ON Semiconductor®

Closing the Feedback Loop through Simulation and Analysis

Christophe Basso – Technical Fellow

IEEE Senior Member

Course Agenda

- ☐ Blocks in a Switching Converter
- ☐ Introduction to Small-Signal Modeling
- ☐ Analytical Analysis of an Output Stage
- ☐ Simulation Models - Averaged or Switched?
- ☐ Crossover Frequency and Phase Margin
- ☐ Compensation Strategy
- ☐ Experiments on Prototypes
- ☐ Conclusion

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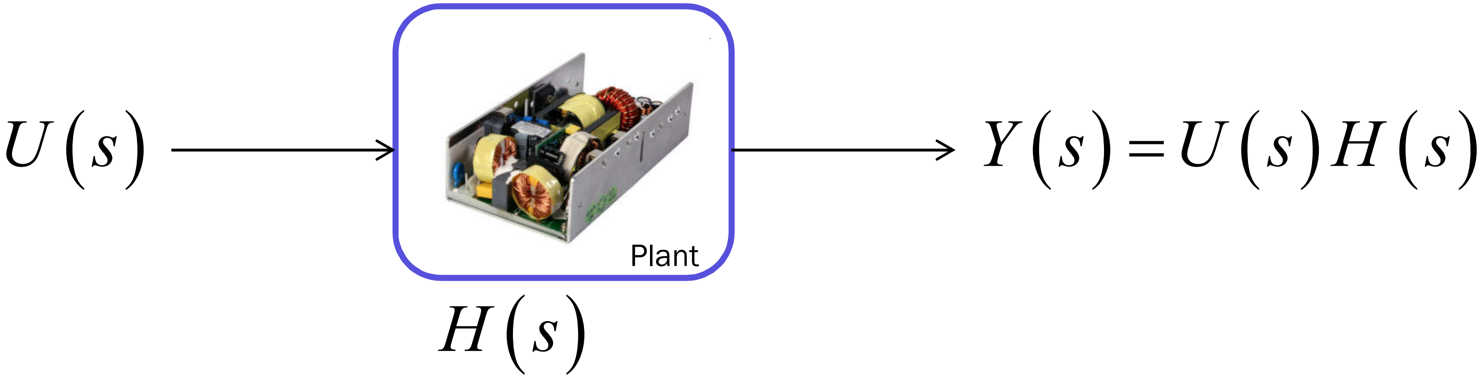
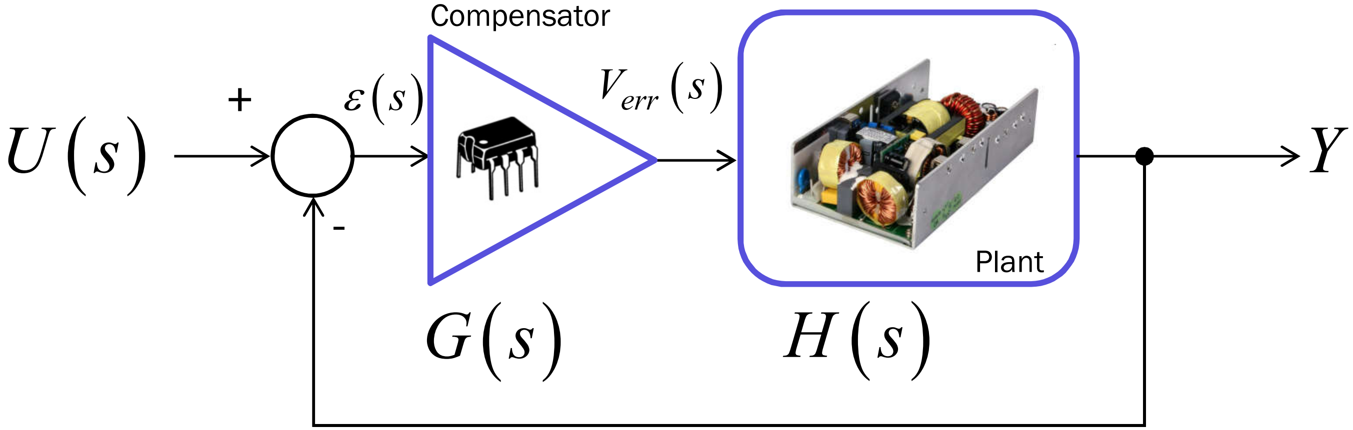
What is a Control System?

