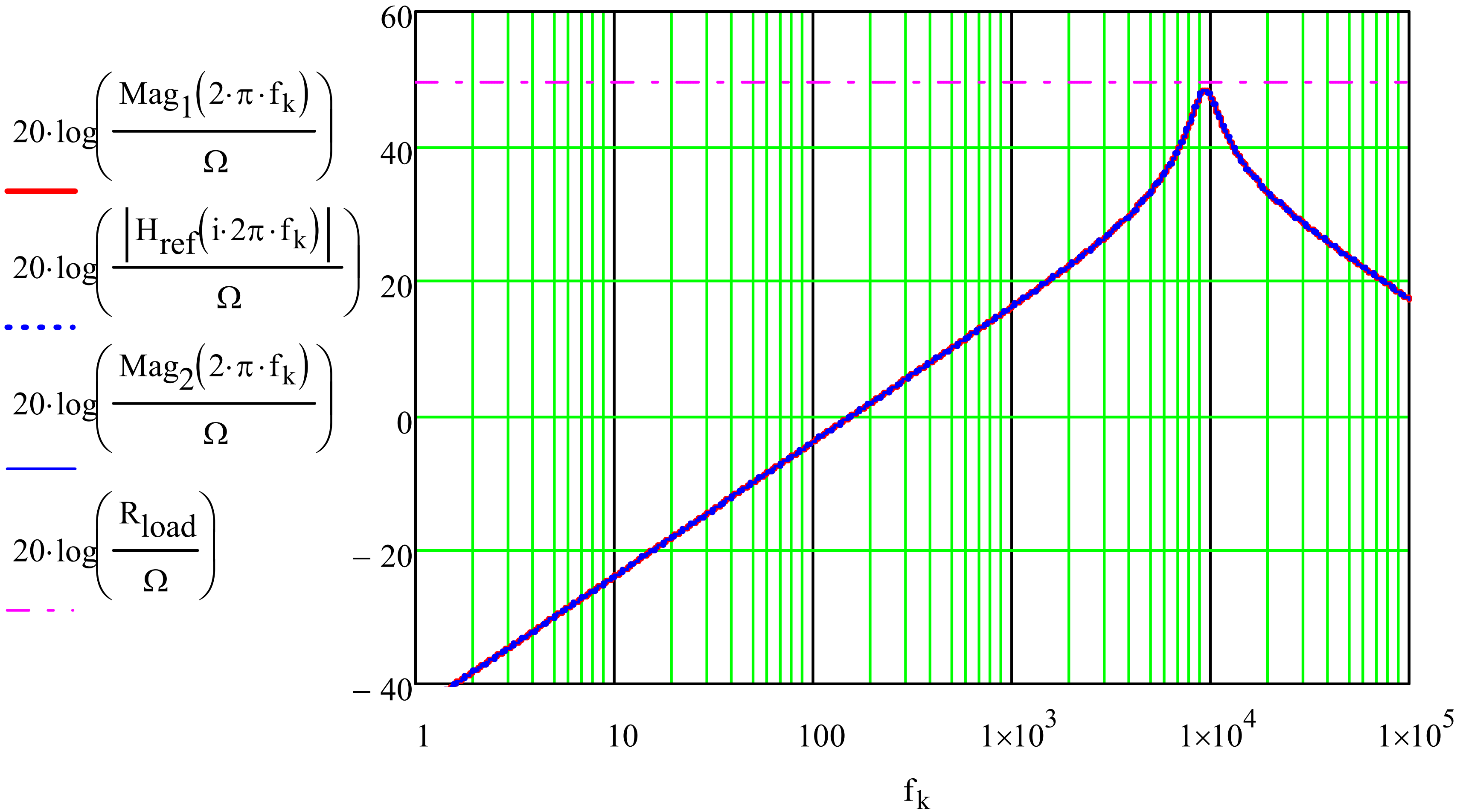
$$Z_{out}(s) = \frac{p(s)\left[1 + nQp(s)\right]}{1 + nQp(s) + (1+n)p(s)^2 + nQp(s)^3}$$

$$\left|Z_{out}(\omega)\right| = R_0\frac{x\sqrt{1 + n^2Q^2x^2}}{\sqrt{\left[1 - (1+n)x^2\right]^2 + \left[xnQ\left(1 - x^2\right)\right]^2}} \qquad x = \frac{\omega}{\omega_0} \longrightarrow \text{mag}_2$$

Design Techniques for Preventing Input-Filter Oscillations in Switched-Mode Regulators, R.D Middlebrook, Powercon, May 1978

Always Run a Sanity Check

- Check if expressions are ok versus Mathcad® calculations



- Compare analytical results versus raw magnitude expression

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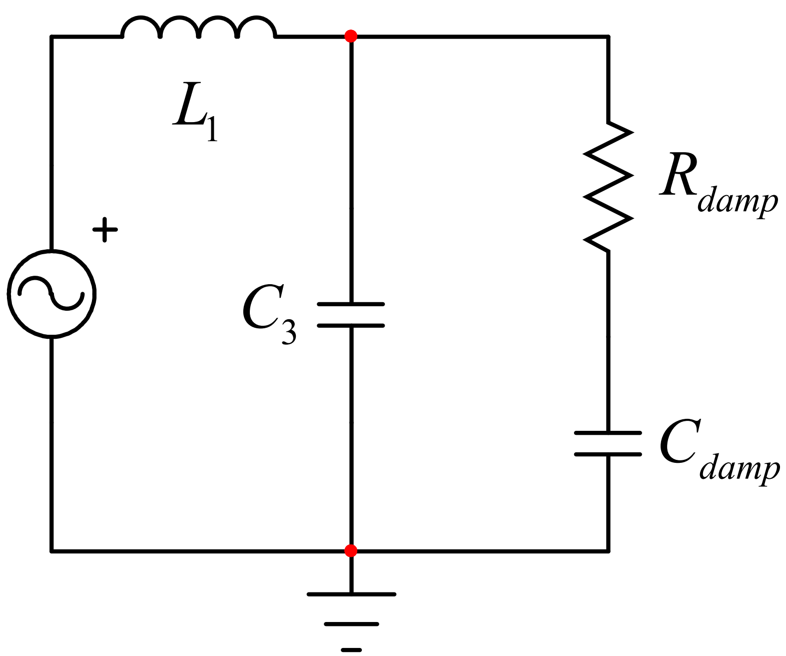
The Damping Resistor Affects Q and ω_0

- The damping resistor value changes the resonant frequency



$R_{damp} \to \infty$

No loss

$Q \to \infty$

$$\omega_0 = \frac{1}{\sqrt{L_1 C_3}}$$

$R_{damp} = 0$

No loss

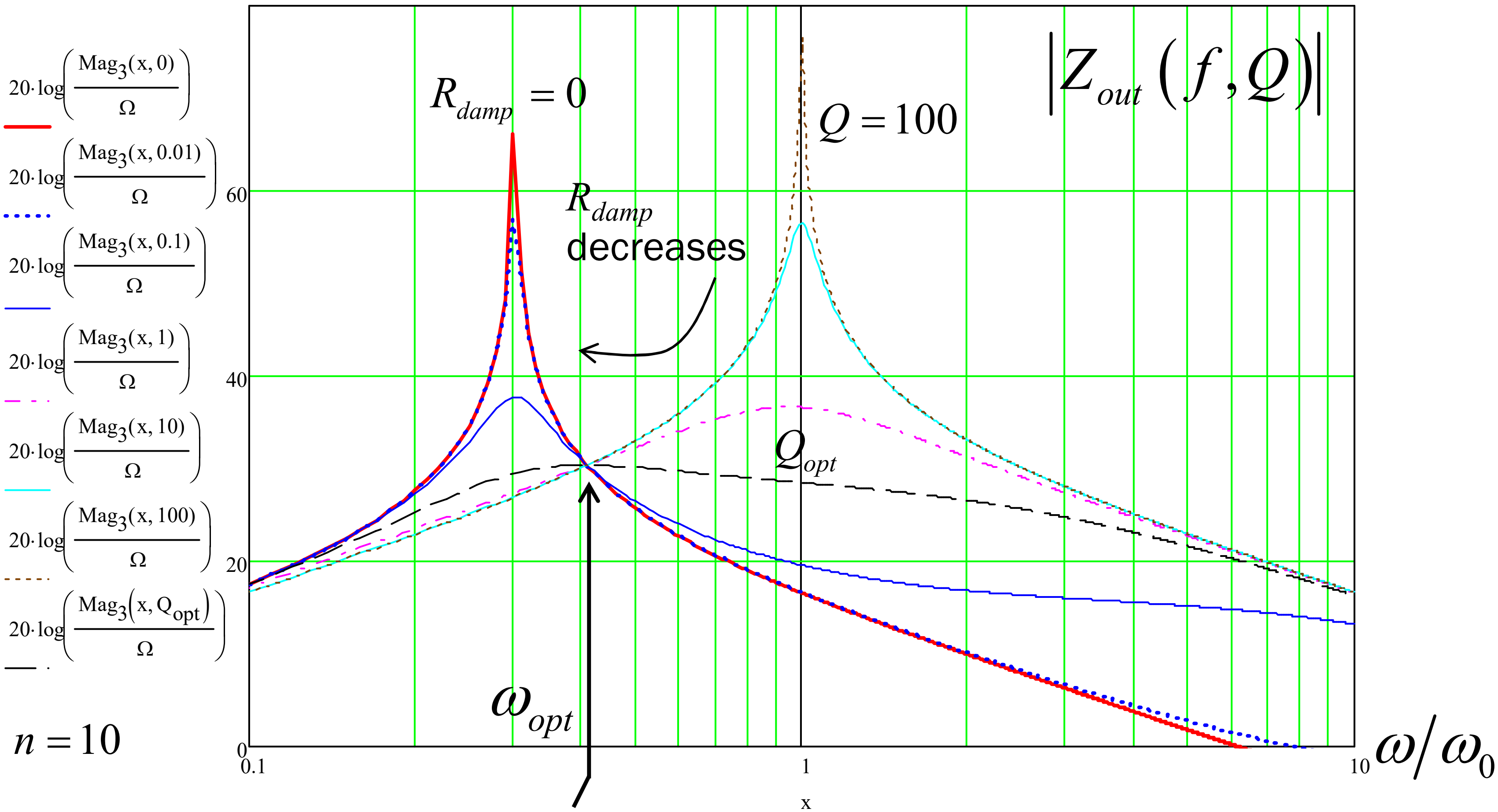
$Q \to \infty$

$$\omega_0 = \frac{1}{\sqrt{L_1\left(C_3 + C_{damp}\right)}}$$

- There must be a $R_{damp}C_{damp}$ couple optimizing Q
  - What is the minimum in the maximum (minmax) peaking of $|Z_{out}|$?
  - For what optimum Q value does it occur and at what frequency?

Changes Induced by Damping Resistor

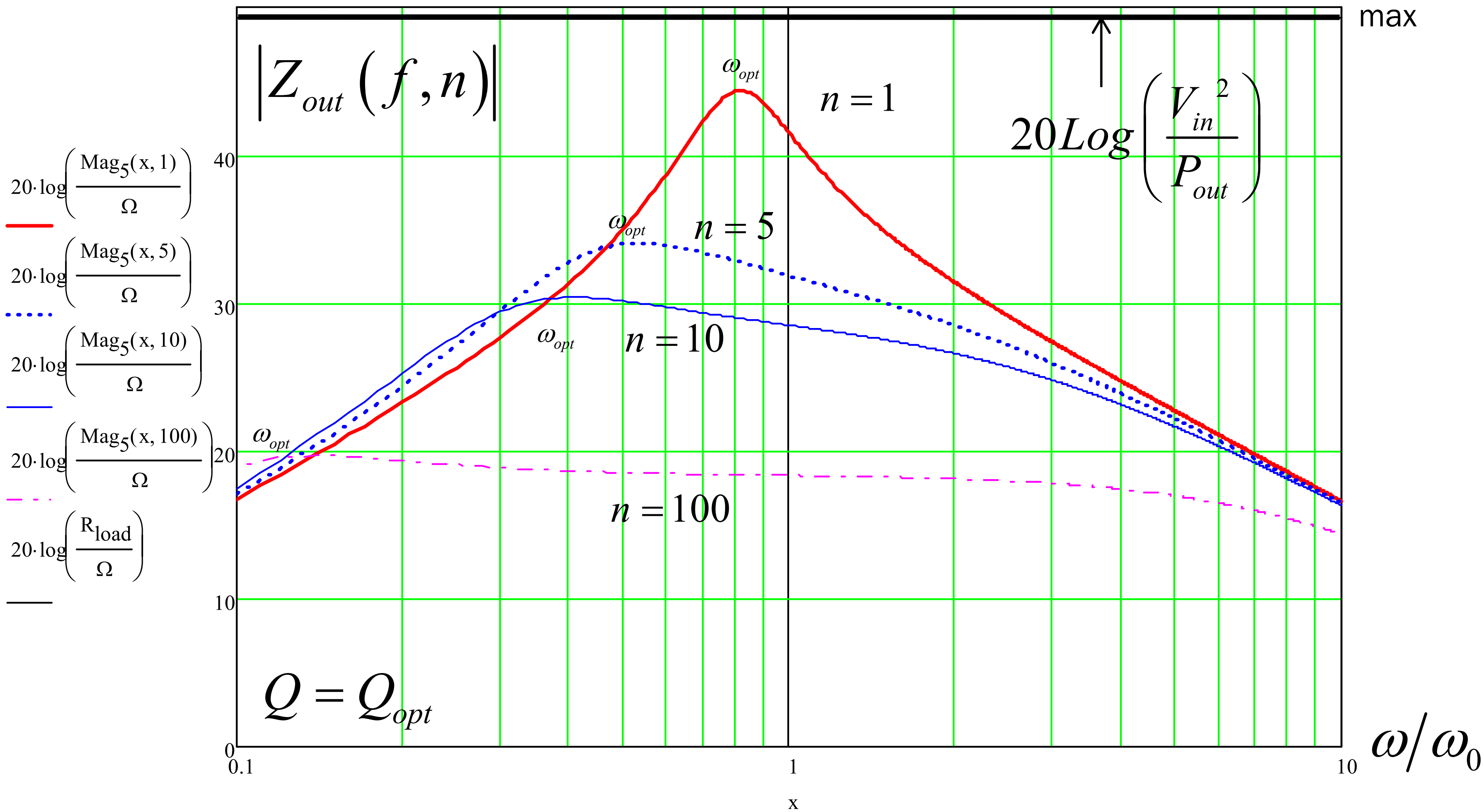
- Plot magnitudes versus various Q factors, C_{damp} = 10C_3



This point is independent of Q → It is the point at which peaking is the lowest!

Optimum Peak Depends on C_{damp}

- Optimum Z_{out} magnitudes versus different values of n



- There is an optimum R_{damp} and n to match a given Z_{out}

Determine the Optimum Frequency

- ❑ The frequency at which the minmax occurs is immune to Q
- ➢ Calculate the sensitivity of Z_{out} to Q and cancel it
- ➢ Work on $Z_{out}{}^2$ to get rid of square roots

$$\frac{d}{dQ}\left(Z_{out}(Q)^2\right)=\frac{d}{dQ}\left(R_0{}^2\frac{x^2\left(1+n^2Q^2x^2\right)^2}{\left[1-(1+n)x^2\right]^2+\left[xnQ\left(1-x^2\right)\right]^2}\right)$$

$$\frac{d}{dQ}\left(Z_{out}(Q)^2\right)=\frac{2Qn^3x^6\left(nx^2+2x^2-2\right)}{D(Q)}=0$$

$$\left.\begin{aligned}
&nx^2+2x^2-2=0\\
&x^2(n+2)=2\\
&x=\sqrt{\frac{2}{2+n}}\\
&x=\frac{\omega}{\omega_0}
\end{aligned}\right\}$$

The point at which Q_{opt} occurs is: $\omega_{opt}=\sqrt{\frac{2}{2+n}}\omega_0=\sqrt{\frac{2}{(2+n)L_1C_3}}$

Calculate the Magnitude at ω_{opt}

- Update the magnitude definition to have $|Z_{out}|$ at ω_{opt}

$$\left|Z_{out}(\omega)\right| = R_0 \frac{x\sqrt{1+n^2Q^2\boxed{x}^2}}{\sqrt{\left[1-(1+n)x^2\right]^2+\left[xnQ\left(1-x^2\right)\right]^2}} \qquad x=\frac{\omega_{opt}}{\omega_0}=\sqrt{\frac{2}{2+n}}$$

$$\left|Z_{out}\left(\omega_{opt}\right)\right| = \frac{\sqrt{2(2+n)}}{n}R_0 = \frac{\sqrt{2(2+n)}}{n}\sqrt{\frac{L_1}{C_3}}$$

This is the value of $|Z_{out}|$ at ω_{opt}

- We want to minimize $|Z_{out}|$ at ω_{opt}
  - Differentiate $Z_{out}{}^2$ with respect to x^2, find the optimum Q
    - Replace $A = x^2$ and $\sqrt{A} = x$

$$\frac{d}{dA}\left|\frac{Z_{out}(A)}{R_0}\right|^2 = \frac{d}{dA}\frac{A\left(1+n^2Q^2A\right)}{\left[1-(1+n)A\right]^2+\left[n\sqrt{A}Q(1-A)\right]^2}=0$$

Determine the Optimum Value of Q

- Apply brute-force differentiation with Mathcad ®

$$\frac{d}{dA}\left|\frac{Z_{out}(A)}{R_0}\right|^2 = -\frac{(AQn)^4 + 2A^3Q^2n^2 - A^2Q^4n^4 + 2A^2Q^2n^3 + A^2n^2 + 2A^2n + A^2 - 2AQ^2n^2 - 1}{D} = 0$$

Extract Q

$\downarrow$

$$(AQn)^4 + 2A^3Q^2n^2 - A^2Q^4n^4 + 2A^2Q^2n^3 + A^2n^2 + 2A^2n + A^2 - 2AQ^2n^2 - 1 = 0$$

$$Q_{opt} = \sqrt{\frac{\sqrt{-A^3n^5\left(A^3n - 2A - 2A^2 + 2A^3 - 2An + 2\right)} - An^2 + A^2n^3 + A^3n^2}{A^2n^4 - A^4n^4}}$$

$$A = \frac{2}{2+n} \quad \Rightarrow \quad Q_{opt} = \sqrt{\frac{3n^2 + 10n + 8}{2n^2(n+4)}} = \boxed{\sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}}}$$