- ☐ An *open-loop* system links the output to the control variable



$$Y(s)=U(s)H(s)$$

- ☐ A *control system* observes the output and minimizes errors

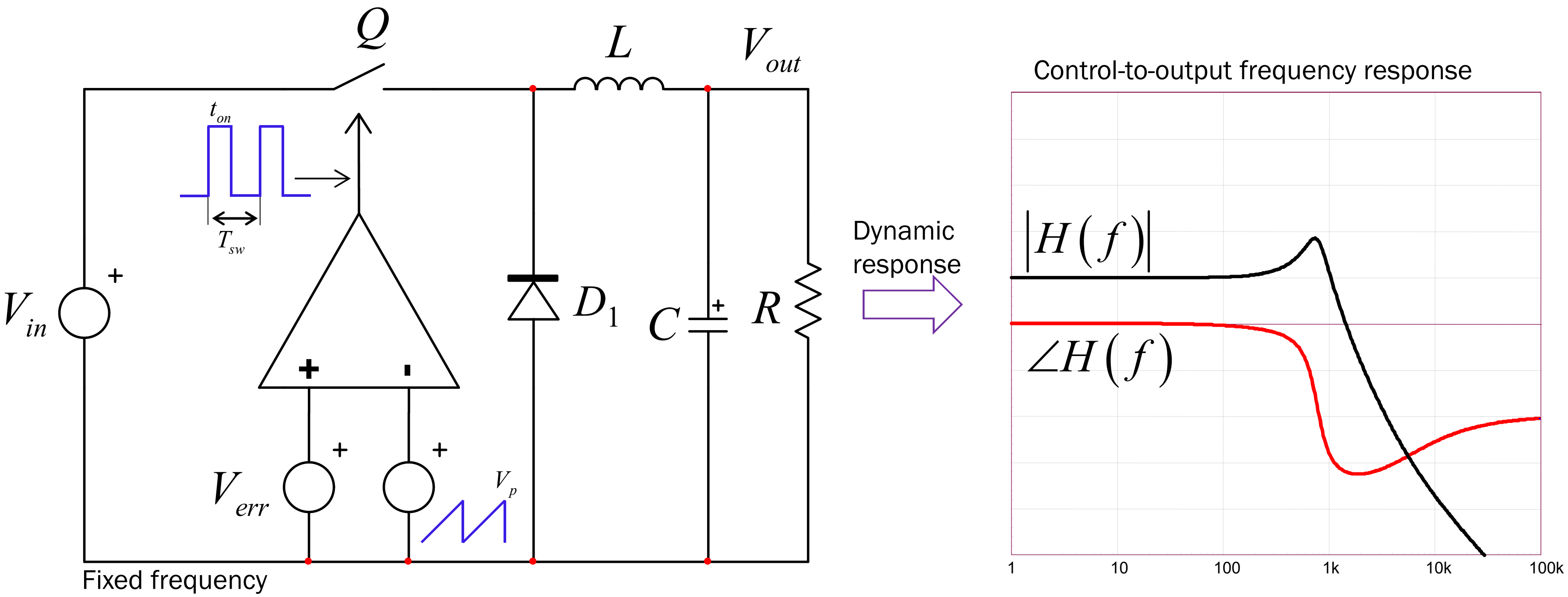


$$Y(s)=\underset{\underset{[U(s)-Y(s)]G(s)}{\uparrow}}{V_{err}}(s)H(s)$$

input output

Elements Found in a Switching Converter

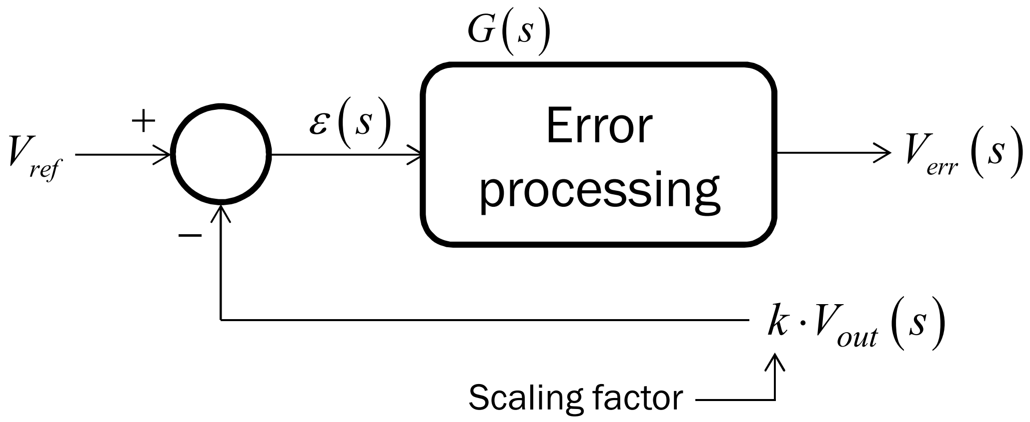
- The control system – your converter – is made of various blocks
  - The plant is the power stage of the buck, boost, LLC etc.
  - The control variable is D, the duty ratio, the response is V_{out} or I_{out}



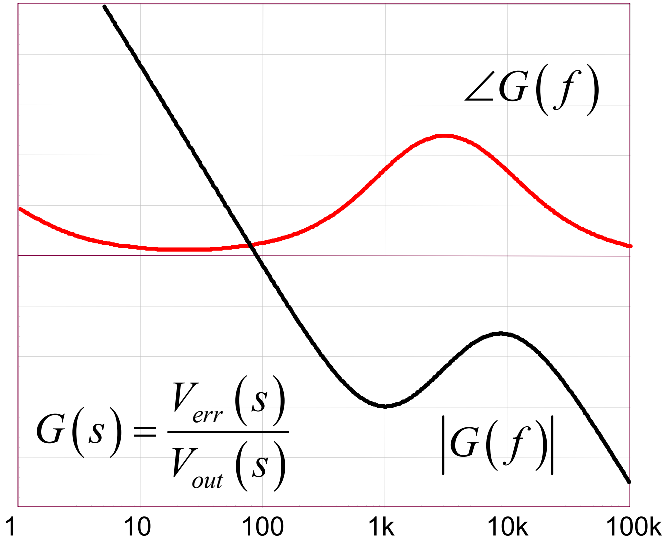
- You need the control-to-output response of the plant beforehand

The Compensator – Shaping the Loop

- ☐ The **compensator** builds the error variable and ensures stability
- ➢ Insert poles and zeros to build the compensation strategy
- ➢ Choose how to cross over at f_c with phase and gain margins



Compensator dynamic response



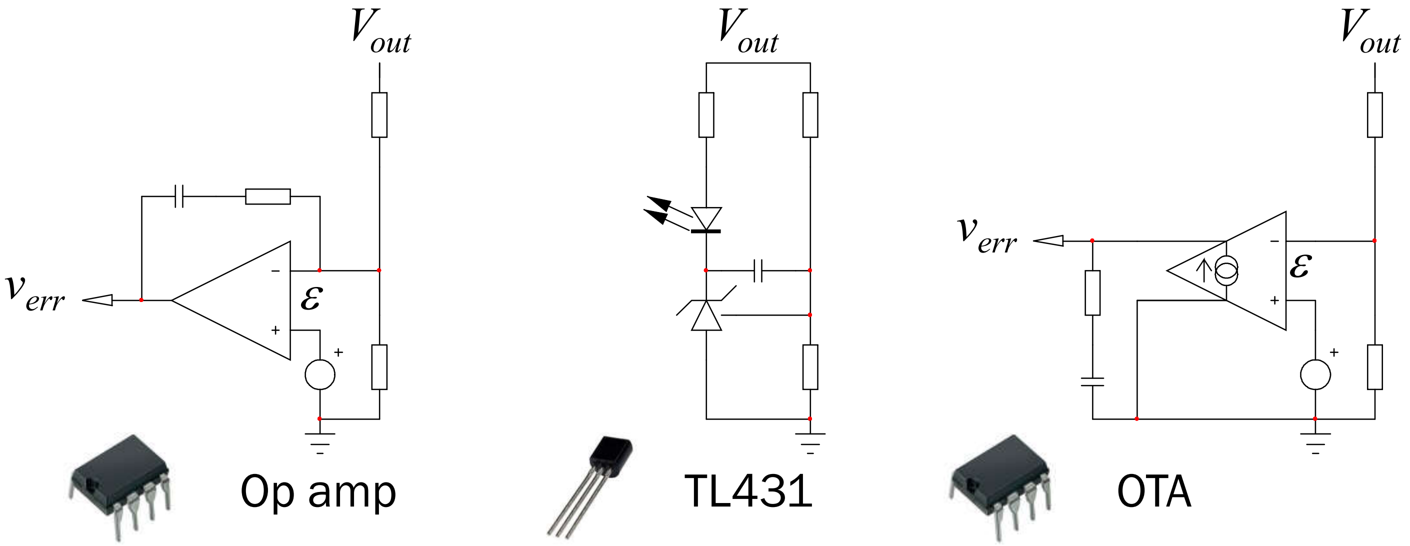
$$G(s) = \frac{V_{err}(s)}{V_{out}(s)}$$

- ☐ The block amplifies and shapes the error ε between V_{ref} and V_{out}
- ➢ Minimize the error between the setpoint and the output

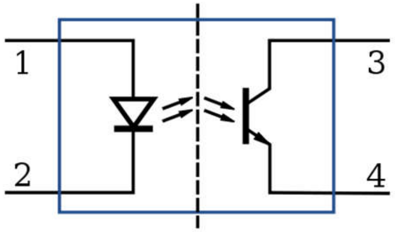
For a comprehensive analysis see APEC 2012 seminar: *The Dark Side of Loop Control Theory*

Amplifiers for the Compensator

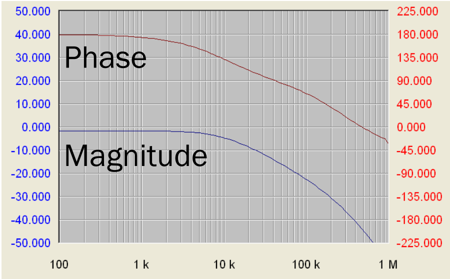
- ☐ The compensator is implemented with an error amplifier



- ☐ It is often associated with an optocoupler for isolation

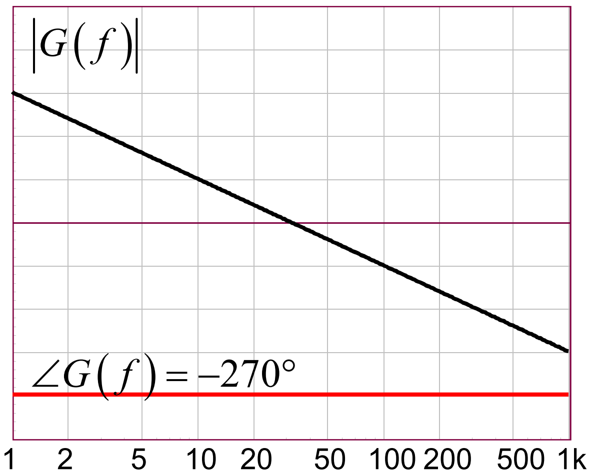


Adds its own response

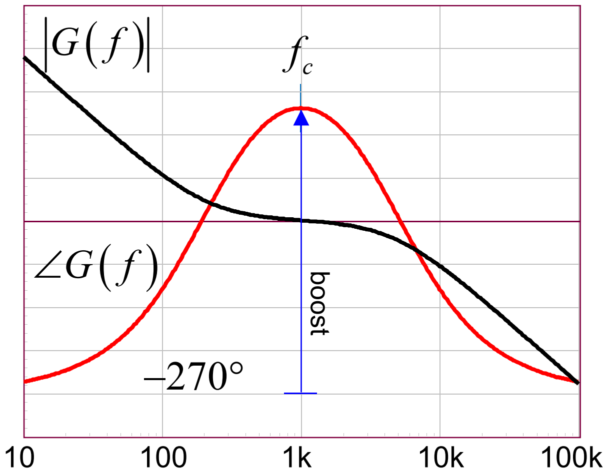


Shaping the Compensator Response

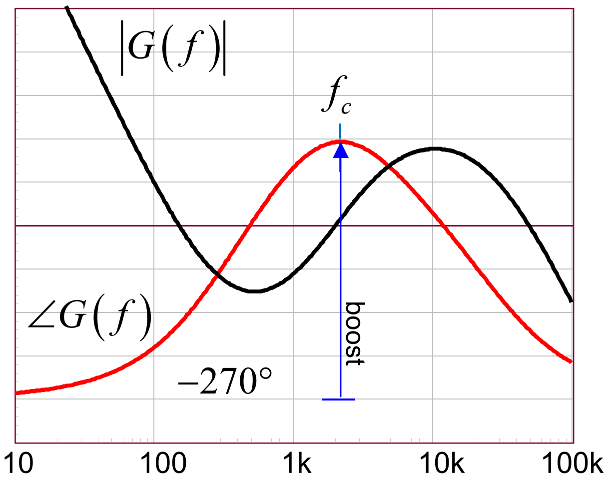
- Choose the closed-loop response by placing poles, zeros and gains



Type 1 – no boost



Type 2 – up to 90°



Type 3 – up to 180°

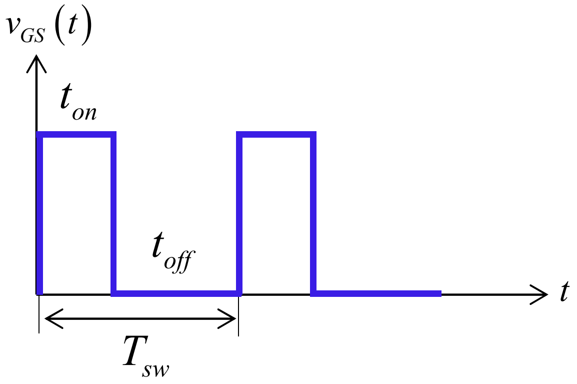
$$G_1(s) = -\frac{1}{\frac{s}{\omega_{po}}}$$

$$G_2(s) = -G_0 \frac{1+\frac{s_{z_1}}{s}}{1+\frac{s}{s_{p_1}}}$$

$$G_3(s) = -G_0 \frac{\left(1+\frac{s_{z_1}}{s}\right)\left(1+\frac{s}{s_{z_2}}\right)}{\left(1+\frac{s}{s_{p_1}}\right)\left(1+\frac{s}{s_{p_2}}\right)}$$

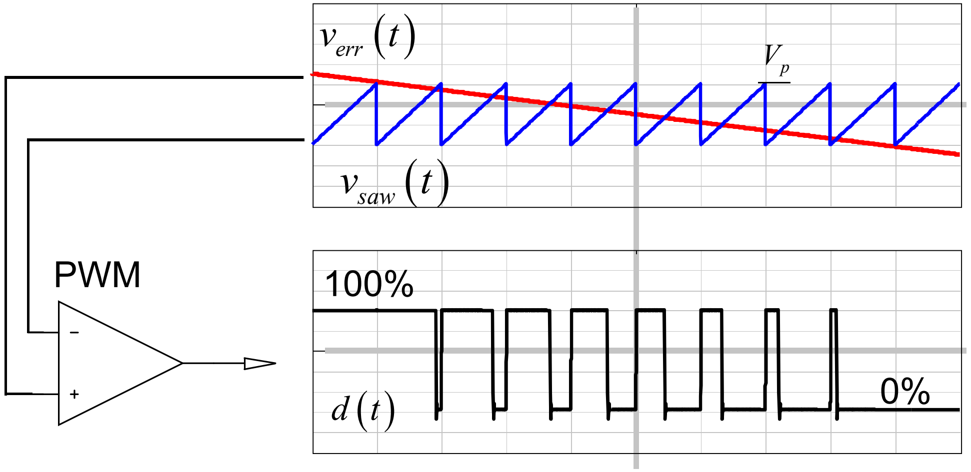
Controlling the Duty Ratio

- ❑ The power stage is controlled via the duty ratio *D*



$$\Rightarrow \quad D = \frac{t_{on}}{T_{sw}} = \frac{3u}{10u} = 30\%$$

- ❑ The pulse width modulator or PWM links *D* to V_{err}



$$v_{saw}(t) = V_p \frac{t}{T_{sw}}$$

Toggling occurs at t_1 when $v_{err}(t_1) = V_p \boxed{\frac{t_1}{T_{sw}}}$ $\leftarrow d(t_1)$