Result from Dr Middlebrook

Apply the Technique to the Buck

- The target is to reduce the filter impedance peak to 0.7 Ω

$$R_0 = \sqrt{\frac{L_f}{C_f}}$$

$$\frac{\left|Z_{out}\right|_{mm}}{R_0} = \sqrt{\frac{2(2+n)}{n^2}}$$

$$n = \frac{R_0\left(R_0 + \sqrt{R_0^{\,2} + 4\left(\left|Z_{out}\right|_{mm}\right)^2}\right)}{\left(\left|Z_{out}\right|_{mm}\right)^2} = 3.5$$

target

$$Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} = 0.65$$

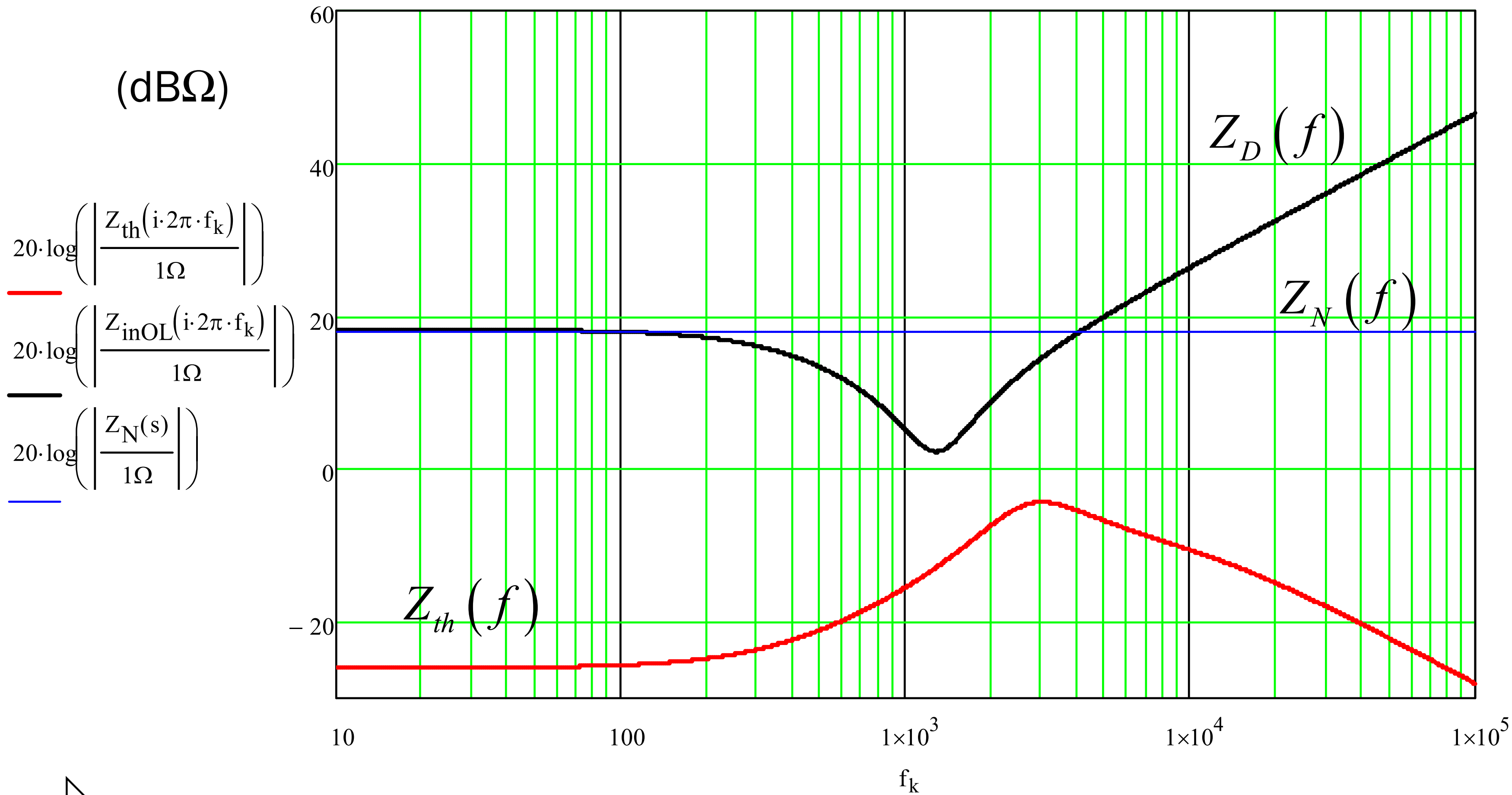
$$R_{damp} = R_0 Q_{opt} = 0.487\ \Omega$$

$$C_{damp} = nC_f = 141\ \mu\mathrm{F}$$

- This is a rather large capacitance value
- ➢ An electrolytic capacitor and its ESR can do the job
- ❖ Watch for temperature effects as ESR increases at low temp!

Check Margins with Z_D and Z_N

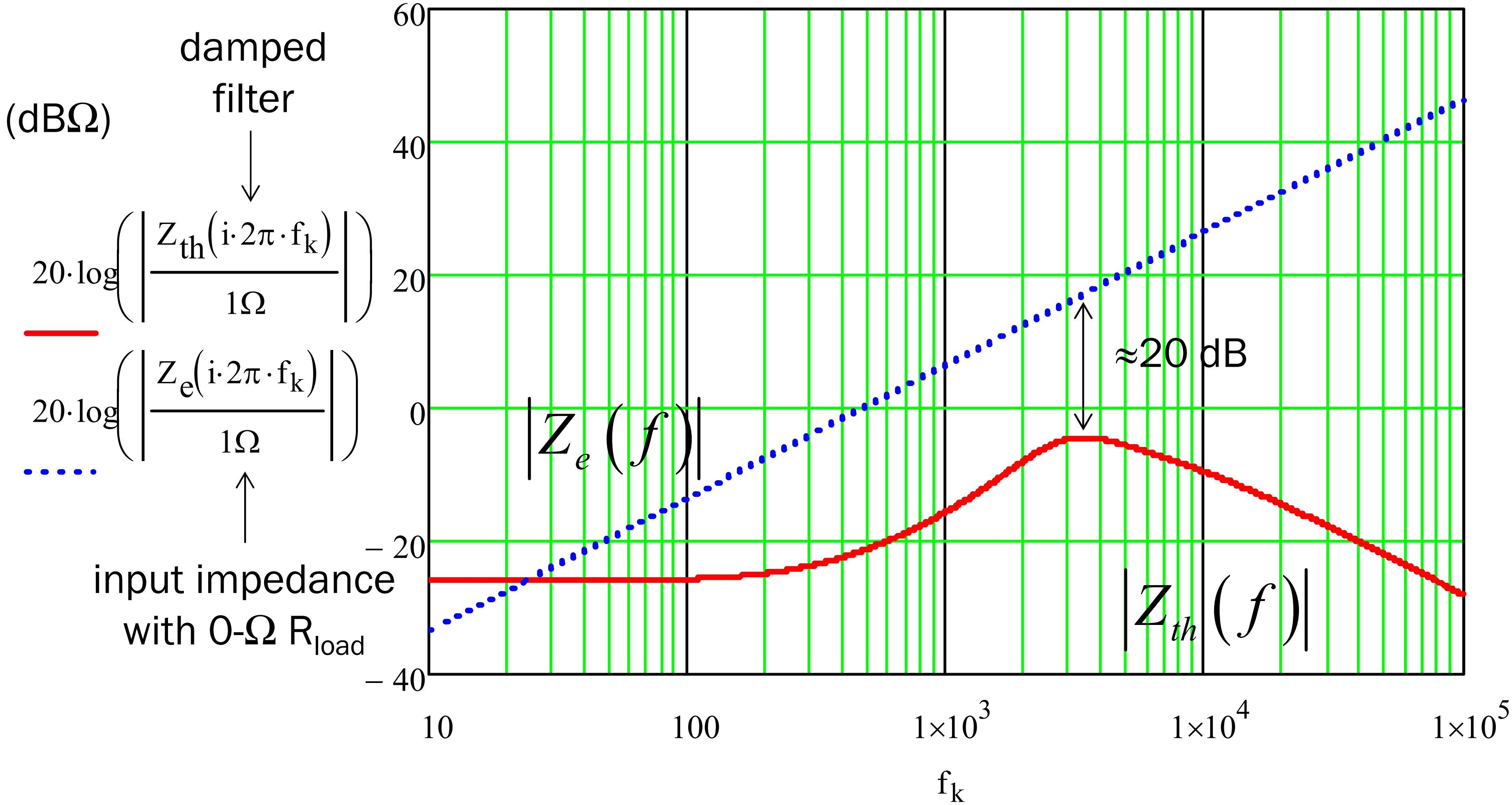
- The overlap is gone for both impedances



Gain and phase distortion of k are minimized

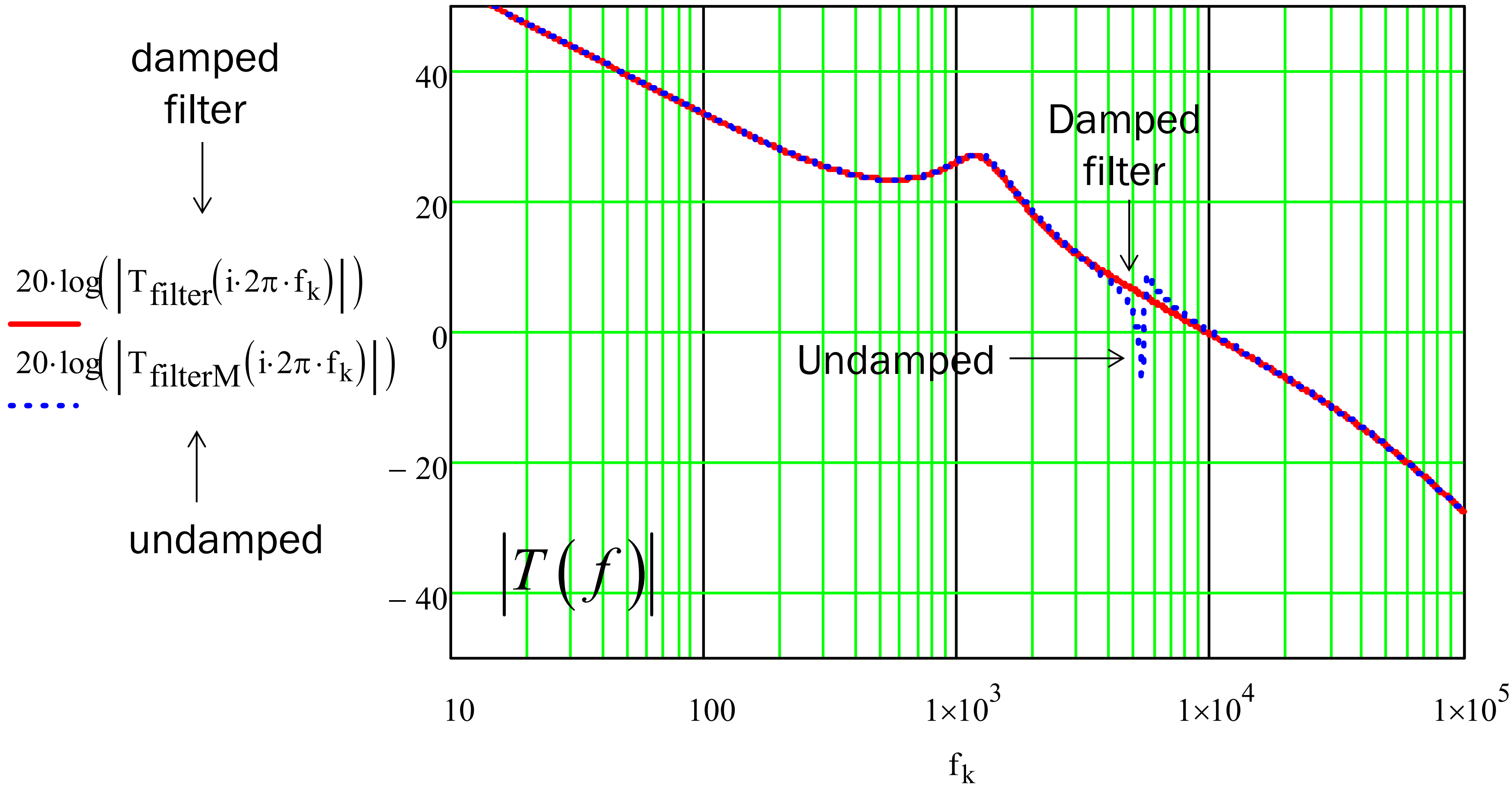
Check Margin with Z_e

- The output impedance should not be affected by the EMI filter



Verify Results after Damping - Magnitude

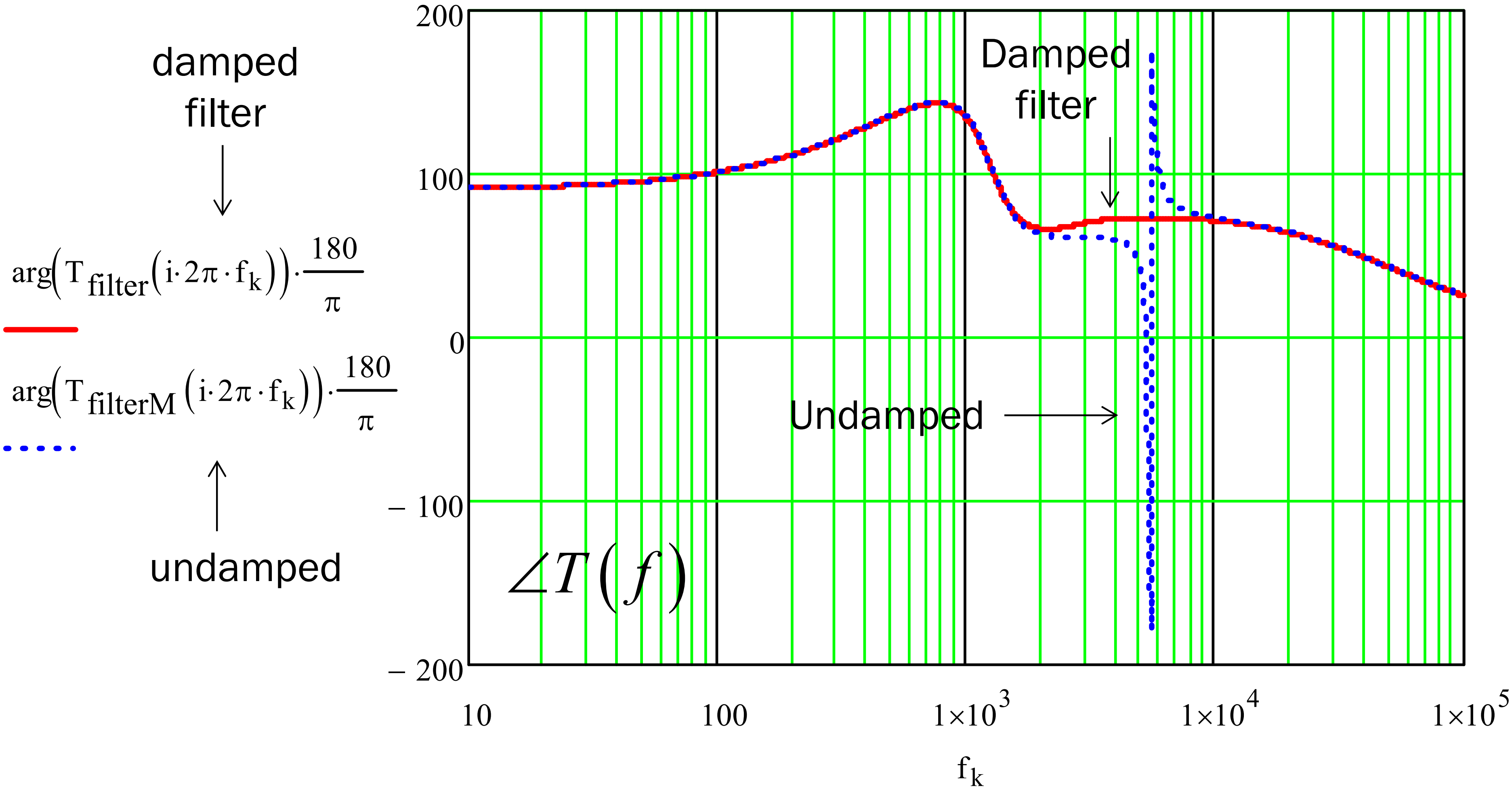
- ❑ Check the resulting control-to-output transfer function



- ❑ Magnitude distorsion has disappeared after damping

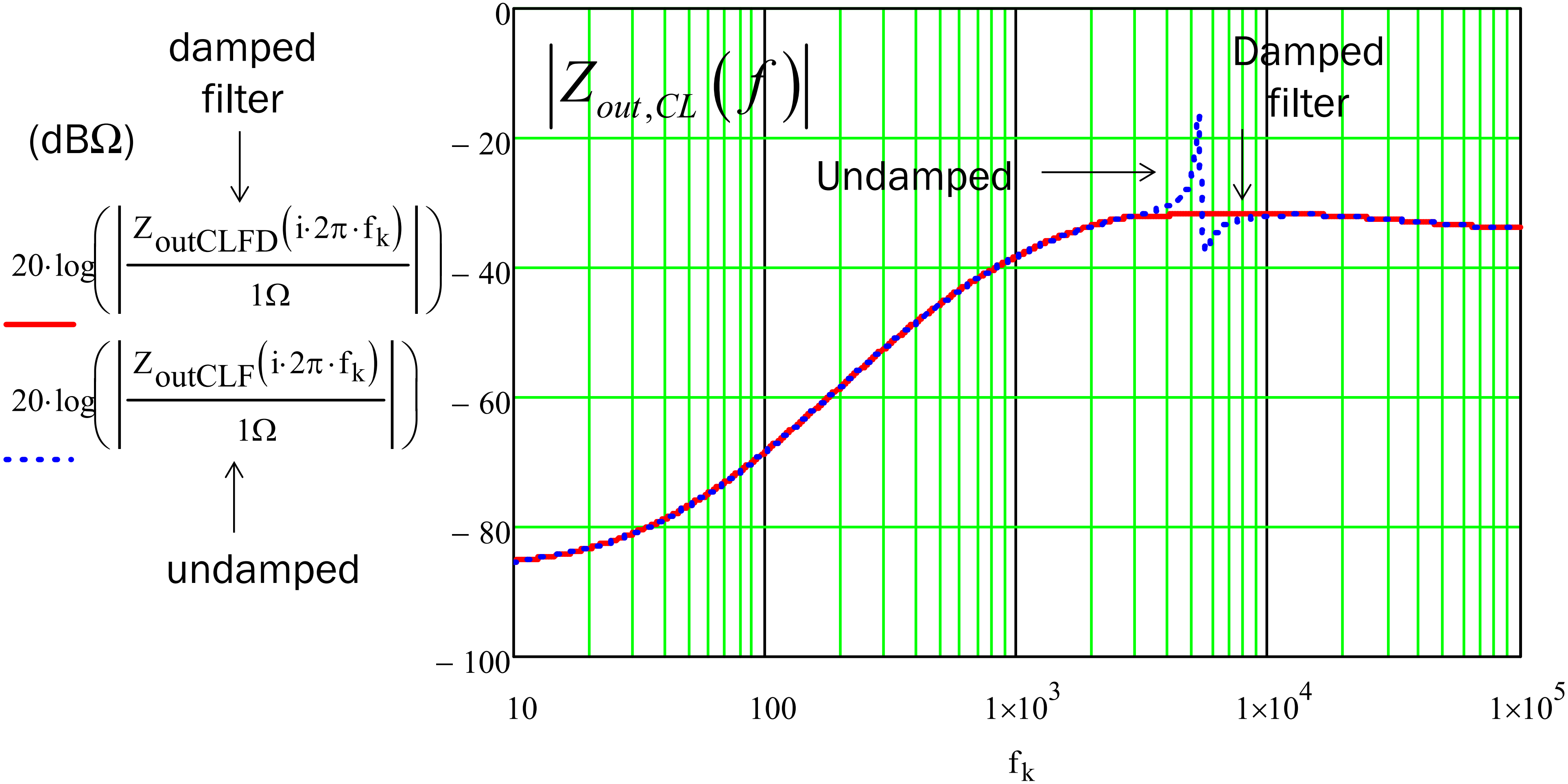
Verify Results after Damping - Phase

- Original phase margin is unaffected after damping



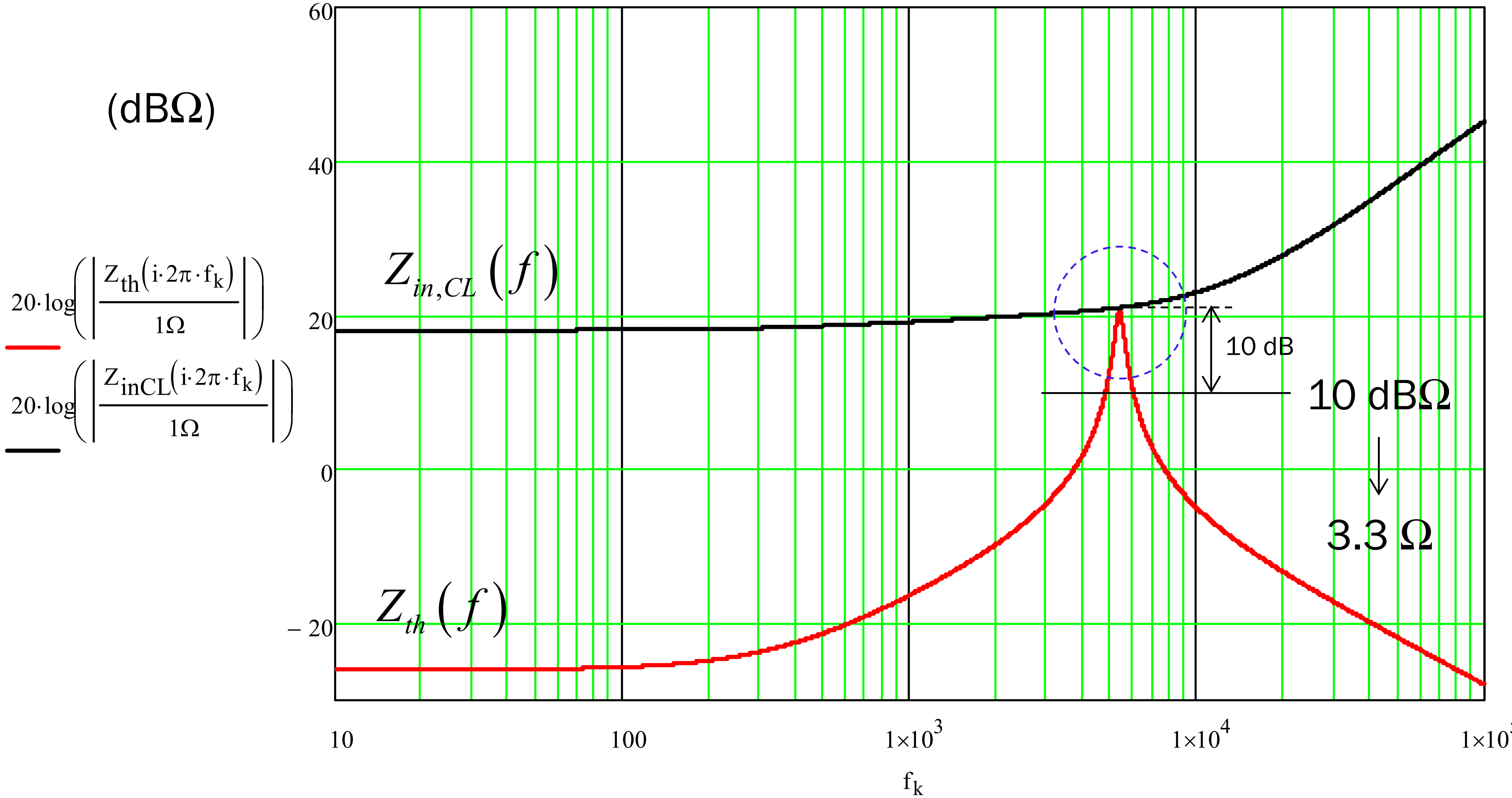
Closed-Loop Output Impedance

- Peaking effects of the EMI filter are now gone



Opting for a Different Strategy

- ❑ What if you only try to get rid of the overlap, ignoring Z_D and Z_N?
- ➢ Plot the closed-loop input impedance and build margin (10 dB)