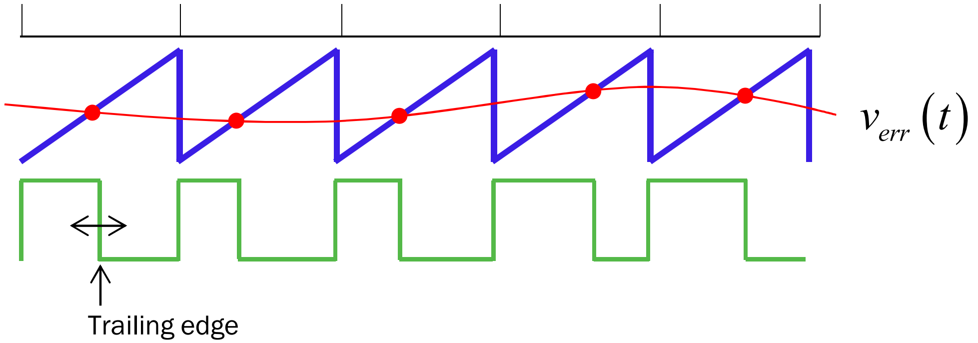
$$v_{err}(t) = V_p d(t)$$

$$\Downarrow$$

$$d(t) = \frac{v_{err}(t)}{V_p}$$

Different Types of Modulators

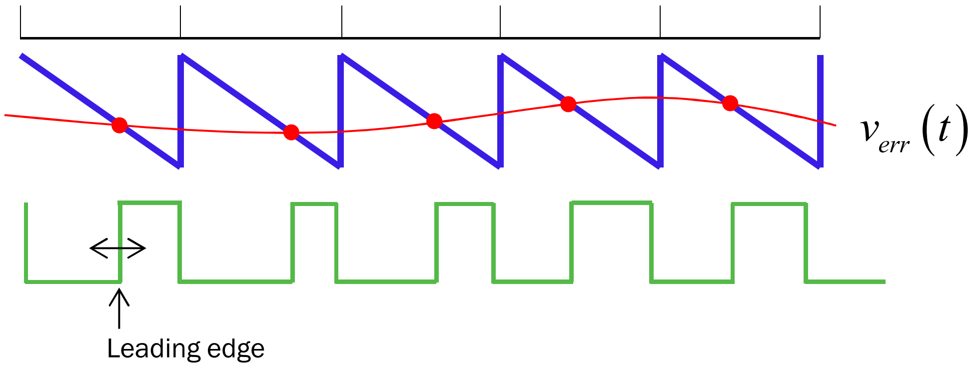
Trailing edge



Clocked turn-on
Fast turn-off
Delayed turn-on

Conventional
ac-dc, dc-dc

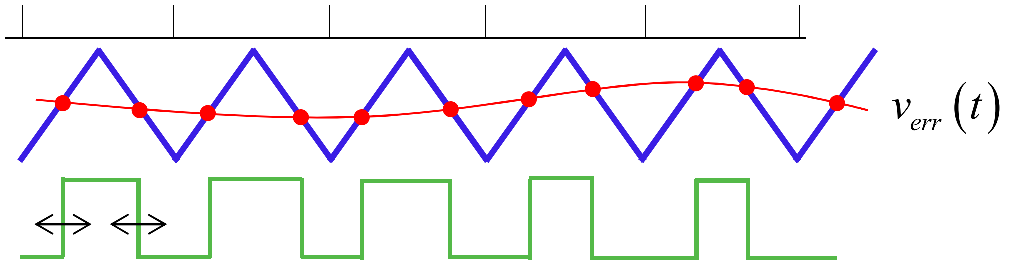
Leading edge



Clocked turn-off
Fast turn-on
Delayed turn-off

Post regulators

Dual edge

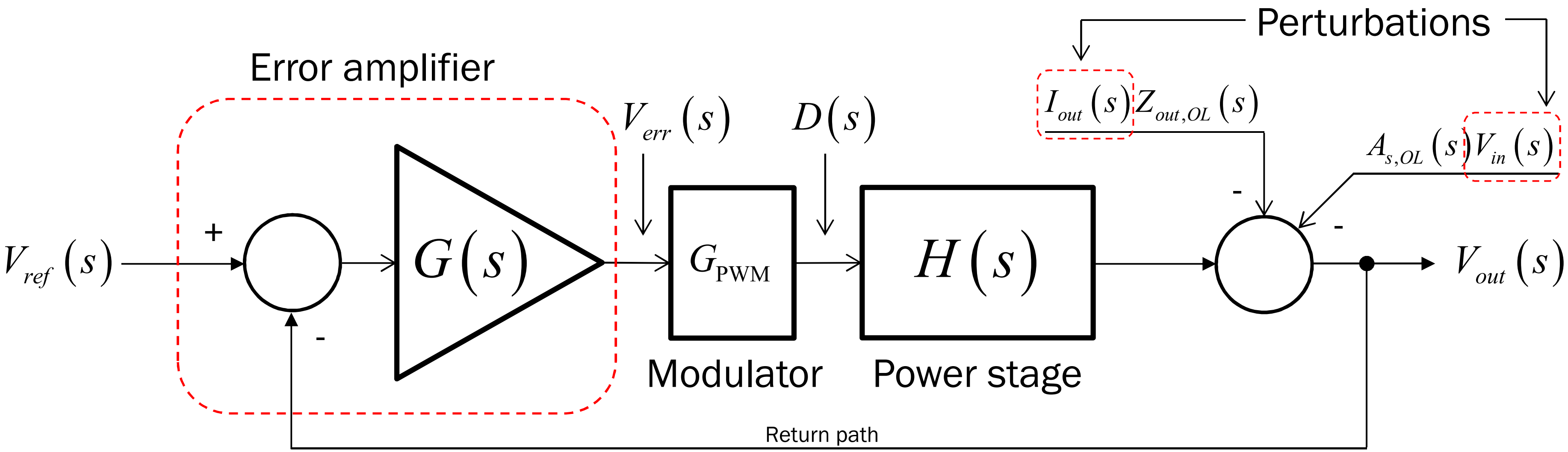


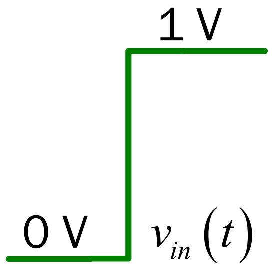
Fast turn-on
Fast turn-off

High-speed
dc-dc

The Complete Picture

- ☐ The whole system is built by associating blocks

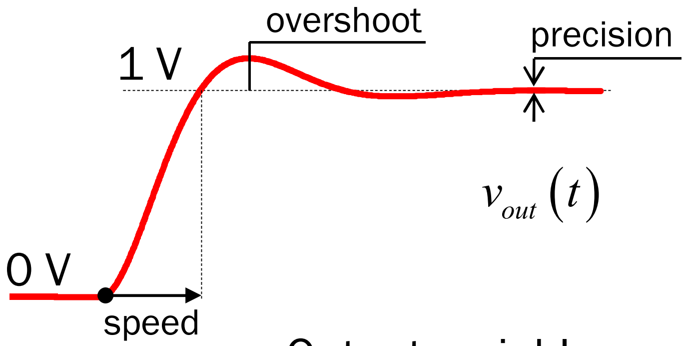




Step input

Compensate for:

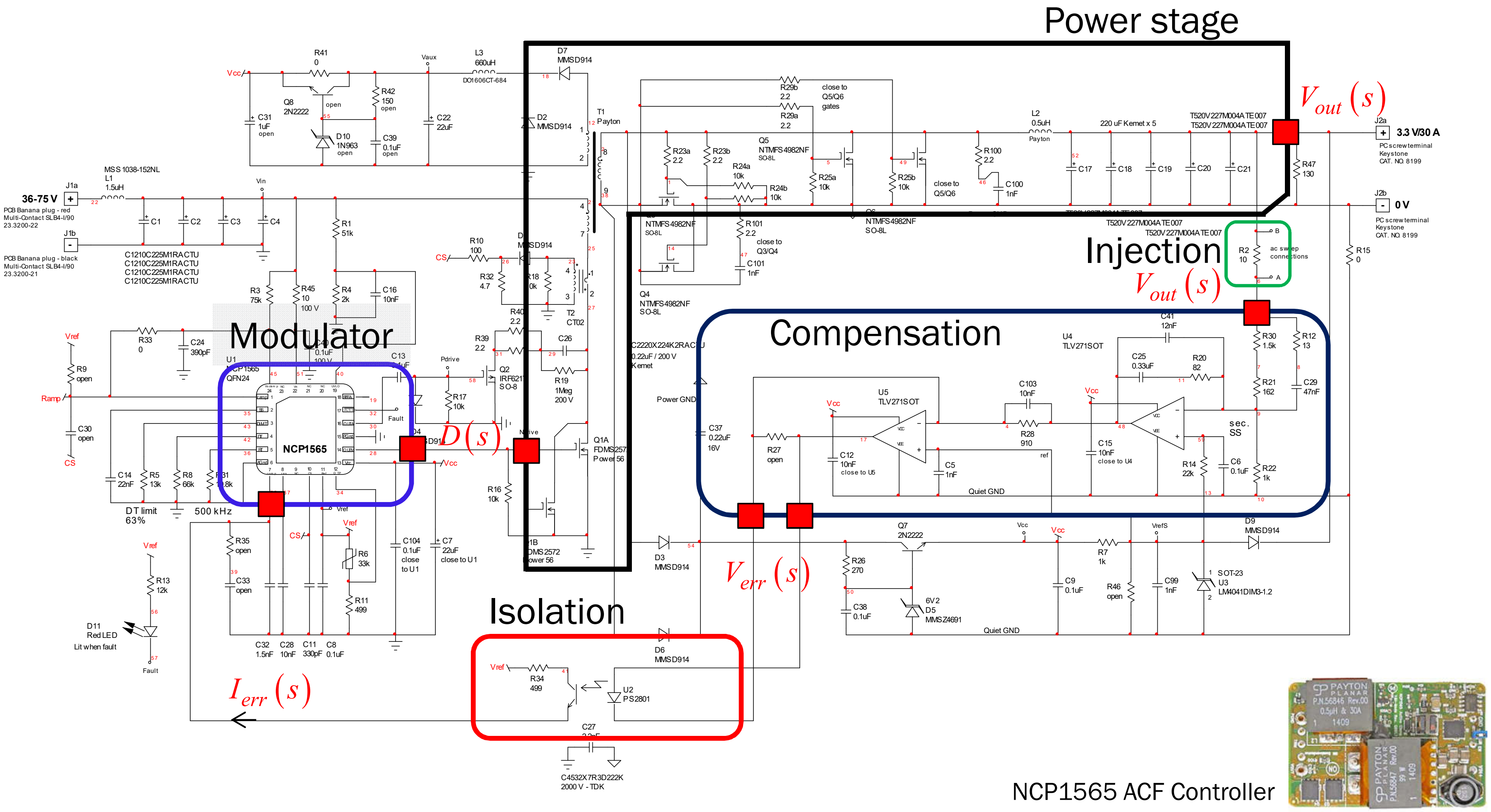
- ✓ Speed
- ✓ Precision
- ✓ Robustness



Output variable

Identifying the Blocks in a Schematic

- ☐ It is important to recognize where the functions appear

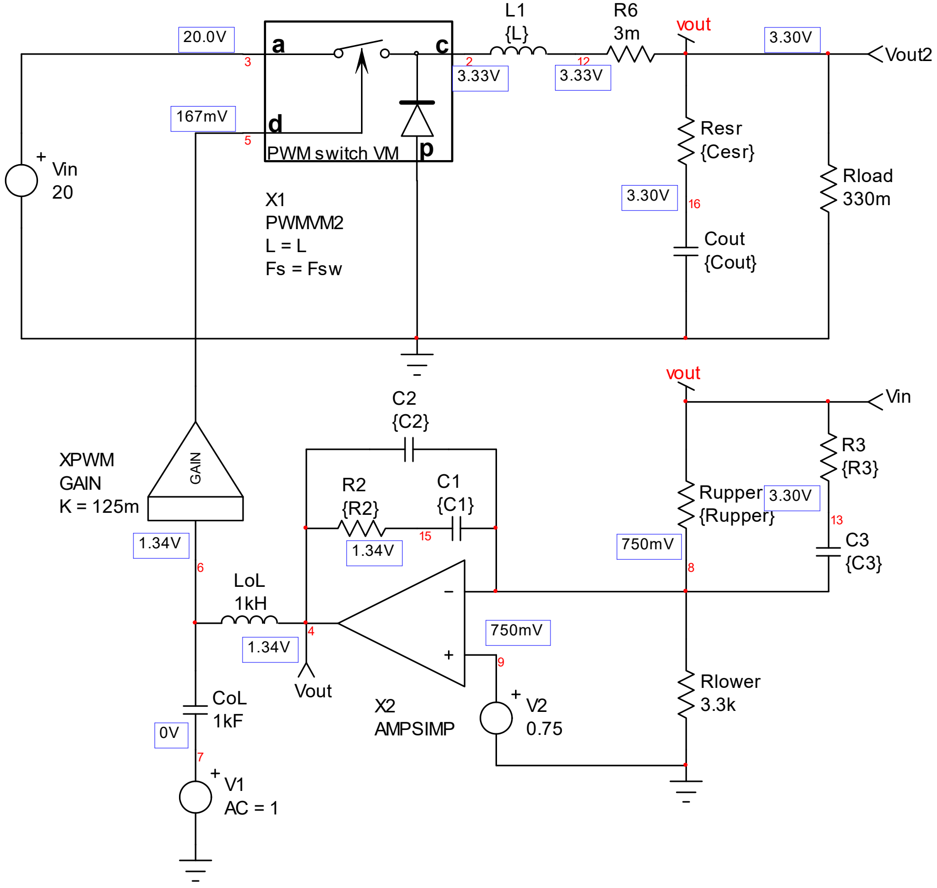


NCP1565 ACF Controller

Power Stage Response Analysis

❑ The power stage response is needed

❖ Simulation with averaged models: the PWM switch

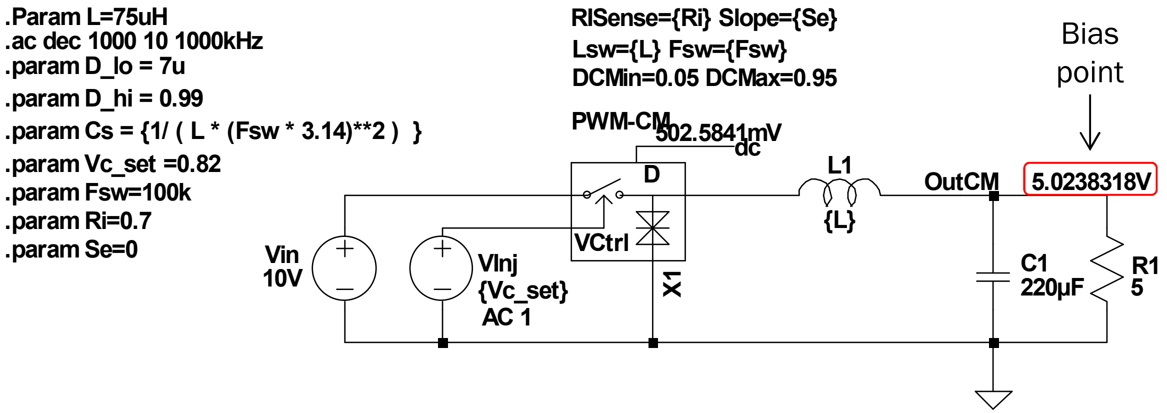


1. Fast (no switching)
2. Small-signal (linear)
3. Easy .PARAM sweep
4. Portable (SPICE2)

- ➢ Power stage model
- ➢ Nonlinearities
- ➢ Convergence issues

Dynamic Response with LTspice®