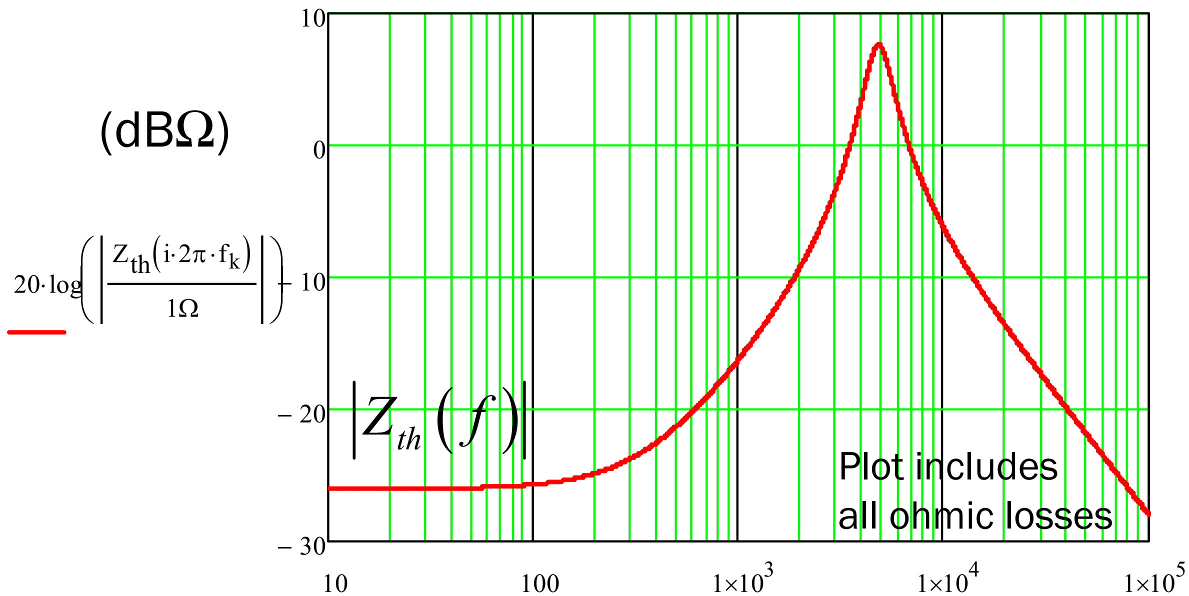
Calculate the New Damping Elements

- Different damping elements are now required

$$n = \frac{R_0\left(R_0 + \sqrt{R_0^{\,2} + 4\left(\left|Z_{out}\right|_{mm}\right)^2}\right)}{\left(\left|Z_{out}\right|_{mm}\right)^2} = 0.5 \qquad Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} = 2.4$$

$\left|Z_{out}\right|_{mm}$ = 3.3 Ω

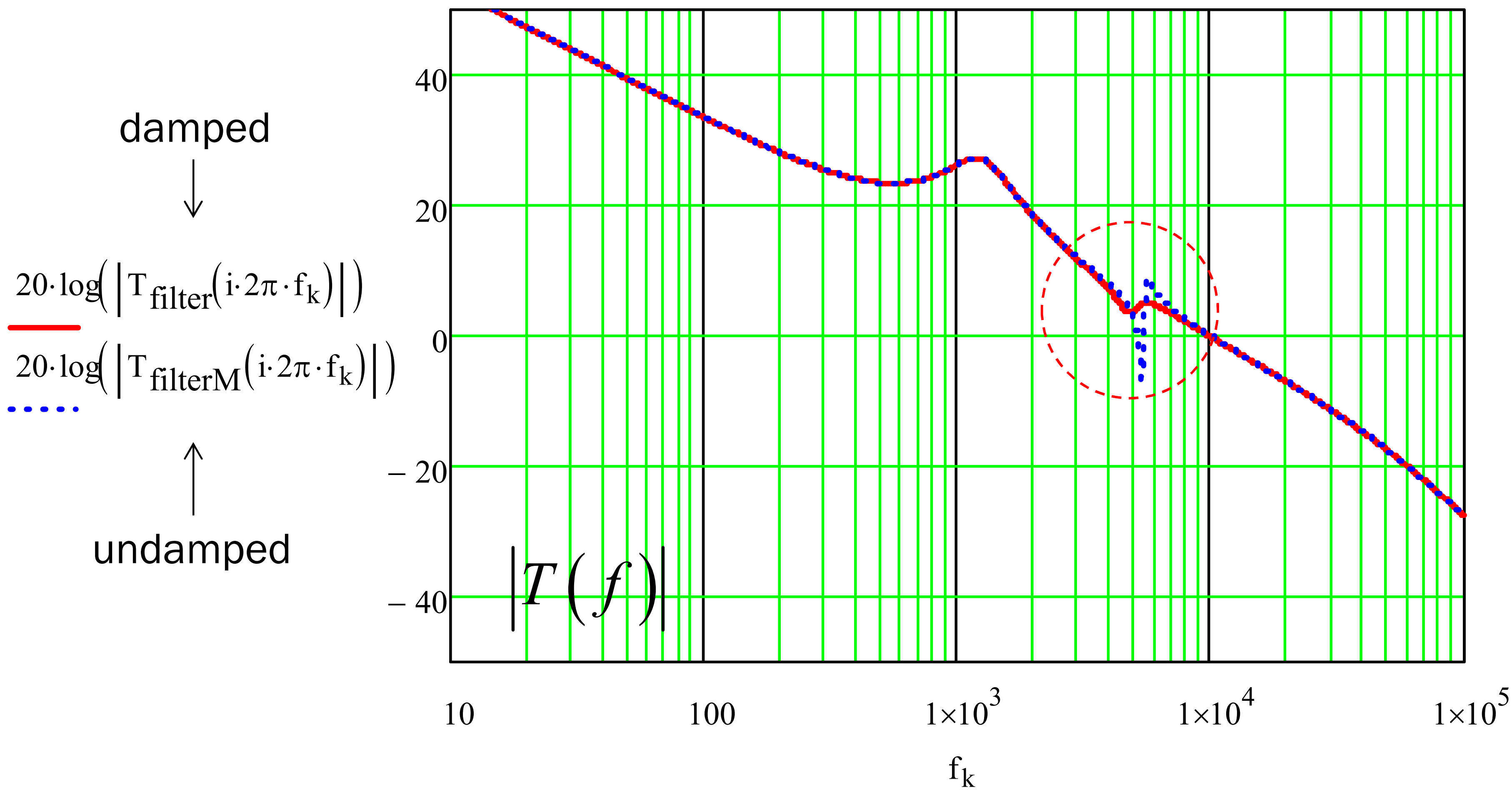
$$R_{damp} = R_0 Q_{opt} = 1.8\ \Omega \qquad C_{damp} = nC_f = 20\ \mu\text{F}$$



Filter output impedance

Verify Results after Damping - Magnitude

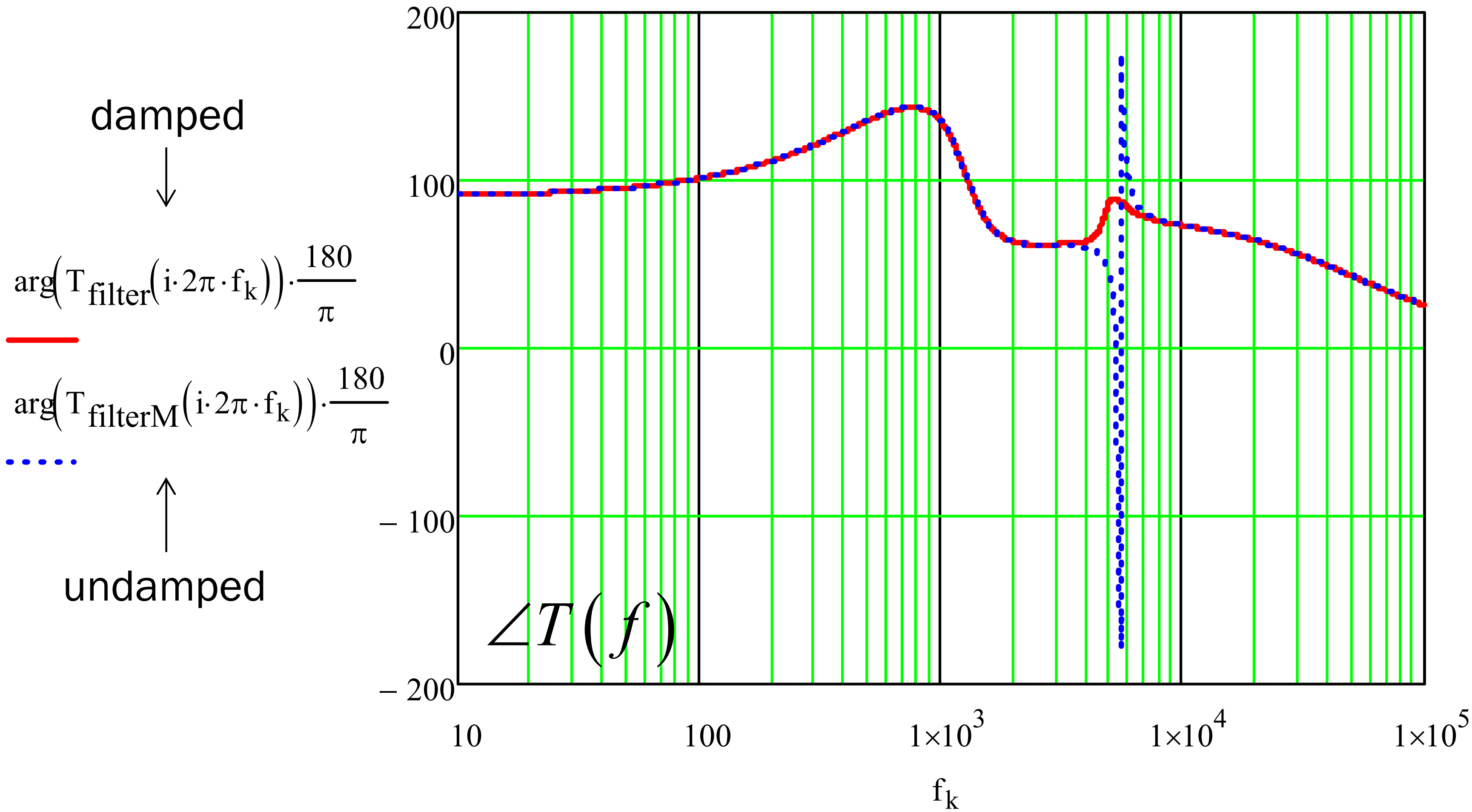
- The new filter effect can be observed when it resonates



- Gain distortion is noticeable before crossover

Verify Results after Damping - Phase

- ❑ Phase distorsion appears but do not jeopardize phase margin



- ❑ Original phase margin is unaffected in this case

Verify Results after Damping - Z_{out}

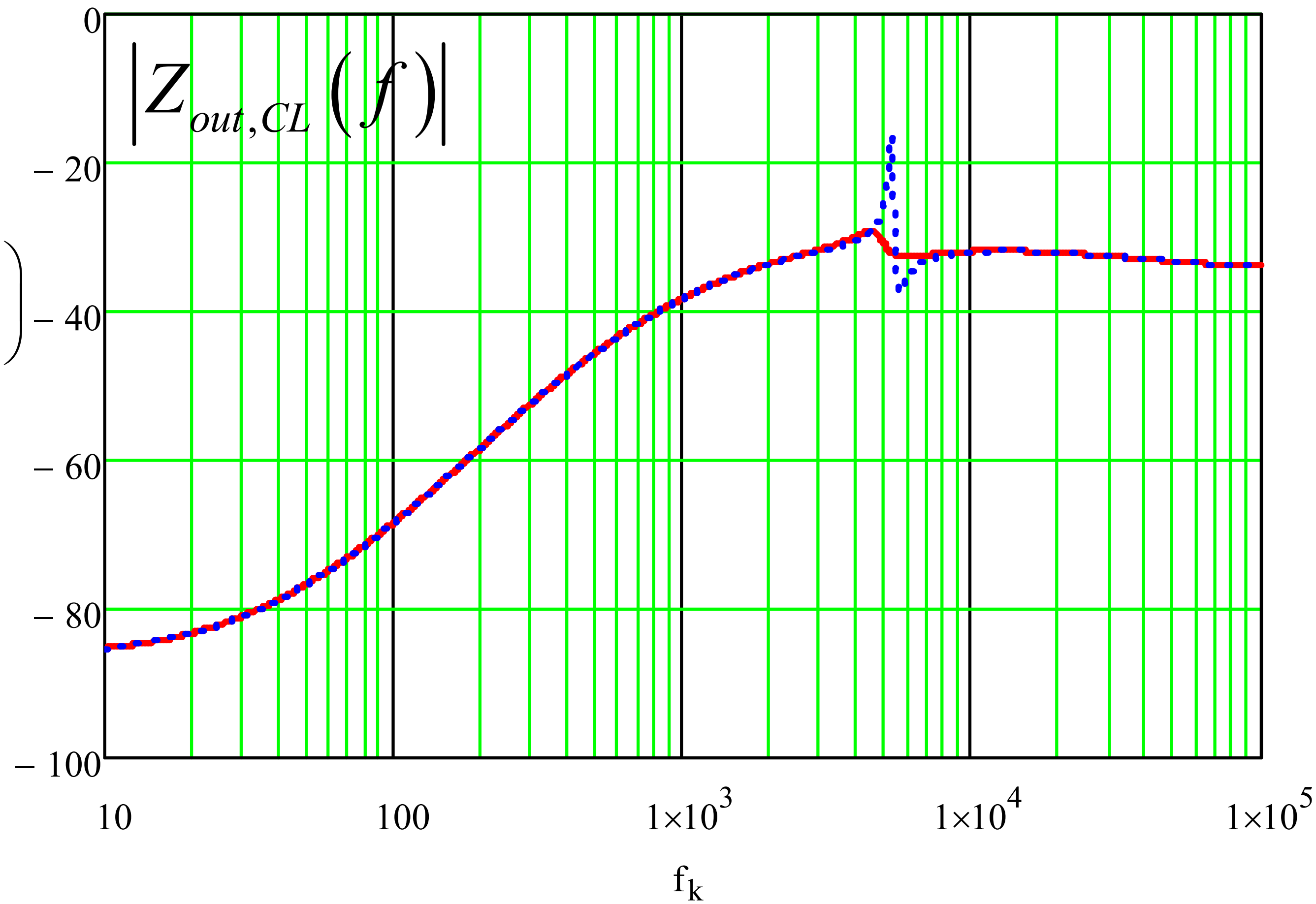
- Peaking can be observed but it remains limited

damped

$$20\cdot\log\left(\left|\frac{Z_{outCLFD}(i\cdot 2\pi\cdot f_k)}{1\Omega}\right|\right)$$

$$20\cdot\log\left(\left|\frac{Z_{outCLF}(i\cdot 2\pi\cdot f_k)}{1\Omega}\right|\right)$$