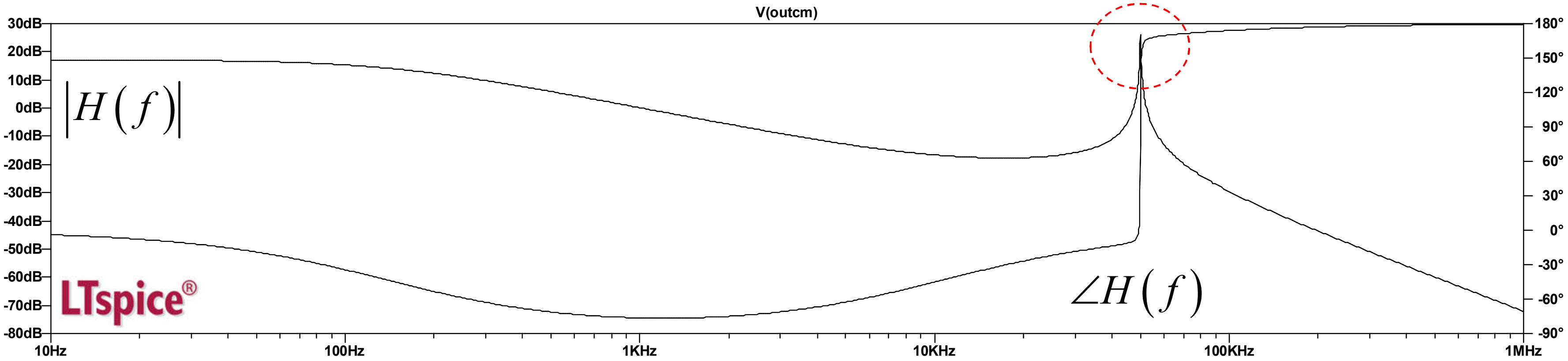
- ☐ Average models are also available with this simulation platform



```
.Param L=75uH
.ac dec 1000 10 1000kHz
.param D_lo = 7u
.param D_hi = 0.99
.param Cs = {1/ ( L * (Fsw * 3.14)**2 ) }
.param Vc_set =0.82
.param Fsw=100k
.param Ri=0.7
.param Se=0
```

```
RISense={Ri} Slope={Se}
Lsw={L} Fsw={Fsw}
DCMin=0.05 DCMax=0.95
PWM-CM
```

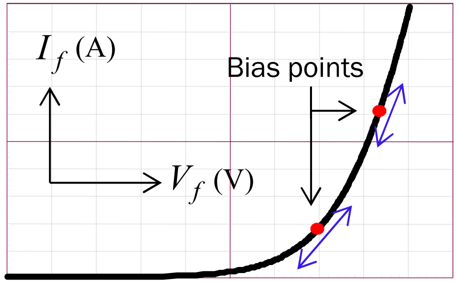
- ☐ Ac response is immediate, predicting sub-harmonic poles in CM



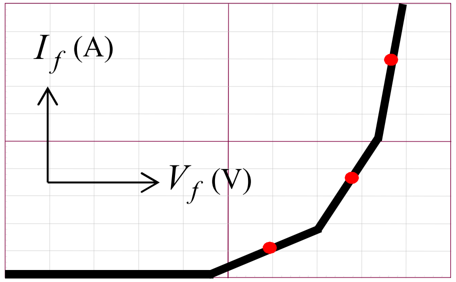
Models available at http://cbasso.pagesperso-orange.fr/Spice.htm

Piece-Wise Linear Simulations

- Simplis® is a PWL simulator: components are always linear

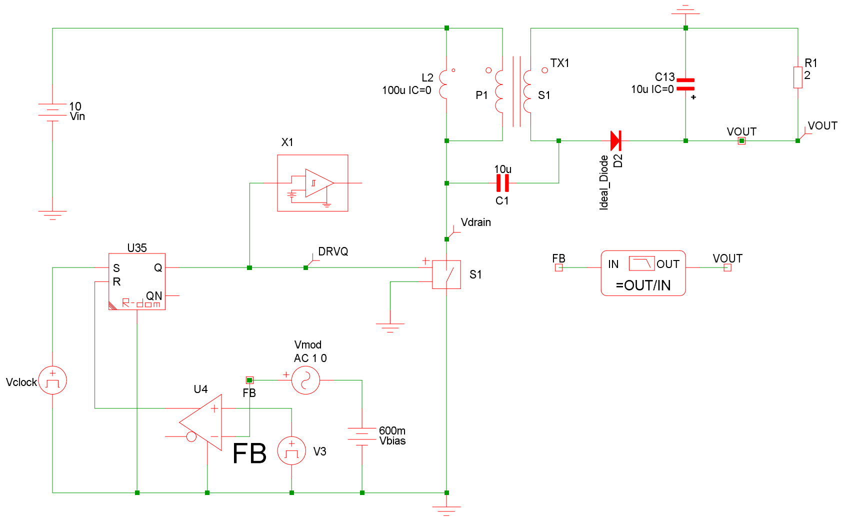


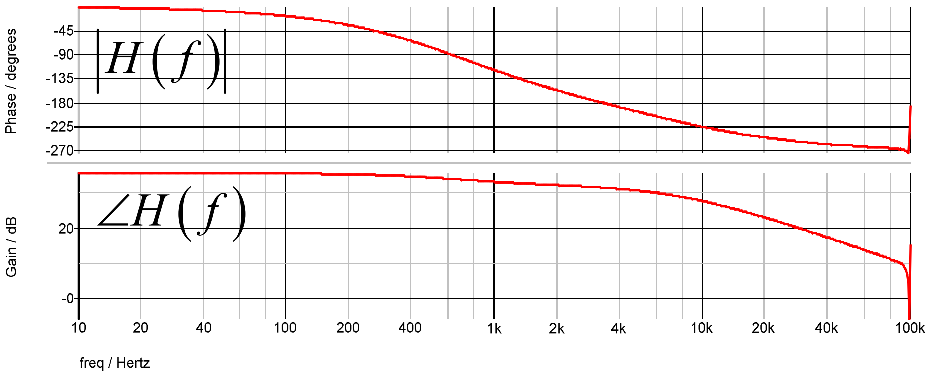
Model must be linearized



Model is already linear

- You can get an ac response from a switching circuit

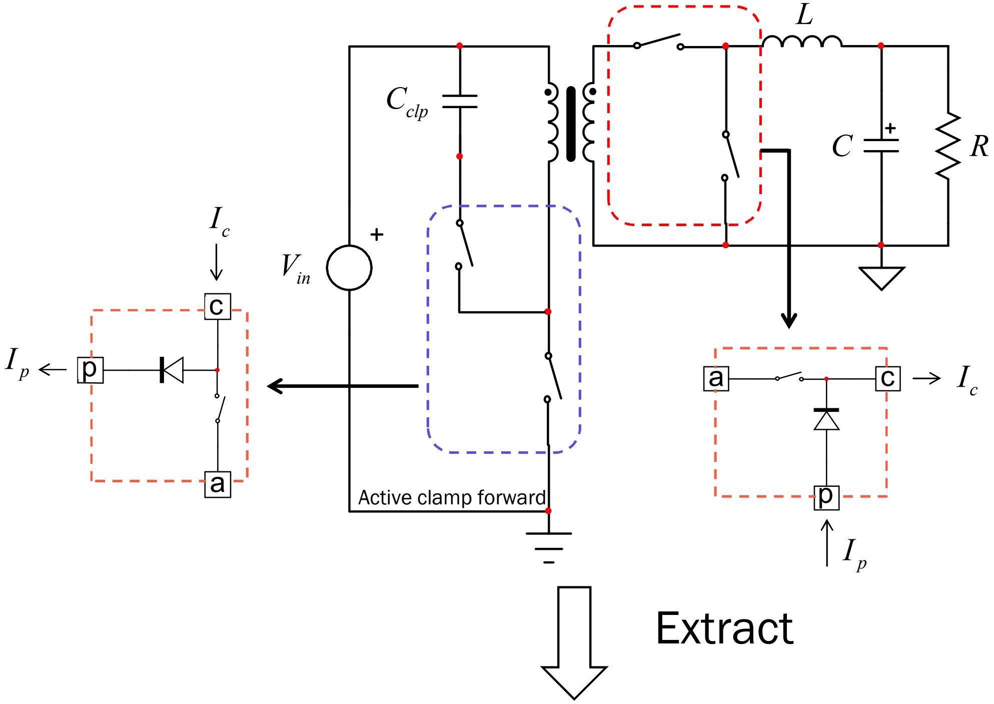




Coupled-inductor SEPIC ac response (3 s)

Equation-Based Analytical Analysis

☐ Identify a small-signal model and derive the transfer function



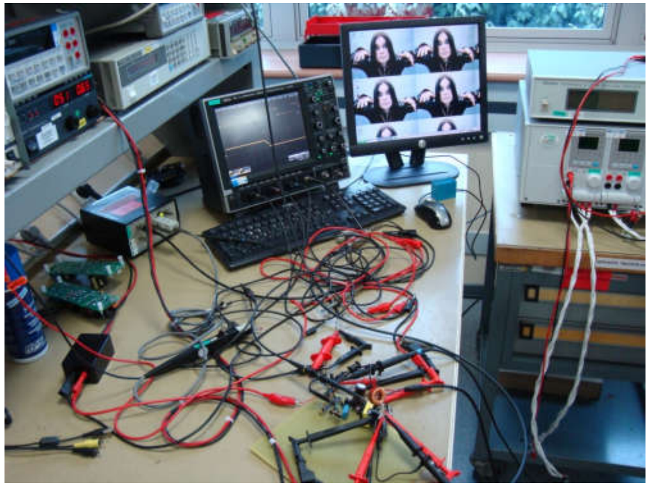
$$\frac{V_{out}(s)}{D(s)} = F_0 \frac{1+\frac{s}{s_{zF}}}{1+\frac{s}{\omega_{0F}Q_F}+\left(\frac{s}{\omega_{0F}}\right)^2} N \left(V_{in} - D_0 r_{on1} M_0 \frac{sC_{clp}}{1+\frac{s}{\omega_{0M}Q_M}+\left(\frac{s}{\omega_{0M}}\right)^2}\right)$$

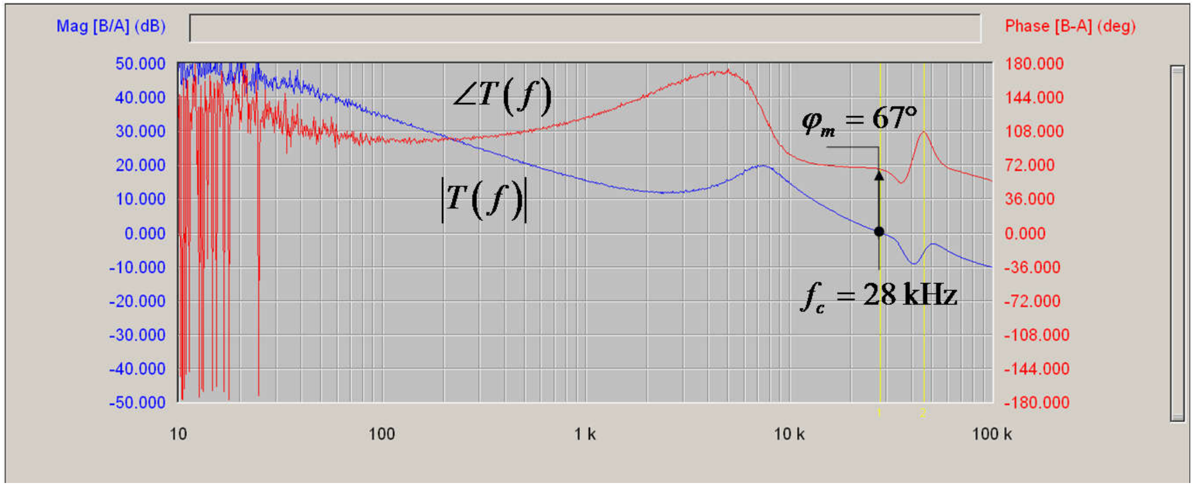
1. Expressions are ready
2. See poles and zeros
3. Parasitic contributions
4. Parametric sweep

- ➢ Scared to try?
- ➢ Have the tools?
- ➢ Valid model?