undamped

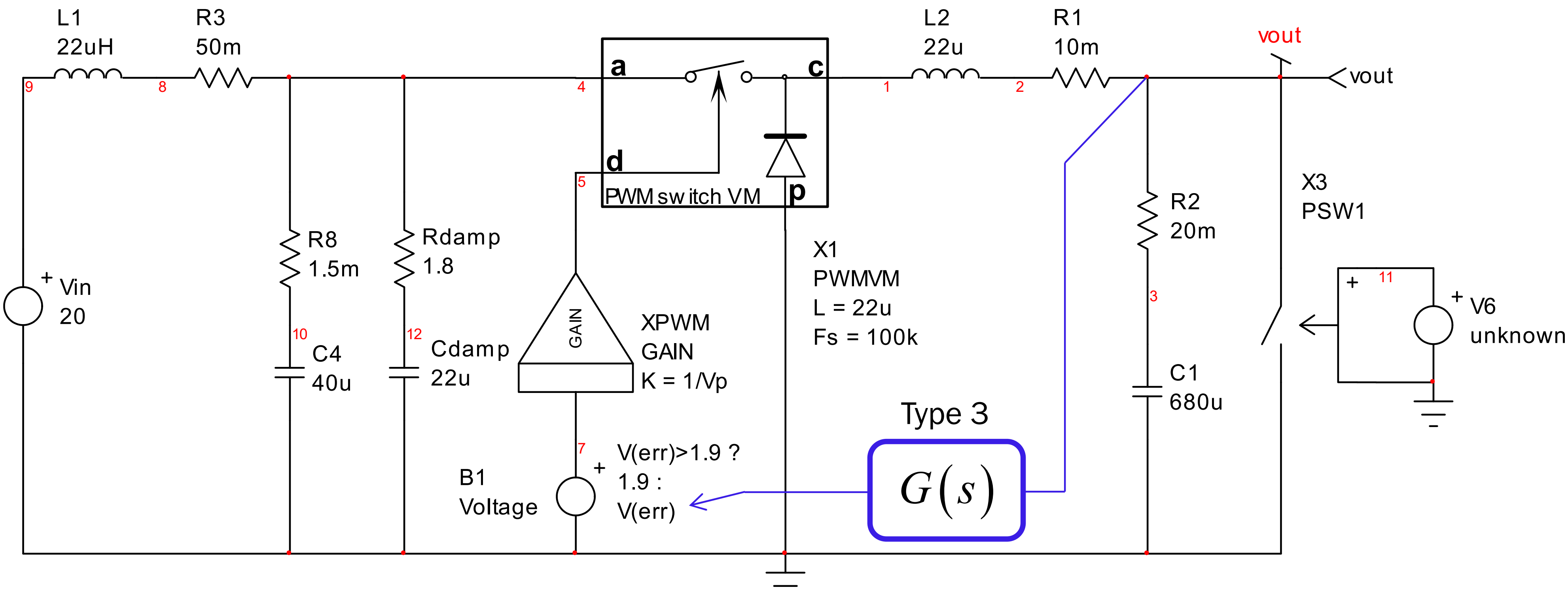


Average Model Simulation Template

- ❑ Both damping strategies can be tested with SPICE

$$R_{damp} = 0.487\ \Omega \qquad R_{damp} = 1.8\ \Omega$$

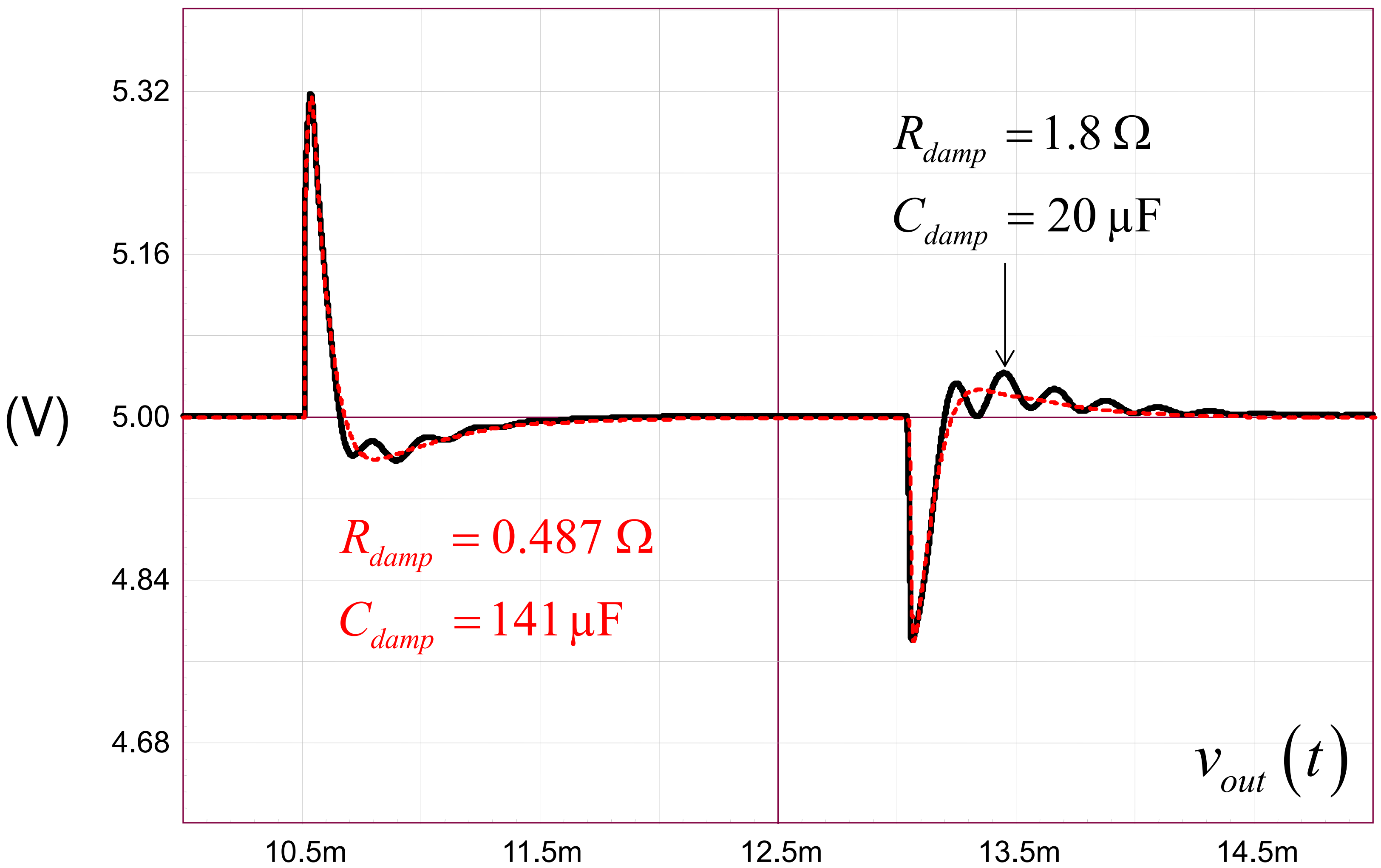
$$C_{damp} = 141\ \mu\text{F} \quad \text{or} \quad C_{damp} = 20\ \mu\text{F}$$



- ❑ The load is stepped from 50 to 100% in 1 µs

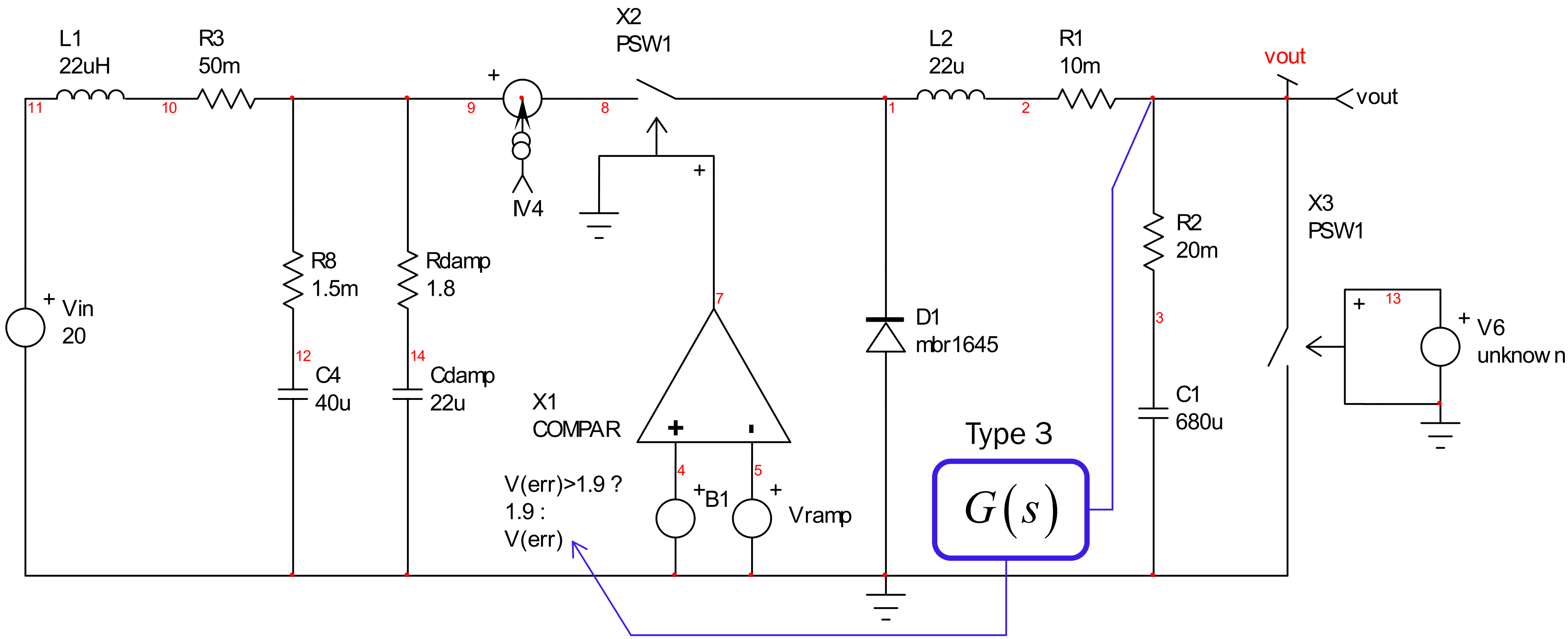
Average Response Shows Oscillations

- Oscillatory but stable response to a load step



Switching Model Simulation Template

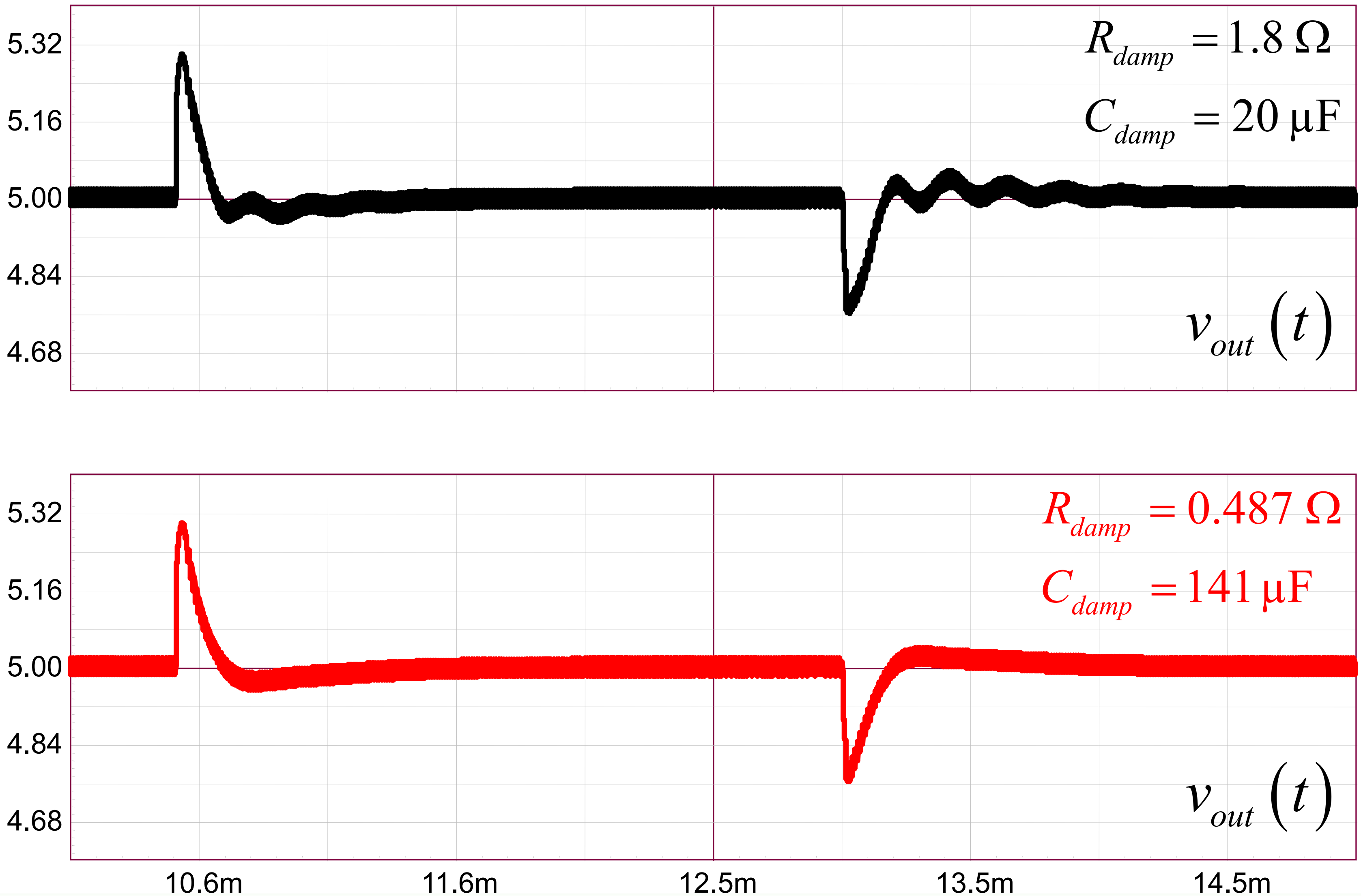
- ❑ This is a simple model compensated by the type III structure



- ❑ You can verify the step response but also the filtered signature

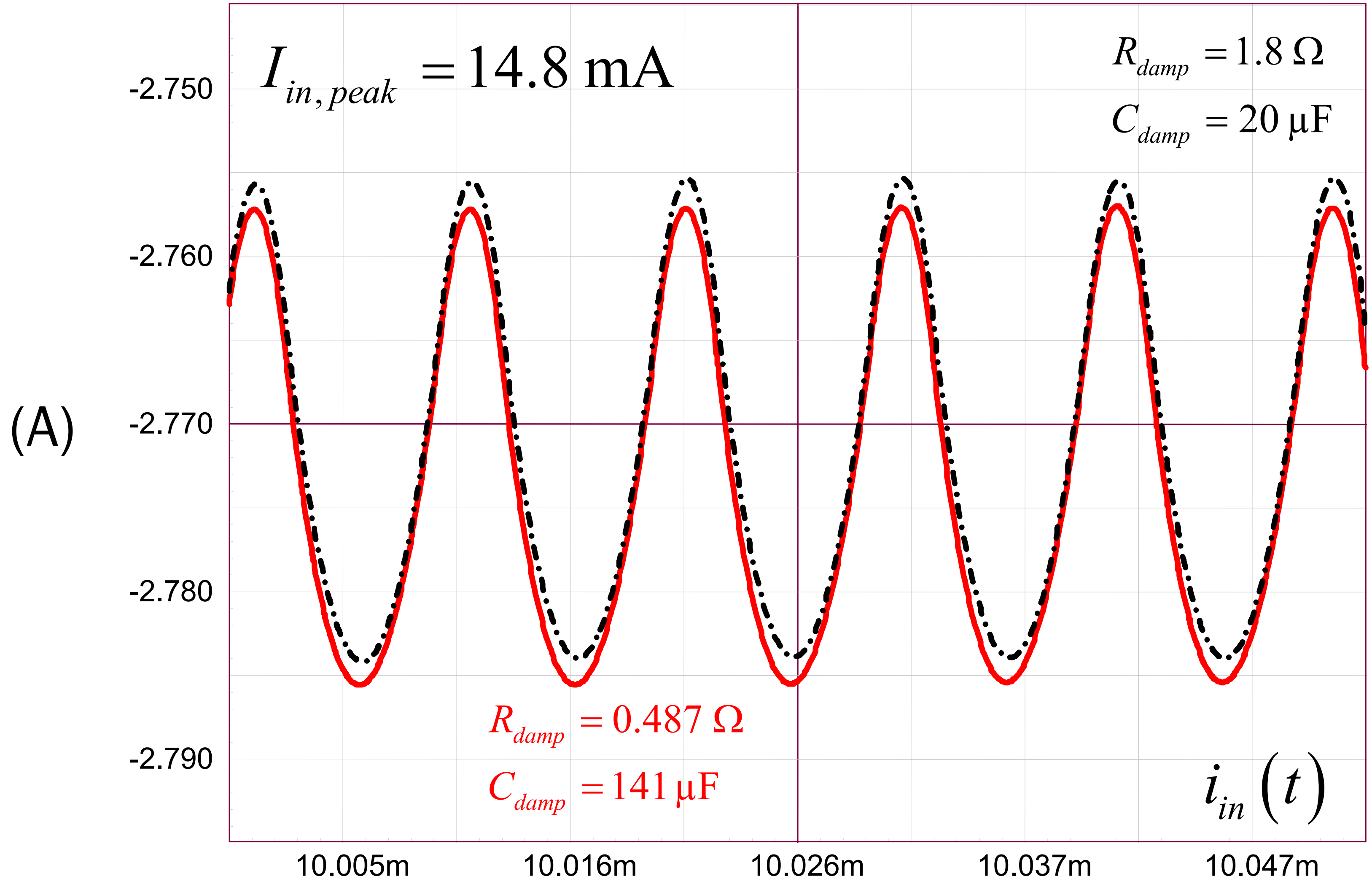
Oscillations in the Switched Model

- Oscillations are present but not too disturbing



Input Current Signature is Good

- No difference in the signature – amplitude is within specs

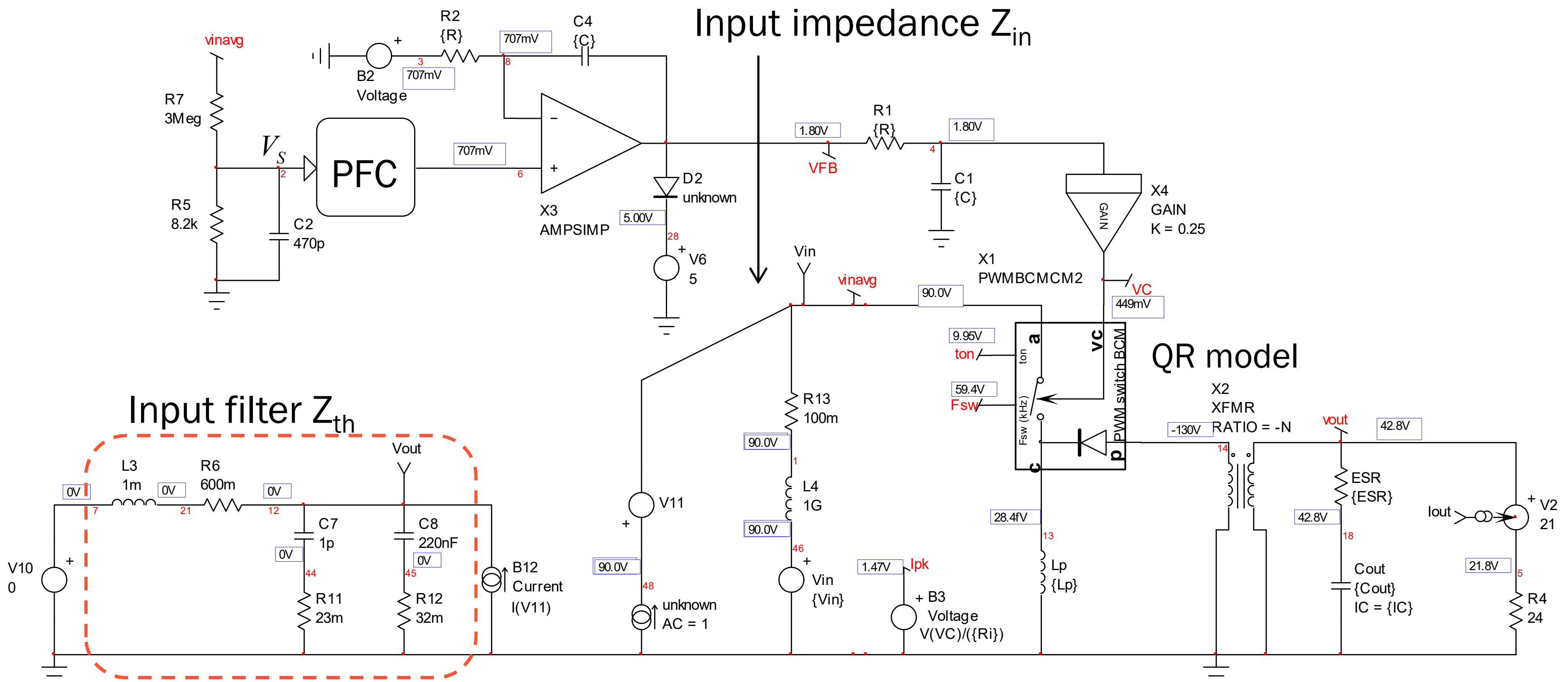


Course Agenda

- ❑ A Switching Regulator as a Load
- ❑ EMI Filter Impact
- ❑ An Introduction to FACTs
- ❑ Buck Converter Input/Output Impedances
- ❑ Filtering the Input Current
- ❑ Damping the Filter
- ❑ Optimum Component Selection
- ❑ **A Practical Case Study**
- ❑ Cascading Converters

A Practical Case with NCL30186

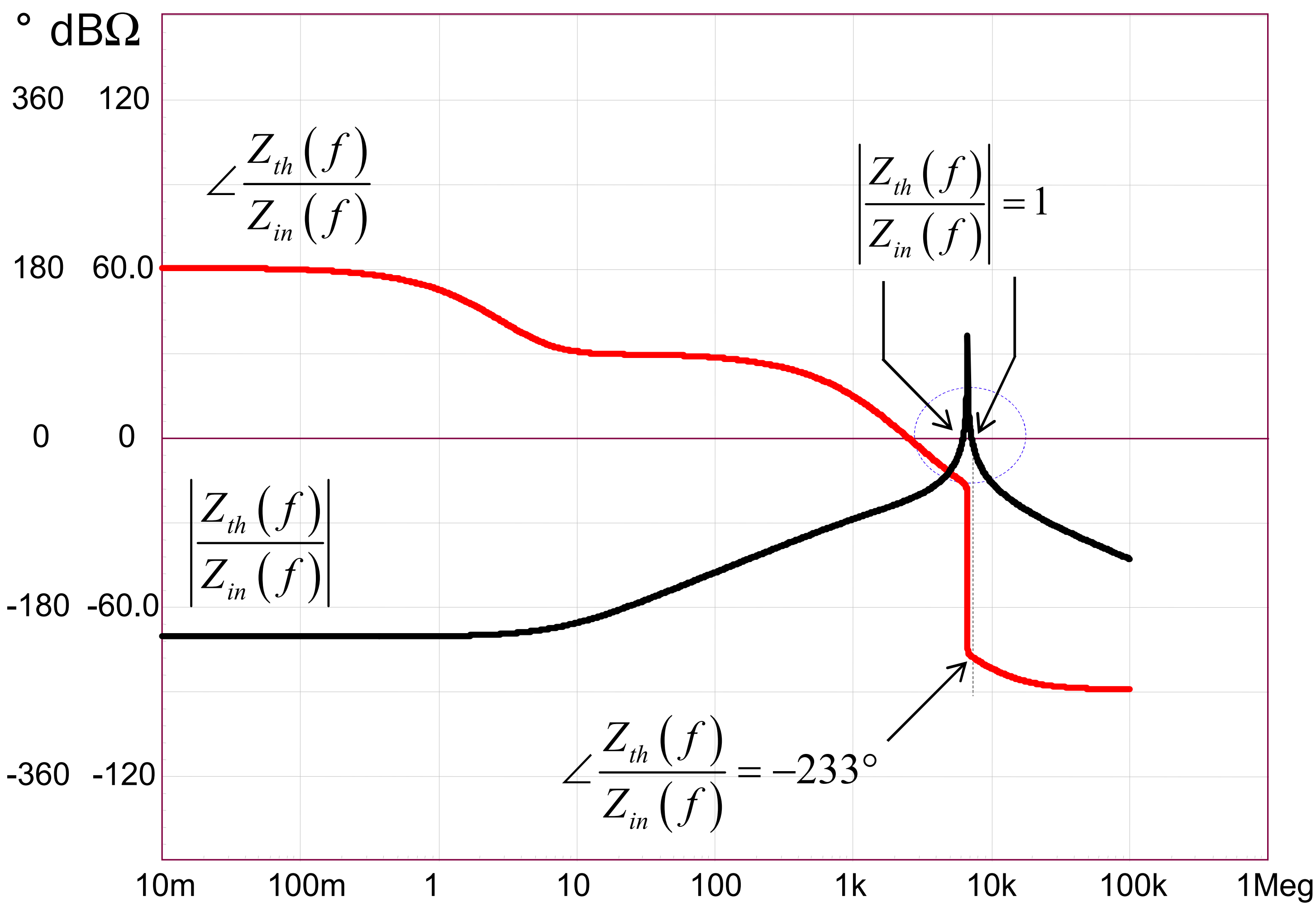
- This LED driver featuring PFC requires the insertion of a filter



Average model by S. Cannenterre

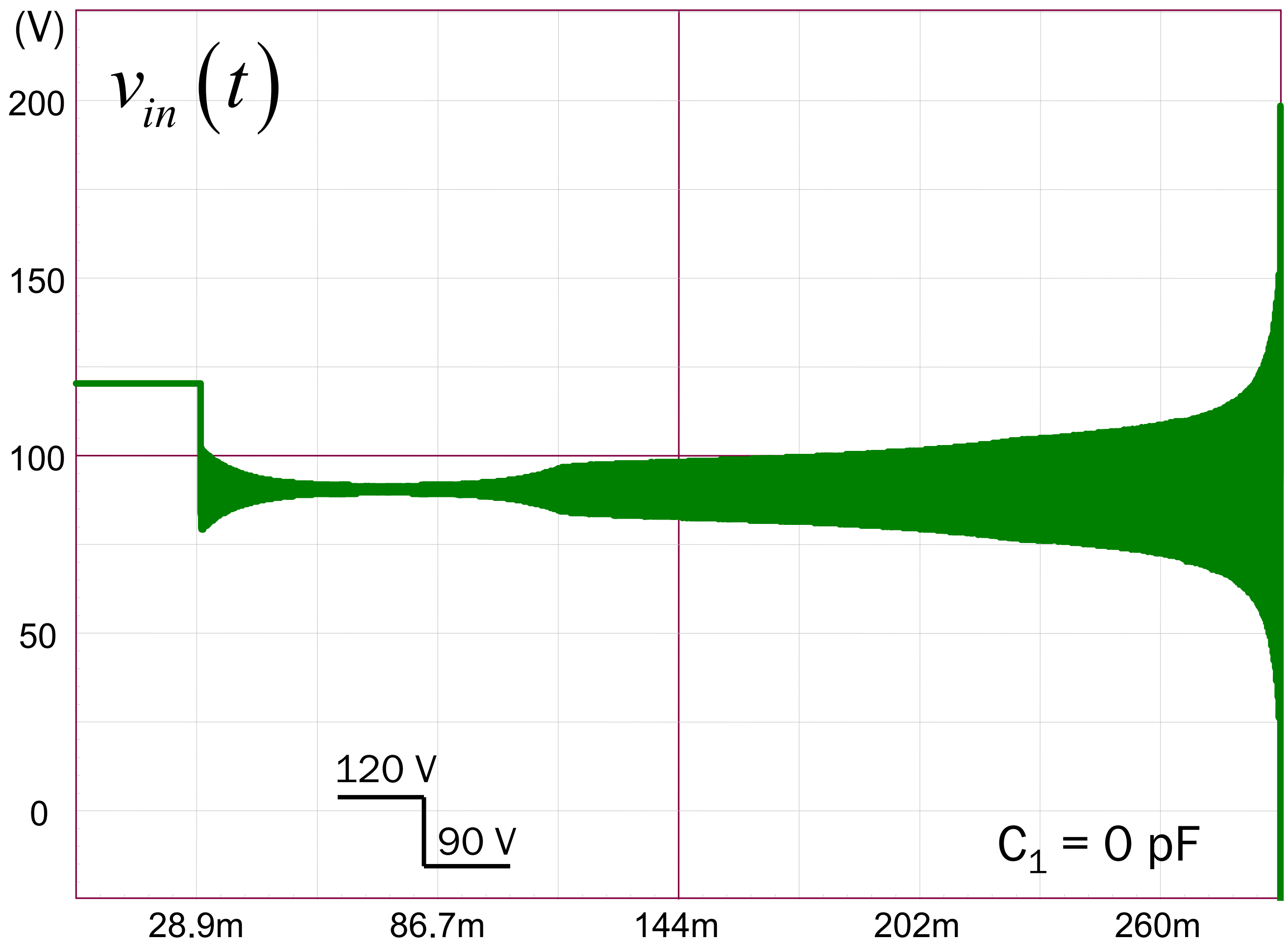
Plot the Impedance Ratio Z_{out}/Z_{in}

- Analysis reveals a negative phase margin: stability issue



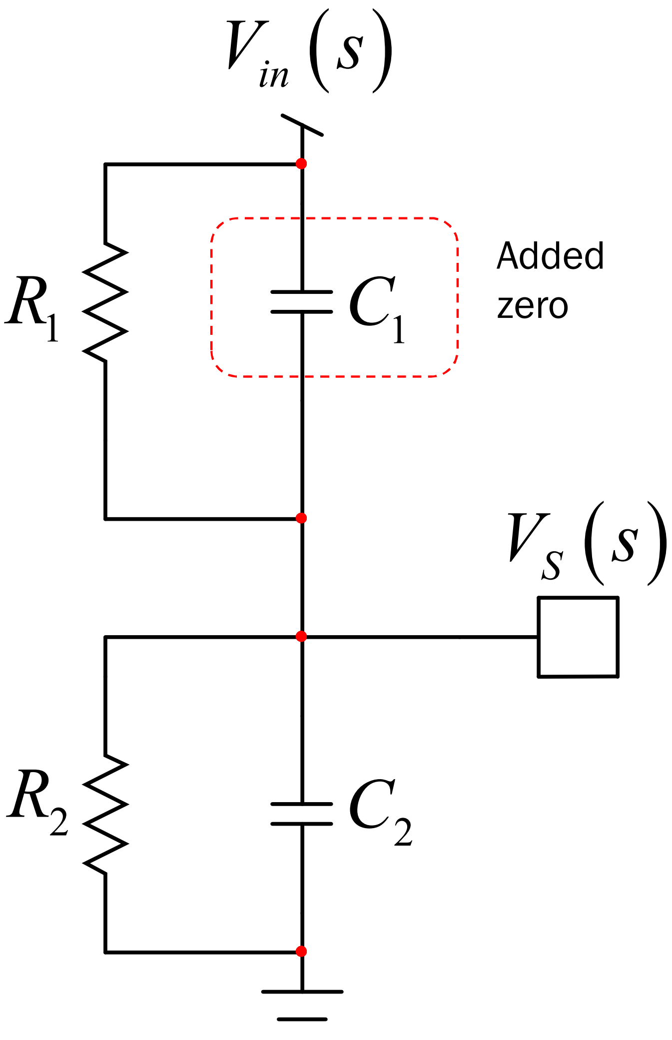
A Simulation Shows Oscillations

- The negative phase margins brings a diverging filter voltage



Act on the PFC Input Voltage

- Reduce the phase stress by inserting a zero in the PFC chain



$$H_0 = \frac{R_2}{R_1 + R_2}$$

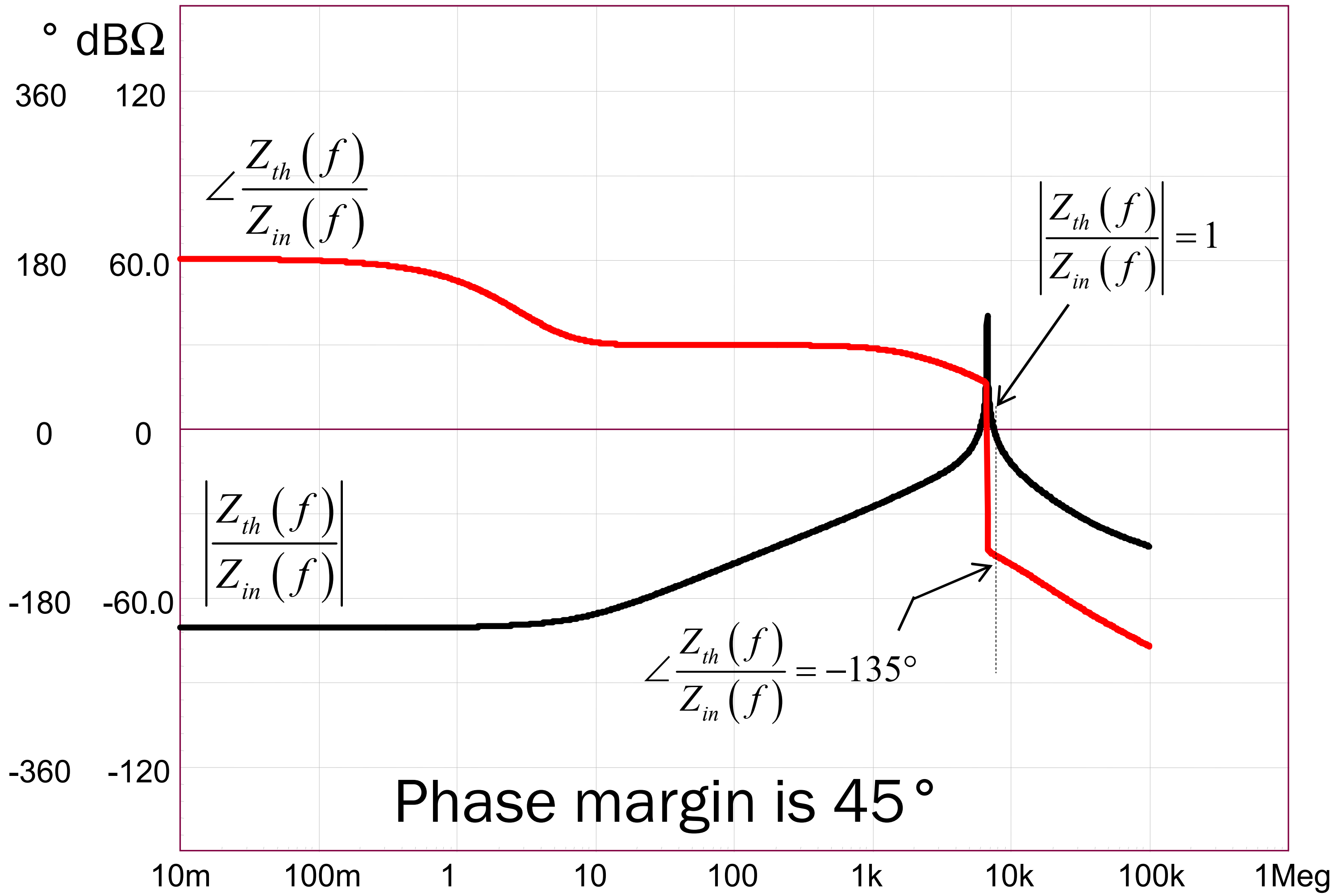
$$\tau_1 = C_1 R_1 \qquad \tau_2 = (R_1 \| R_2)(C_1 + C_2)$$

$$H(s) = H_0 \frac{1 + s\tau_1}{1 + s\tau_2} = H_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}$$

$$\omega_z = \frac{1}{\tau_1} \qquad \omega_p = \frac{1}{\tau_2}$$

Insert the Zero to Reduce Phase Stress

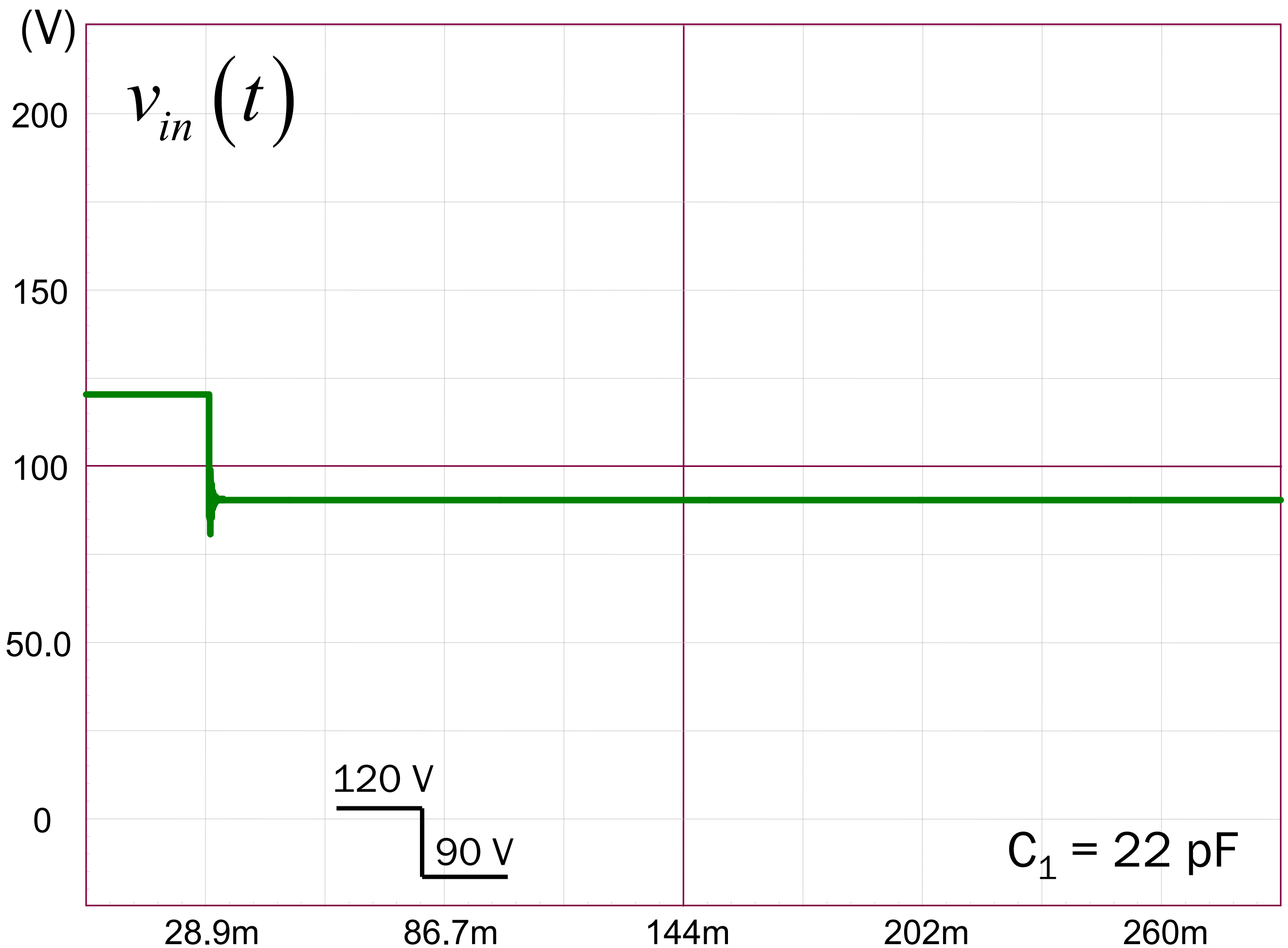
- Adding a zero in the PFC control input brings phase margin



"Interaction between EMI filter and PFC with average current control", G. Piazzi, J. Pomilio, IEEE Transactions on Power Electronics, 1999

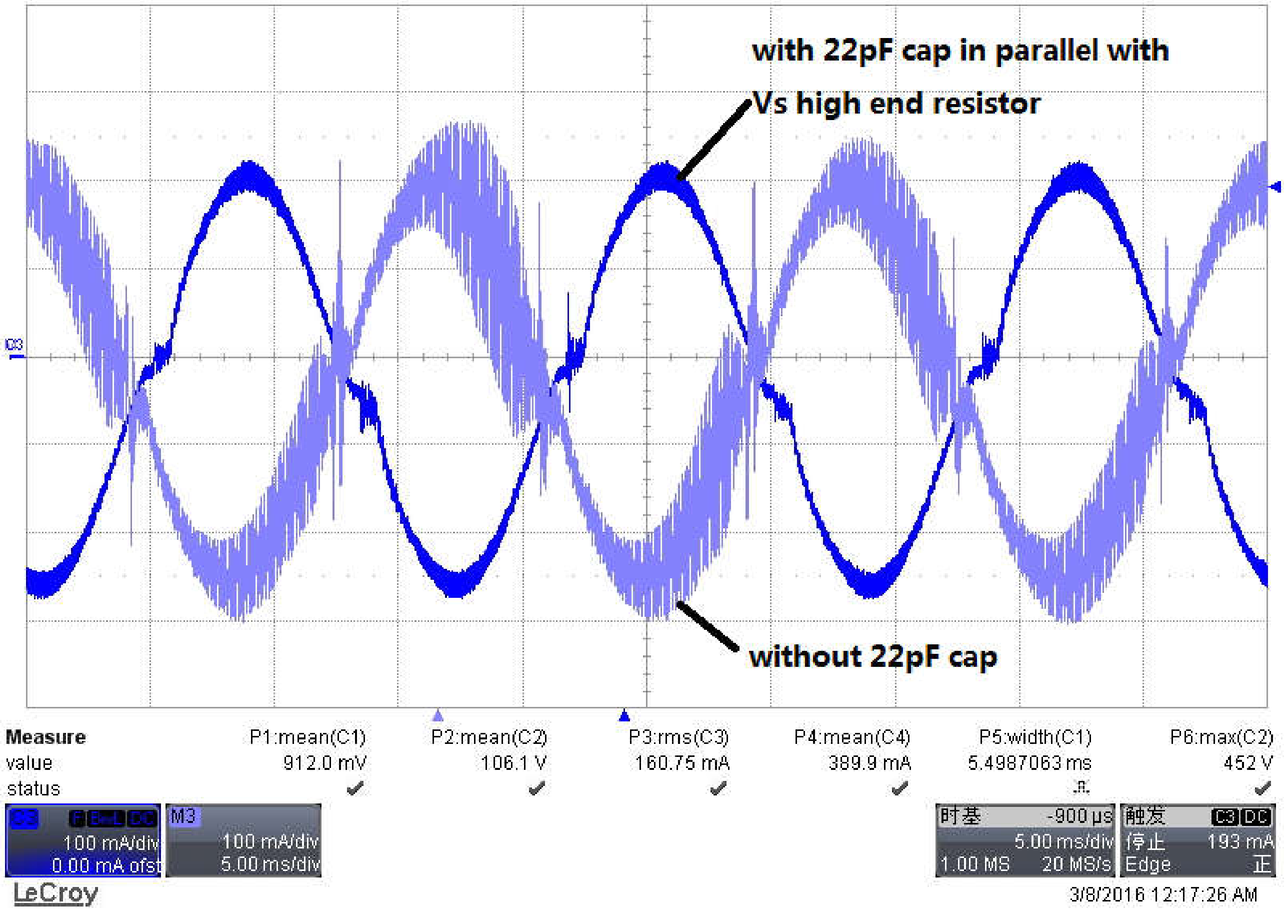
The Zero Insertion Cancels Oscillations

- Adding a zero builds phase margin and tames oscillations



Practical Experiments

- ❑ Placing a zero at 2.5 kHz (22-pF cap.) stops oscillations



- ❑ However, PFC operations are affected: need to damp the filter!

Explore Another Option and Damp Filter

$$R_0 = \sqrt{\frac{L_1}{C_3}} \qquad \frac{\left|Z_{out}\right|_{mm}}{R_0} = \sqrt{\frac{2(2+n)}{n^2}} \longrightarrow n = \frac{R_0\left(R_0 + \sqrt{R_0^{\ 2} + 4\left(\left|Z_{out}\right|_{mm}\right)^2}\right)}{\left(\left|Z_{out}\right|_{mm}\right)^2}$$

$$Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} \qquad R_{damp} = R_0 Q_{opt}$$

$\left|Z_{out}\right|_{mm}$ This is the minimum of Z_{out} peaking to avoid overlap

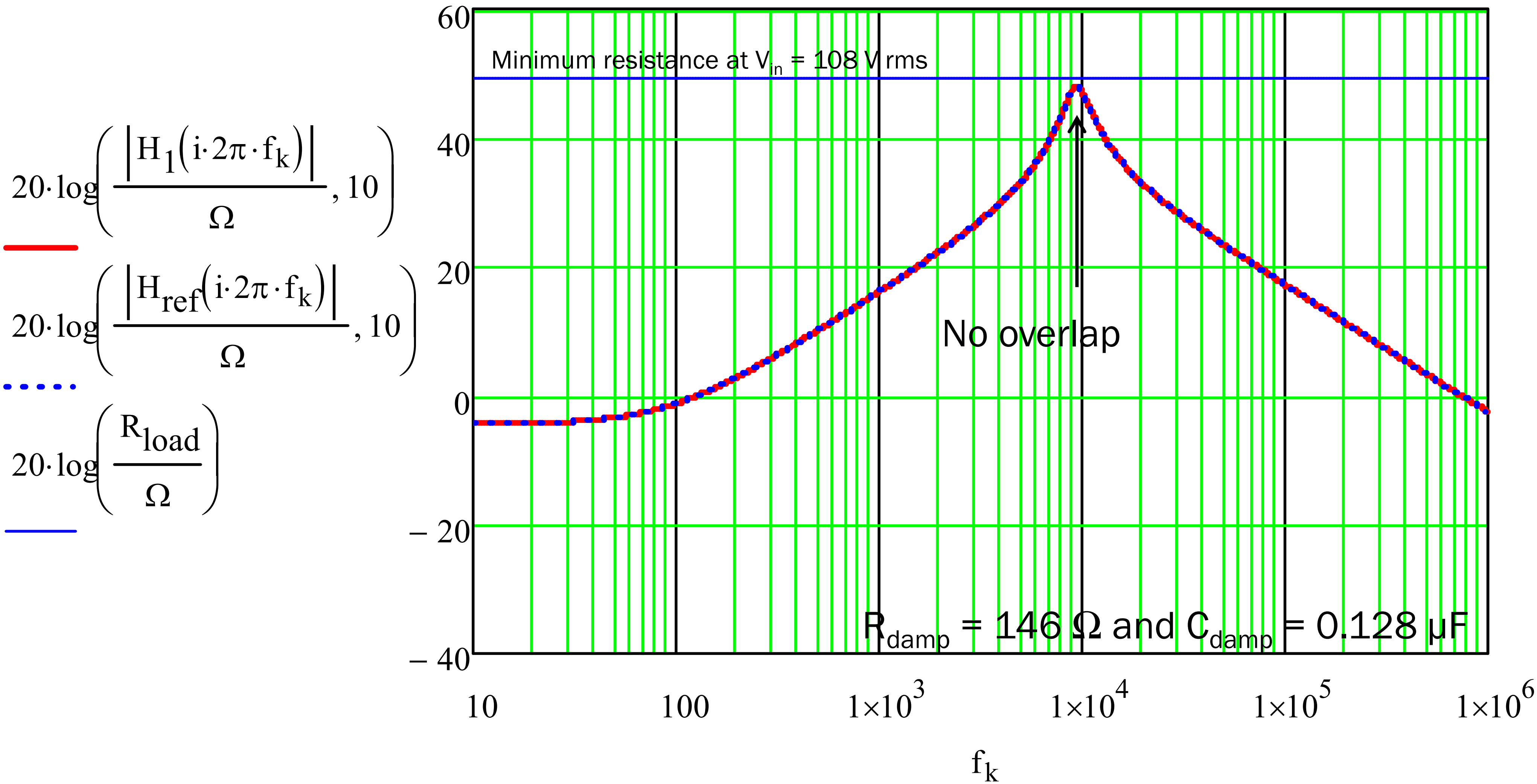
$\longrightarrow \left|Z_{out}\right|_{mm} = k \cdot \frac{V_{in}^{\ 2}}{P_{out}}$ For a 40-W output and a 108-V input:

$\longrightarrow \left|Z_{out}\right|_{mm} = 291\ \Omega$ Assume 10% margin, k = 0.9 $\left|Z_{out}\right|_{mm} = 262\ \Omega$

$$\left.\begin{aligned} n &= 0.584 \\ Q_{opt} &= 2.18 \end{aligned}\right\} \quad \boxed{\begin{aligned} C_{damp} &= n \cdot 220\text{ nF} = 0.128\ \mu\text{F} \\ R_{damp} &= R_0 Q_{opt} = 147\ \Omega \end{aligned}}$$

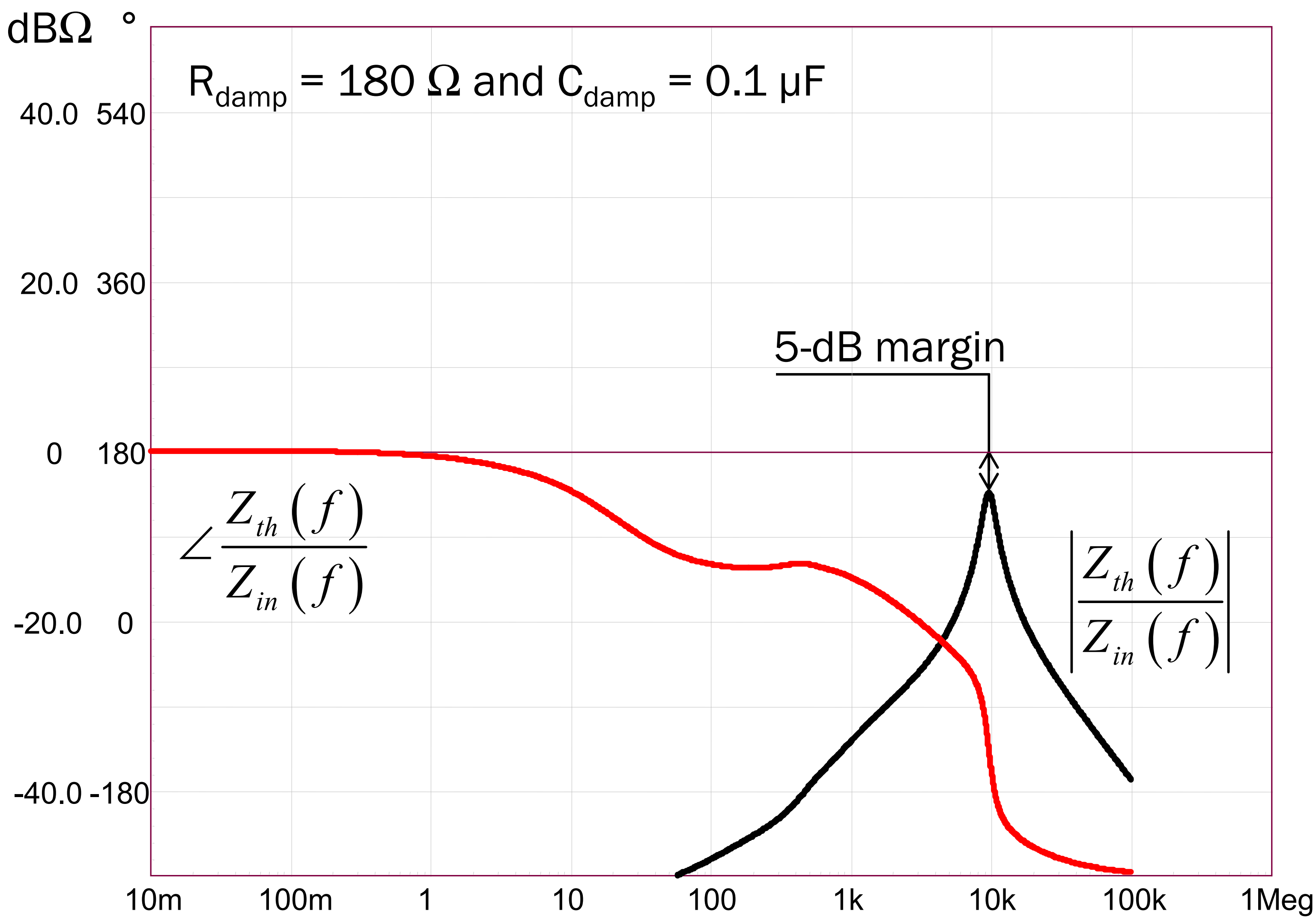
Plots with L_1 = 1 mH and C_3 = 220 nF

- ❑ Check absence of overlap with Mathcad ®



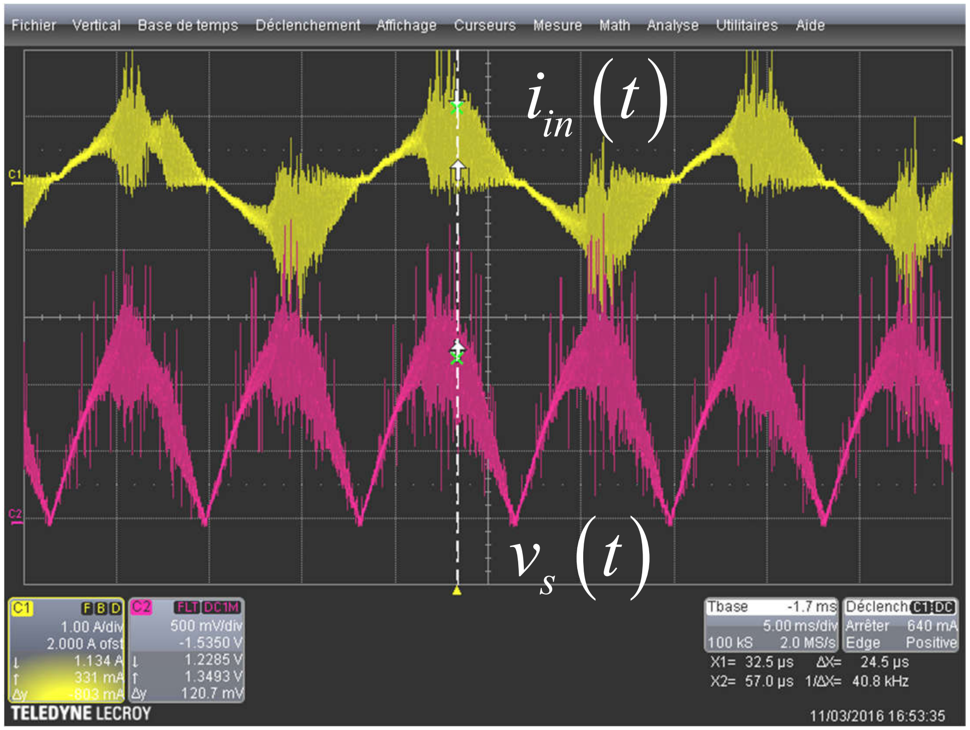
Check Results on the Simulation Template

- Simulation results V_{in} = 108 V rms P_{out} = 40 W: no overlap

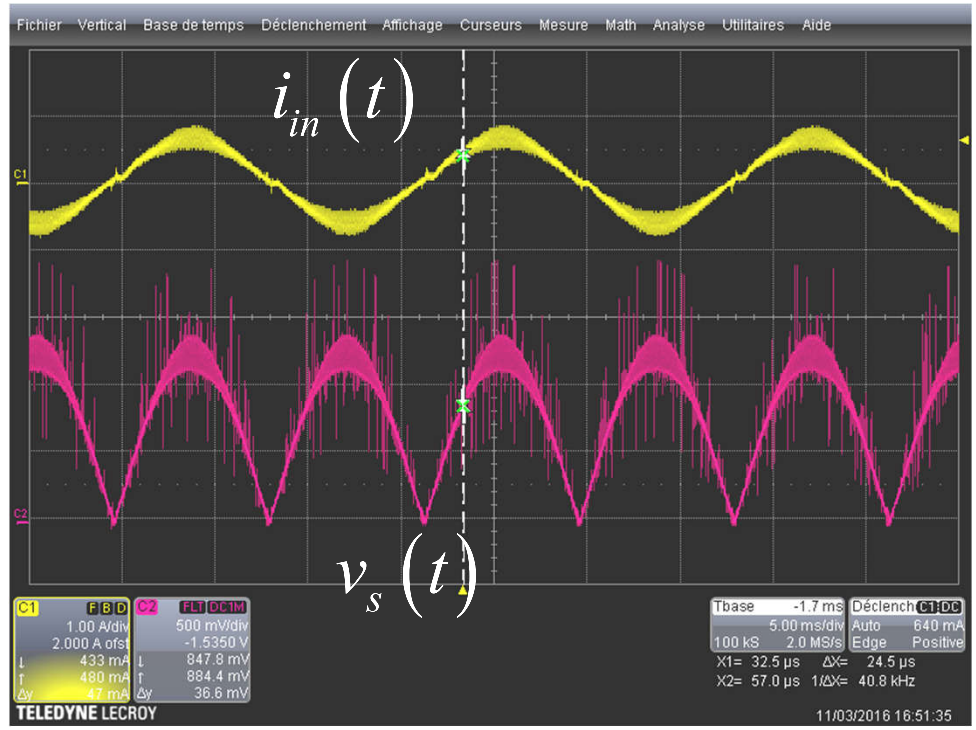


Practical Implementation in the Driver

- Final Q is affected by several other factors:
  - PCB traces, dielectric losses in the caps but also iron losses in the inductor.
  - True peaking is often lower and gives design margin.



No damping
7.5 mH and 220 nF (no C_2)
after bridge. P_{out} > 40 W
V_{in} = 120 V rms



Damping 100 nF + 180 Ω
7.5 mH and 220 nF (no C_2)
after bridge. P_{out} > 40 W
V_{in} = 120 V rms

Curves and experiments by J. Turchi

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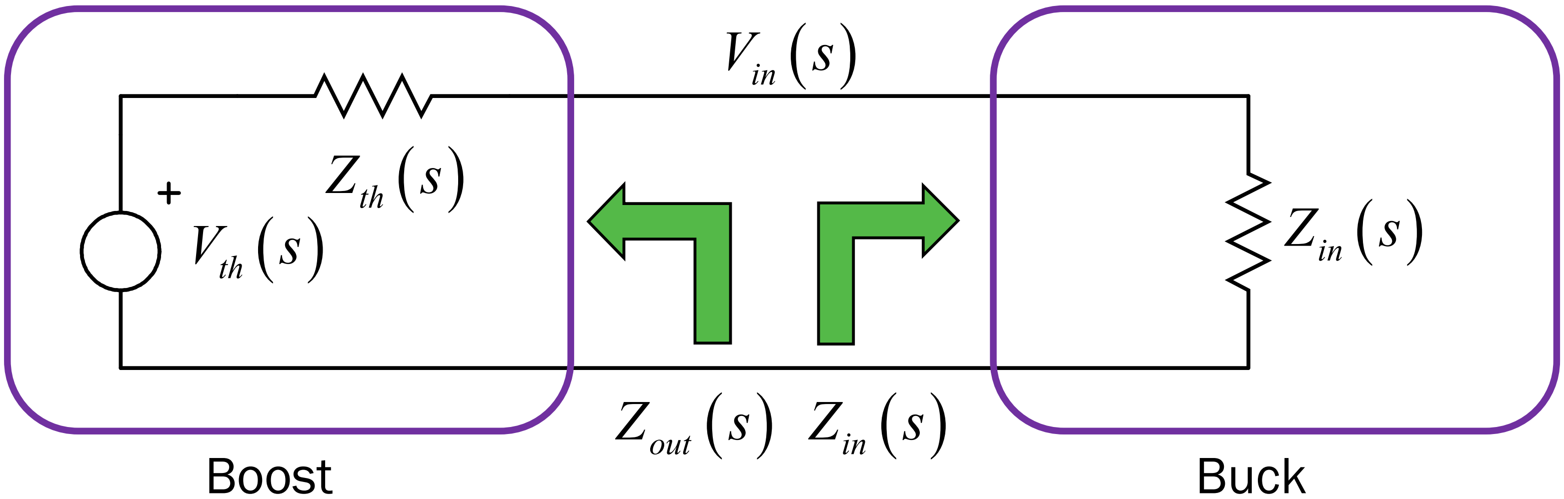
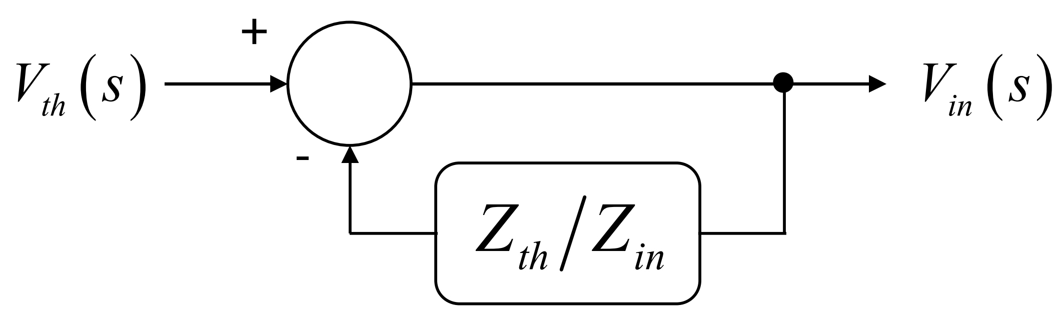
Cascading Converters

- When cascading converters, impedances matter



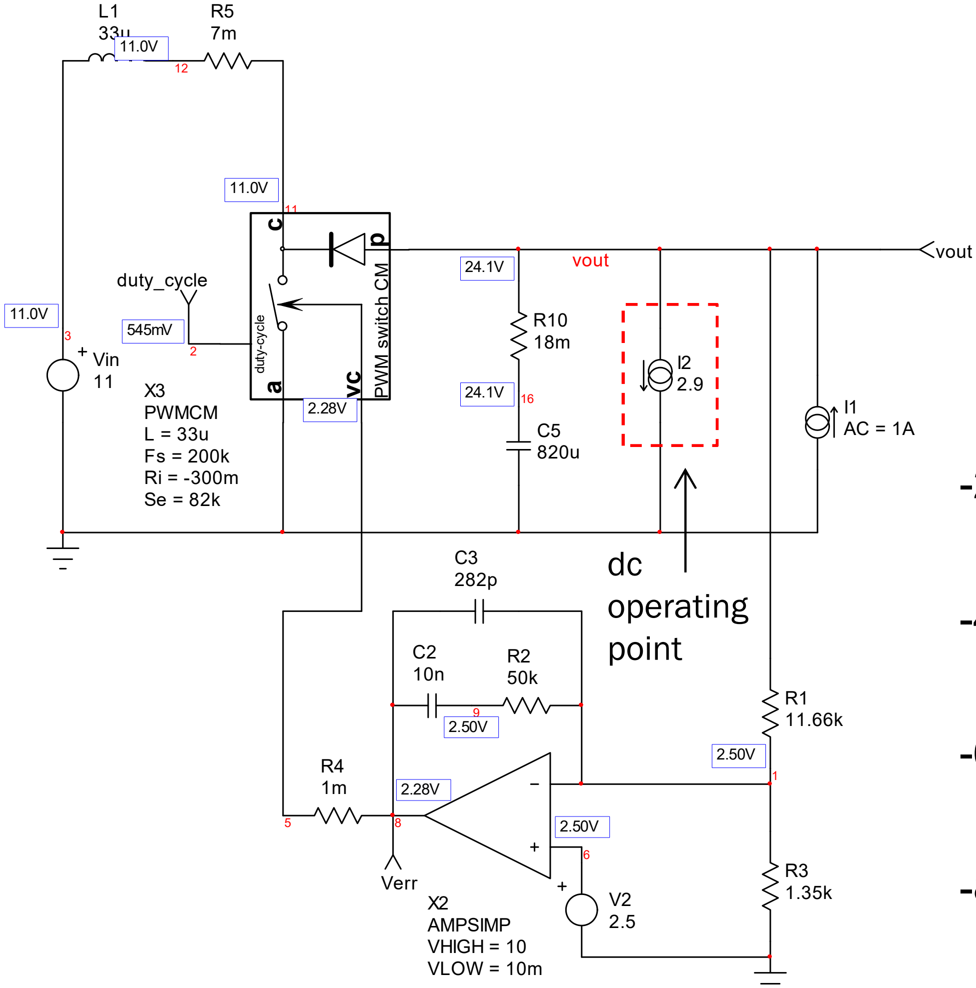
- The system can be also be modeled by a gain T_M

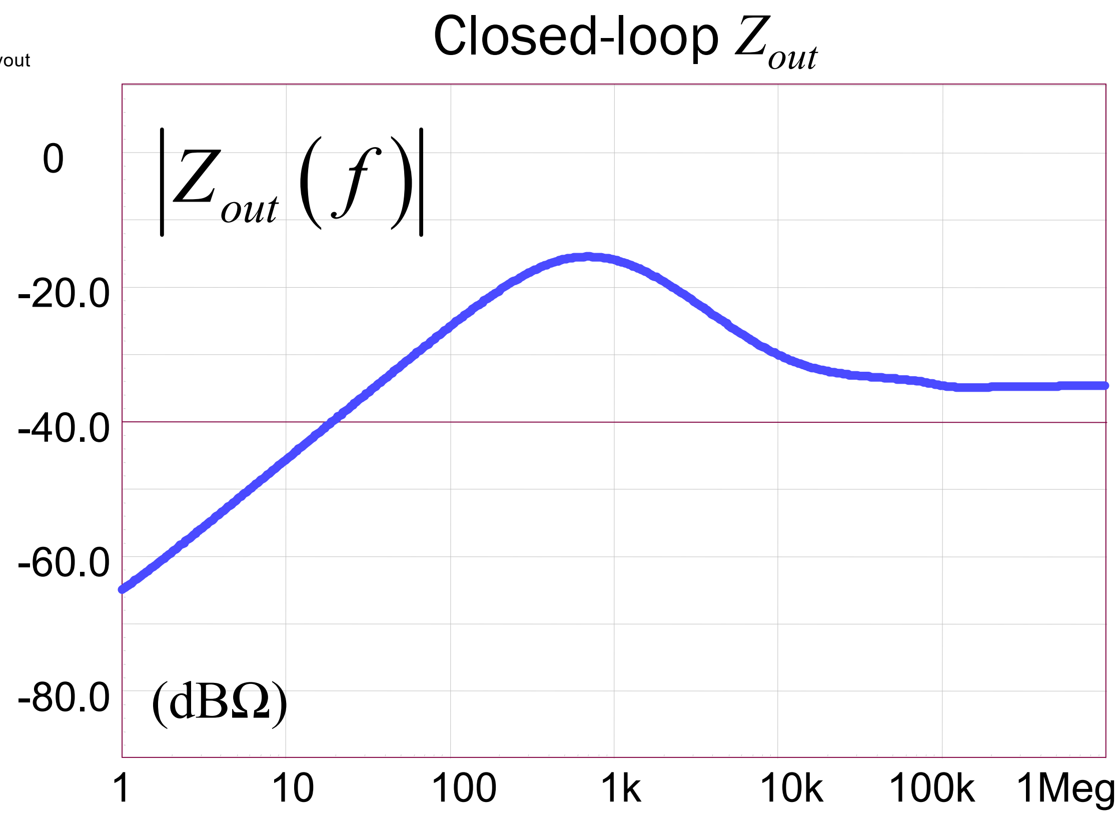
$$V_{in}(s) = V_{th}(s)\frac{1}{1+\underbrace{\boxed{\frac{Z_{th}(s)}{Z_{in}(s)}}}_{T_M(s)}}$$



Use SPICE Models First

- Simulate the boost converter output impedance



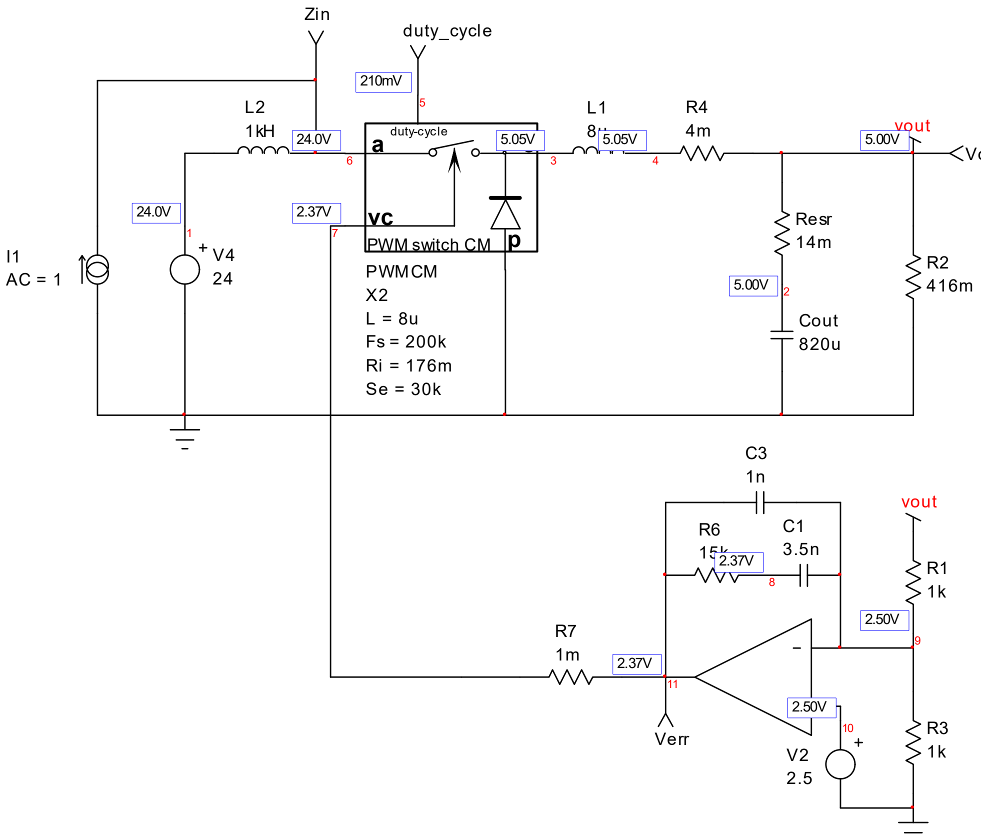


11-15 V/24 V – 3 A

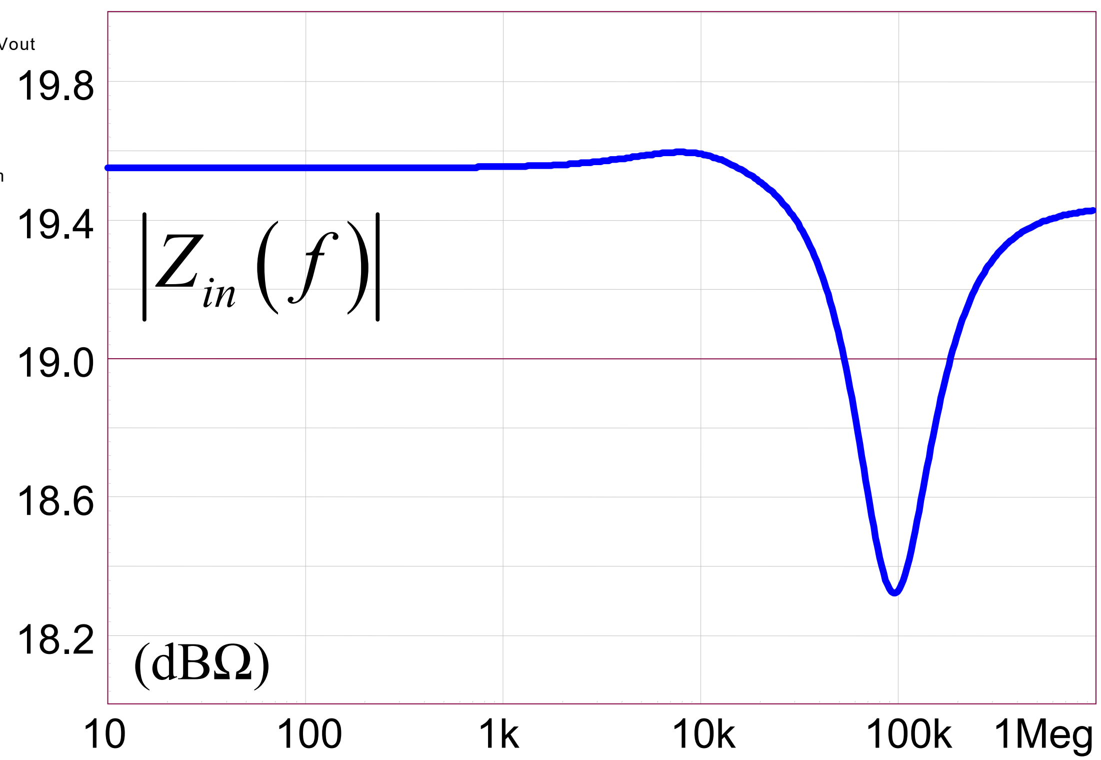
A current source – and not a resistance – ensures the correct bias point for loading the converter during the ac sweep.

Use SPICE Models First

- ❑ Simulate the buck converter input impedance

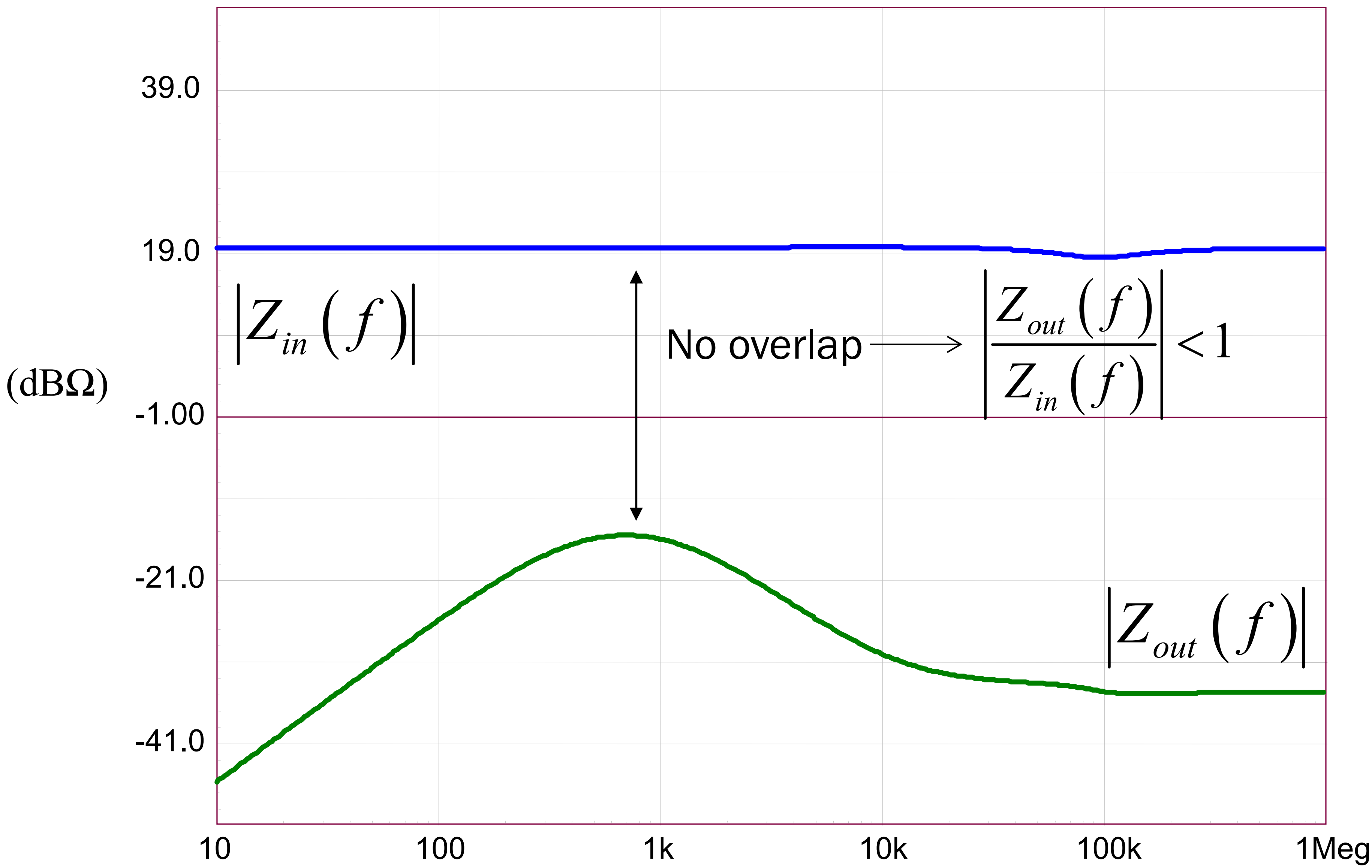


Closed-loop Z_{in}

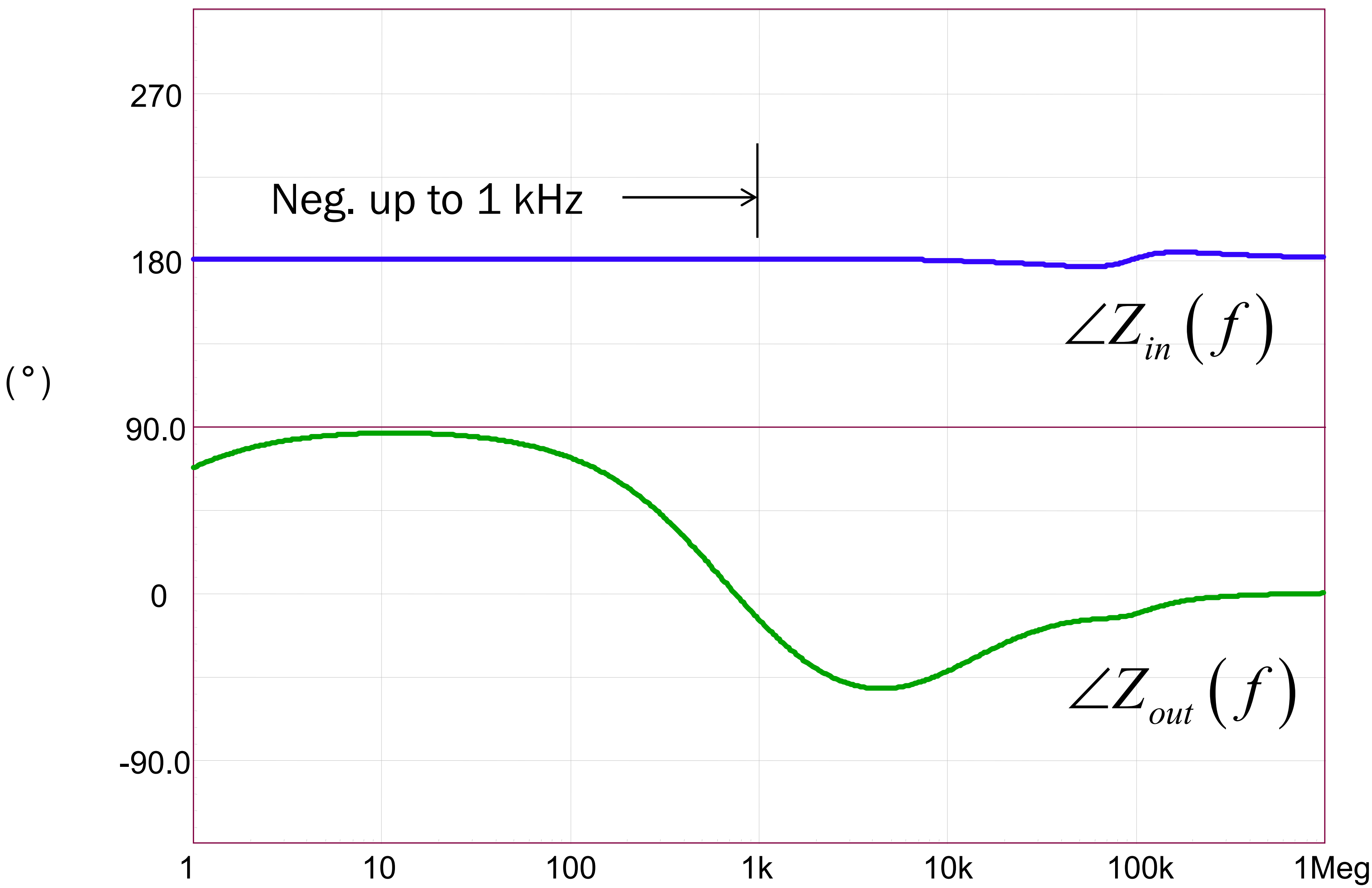


24 V/5 V – 12 A

Compare Magnitude Curves

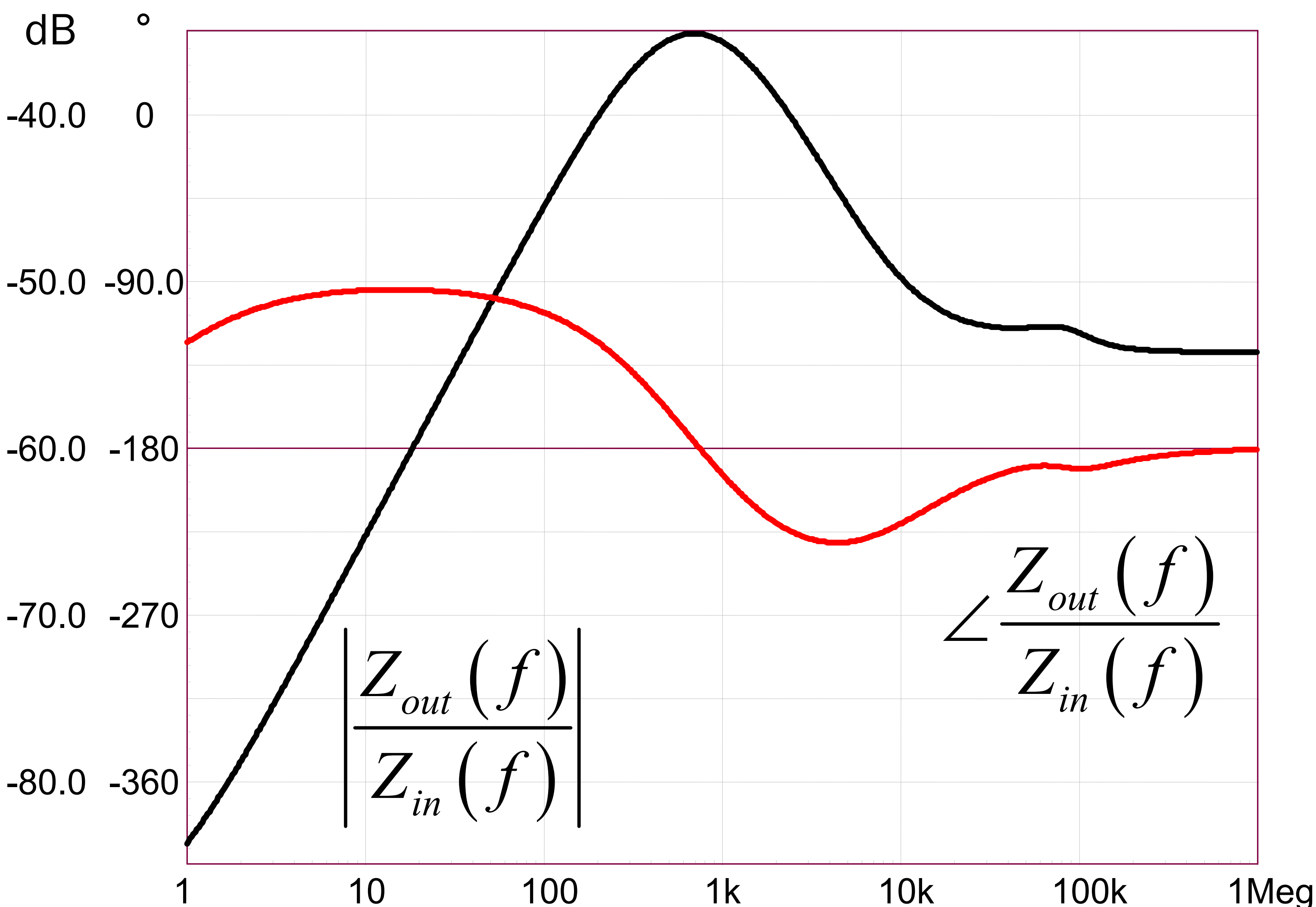


Compare Phase Curves

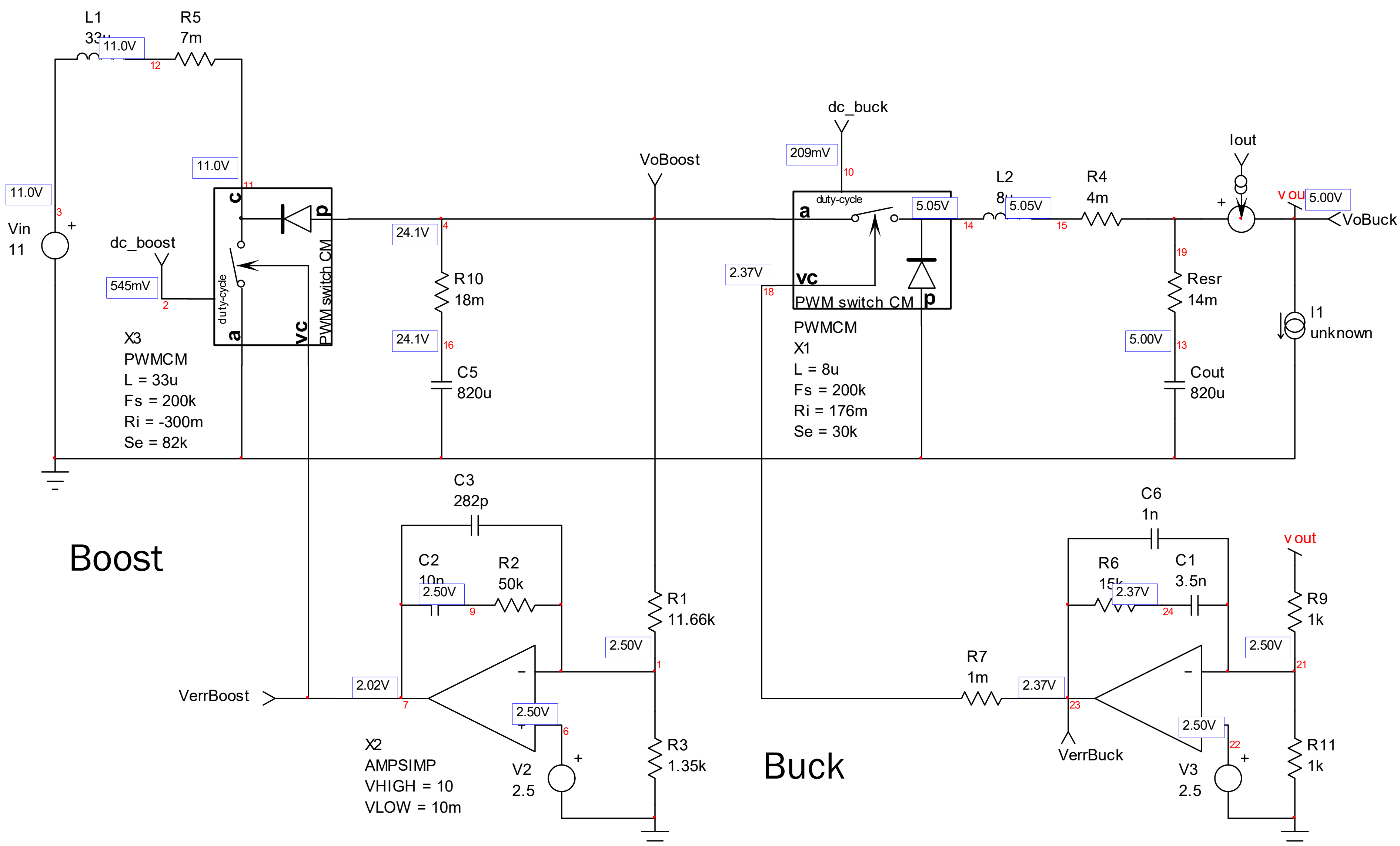


Check Minor Loop Gain

- There is no gain, $\left|Z_{out}\right| << \left|Z_{in}\right|$. The system is stable

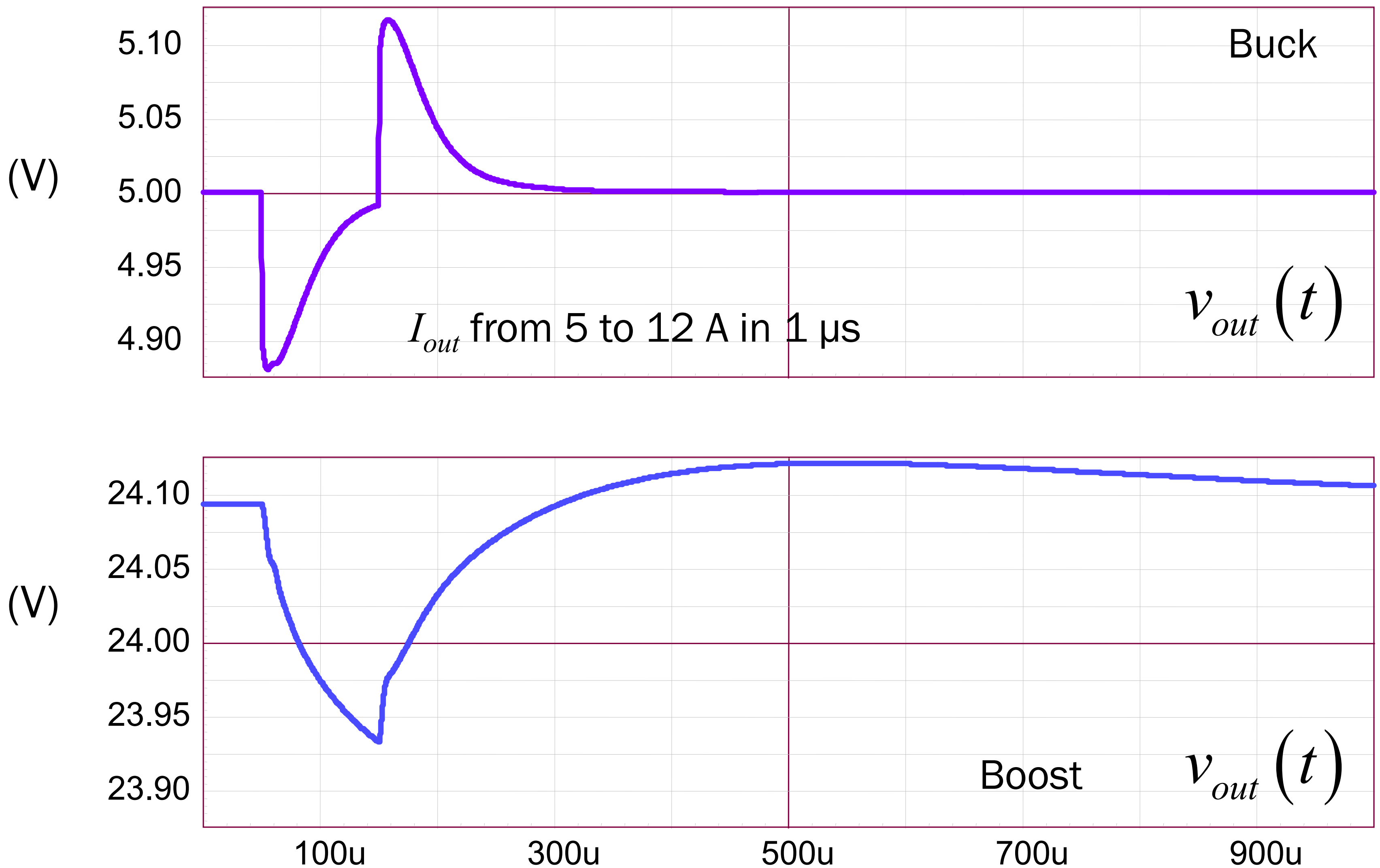


Cascade Converters for Simulation



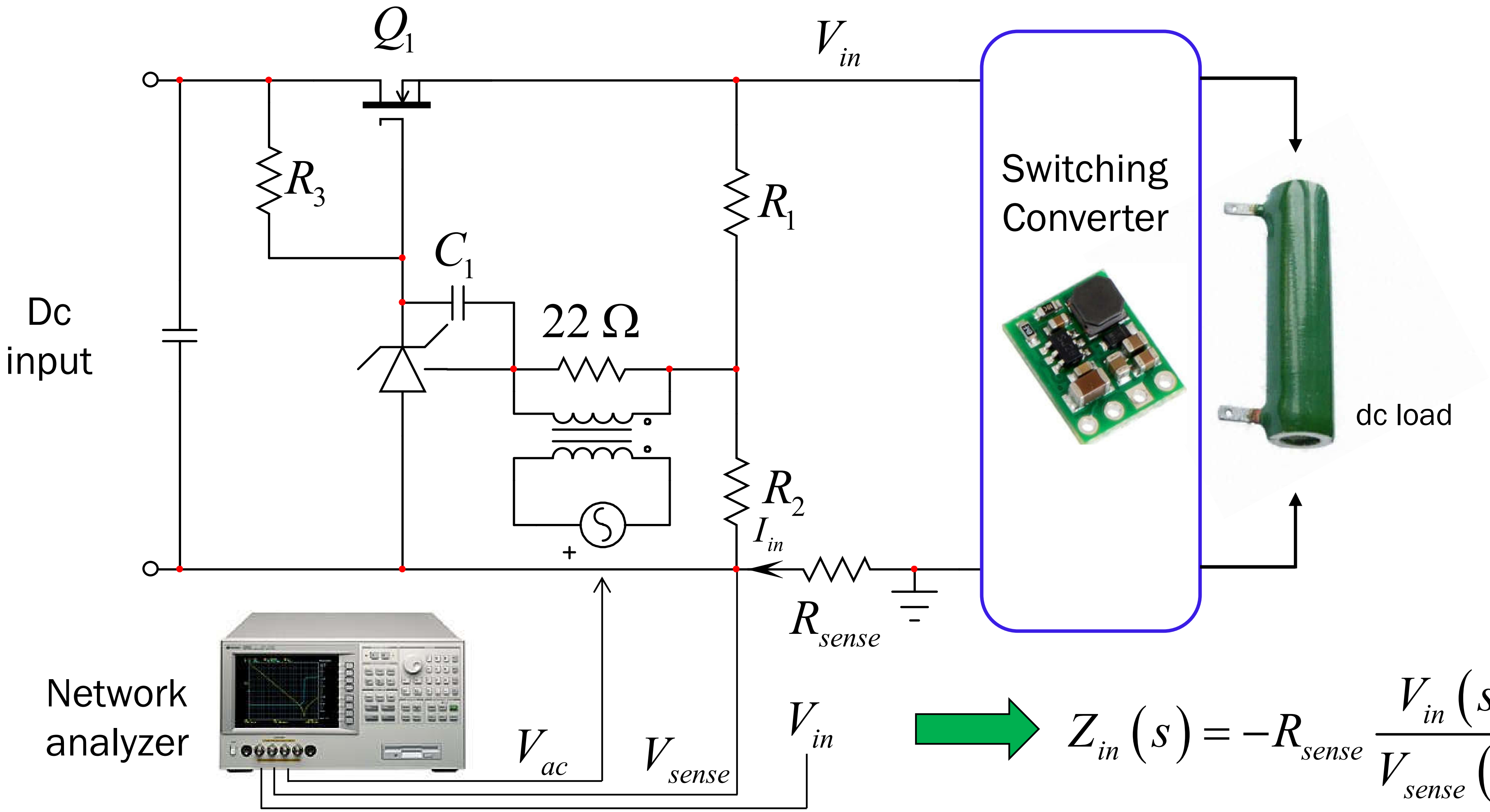
Check Transient Response

- Transient response is good for the buck and boost is stable



Practical Measurements – Z_{in}

- Input impedance measurement requires a dedicated circuit

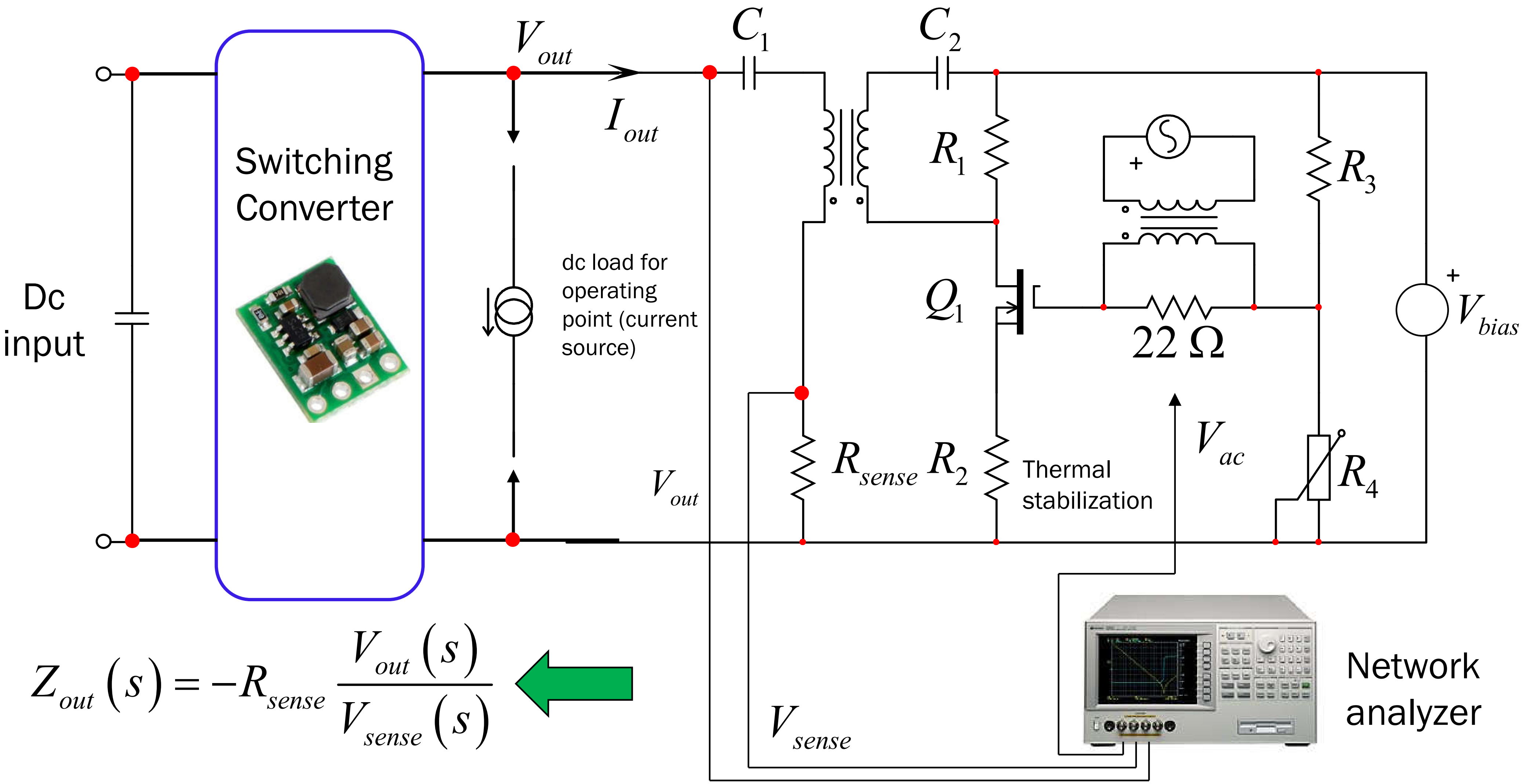


$$Z_{in}(s) = -R_{sense}\frac{V_{in}(s)}{V_{sense}(s)}$$

"Practical Issues of Input/Output Impedance Measurements", Y. Panov, M. Jovanović, IEEE Transactions on Power Electronics, 2005

Practical Measurements – Z_{out}

❑ You need to inject enough current to observe a modulated V_{out}



$$Z_{out}(s) = -R_{sense}\frac{V_{out}(s)}{V_{sense}(s)}$$

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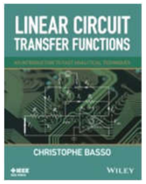
New book!



Conclusion

- ❑ Incremental input resistance of switching converter is negative
- ❑ Inserting a LC filter impacts the switching converter performance
- ❑ You have two design strategies
- ❖ you design the filter together with the converter
- ❖ you add a filter to an unknown-content converter
- ❑ You must determine open- and closed-loop input impedance
- ❑ Cascading converters requires knowledge of Z_{out} and Z_{in}
- ❑ Test in the lab. dynamic responses when the filter is added
- ❑ Explore how damping is maintained at temperature extremes