Bench Experiments

❑ Build a prototype and extract the power stage response





- ❖ You need to have all components on hand (special parts, XFMR)
- ❖ Board can be difficult to measure (isolation path, complex loop)
- ❖ Check if results match the model – refine model and analyses

This is the ultimate stage – do NOT skip it!

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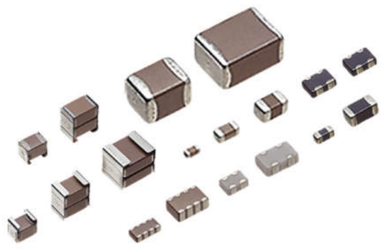
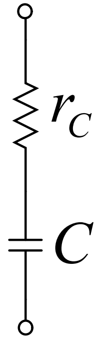
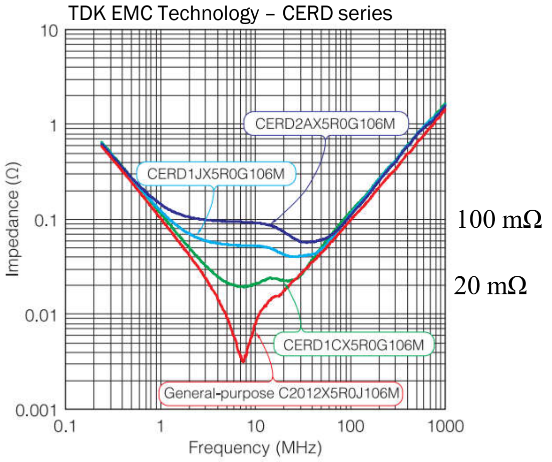
Why do we Need a Transfer Function?

- ☐ The stabilization process starts with the plant response
- ☐ Without an equation, how to identify offenders and neutralize them?

$$H(s) \approx H_0 \frac{1+\frac{s}{\boxed{\omega_{z_1}}}}{1+\frac{s}{\omega_{p_1}}} \frac{1}{1+\frac{s}{\omega_n Q}+\left(\frac{s}{\omega_n}\right)^2}$$







$$\omega_z = \frac{1}{r_C C} \xrightarrow{C=10\,\mu\text{F}} \left.\begin{array}{l} f_z \approx 159\text{ kHz} \\ \\ f_z \approx 796\text{ kHz} \end{array}\right\}$$

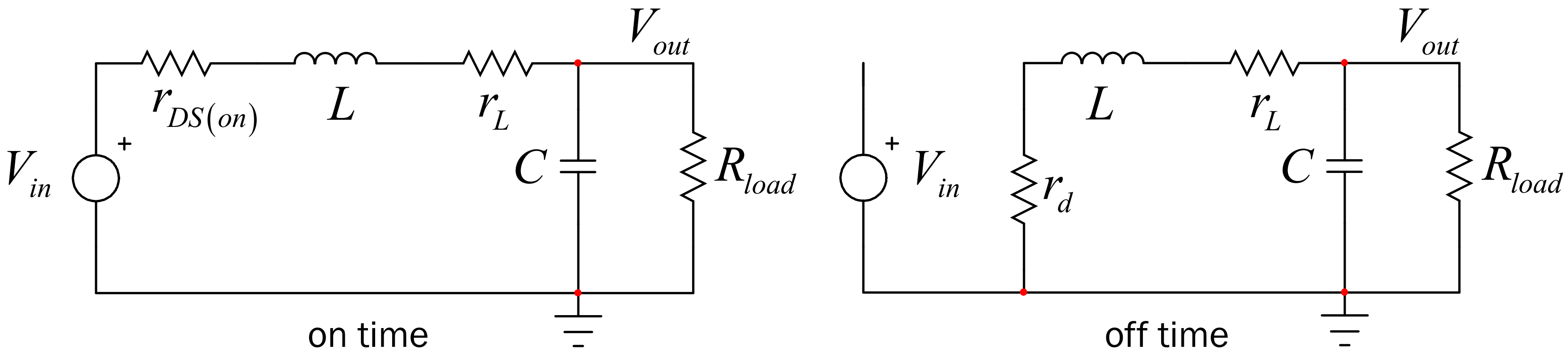
Is stability ensured in all cases?

- ☐ The ESR lumps all losses which vary with temperature and frequency!

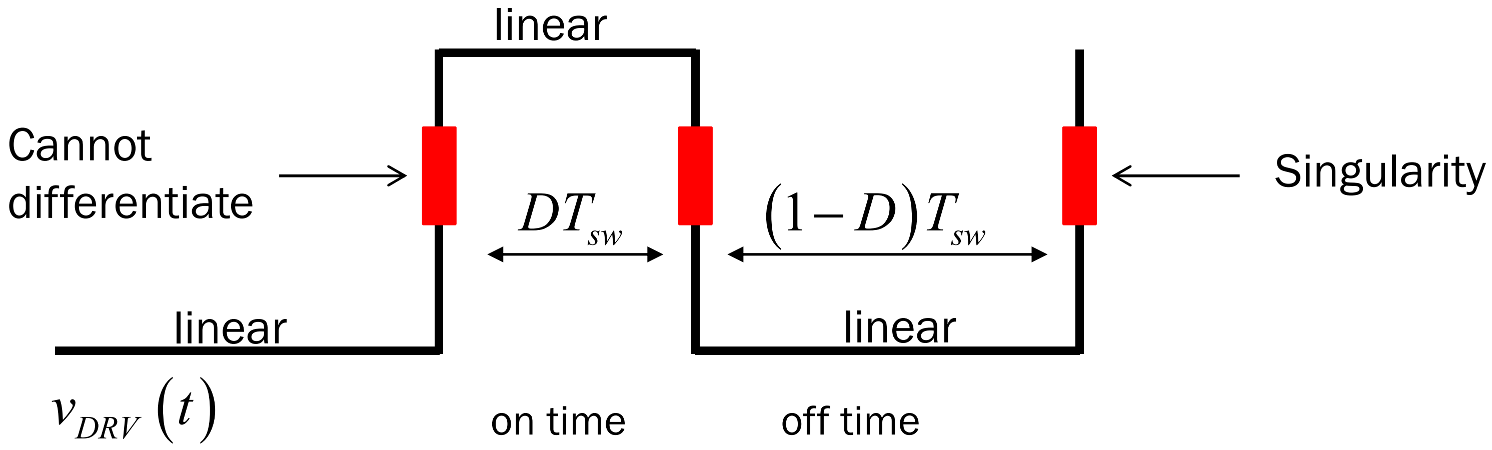
http://www.ieca-inc.com/images/Equivalent_Series_Resistnace_ESR.pdf

How to Model a Switching Converter?

- ☐ A switching converter is made of linear elements



- ☐ The non-linearity or discontinuity is coming from transitions



State Space Averaging (SSA)

- ☐ Despite linear networks, equation is discontinuous in time
- ☐ Introduced in 76, SSA weights on and off expressions

valid during $(1\text{-}D)T_{sw}$

$$\dot{x} = \left[\mathbf{A}_1 D + \mathbf{A}_2\left(1 - D\right)\right]x\left(t\right) + \left[\mathbf{B}_1 D + \mathbf{B}_2\left(1 - D\right)\right]u\left(t\right)$$

valid during DT_{sw}

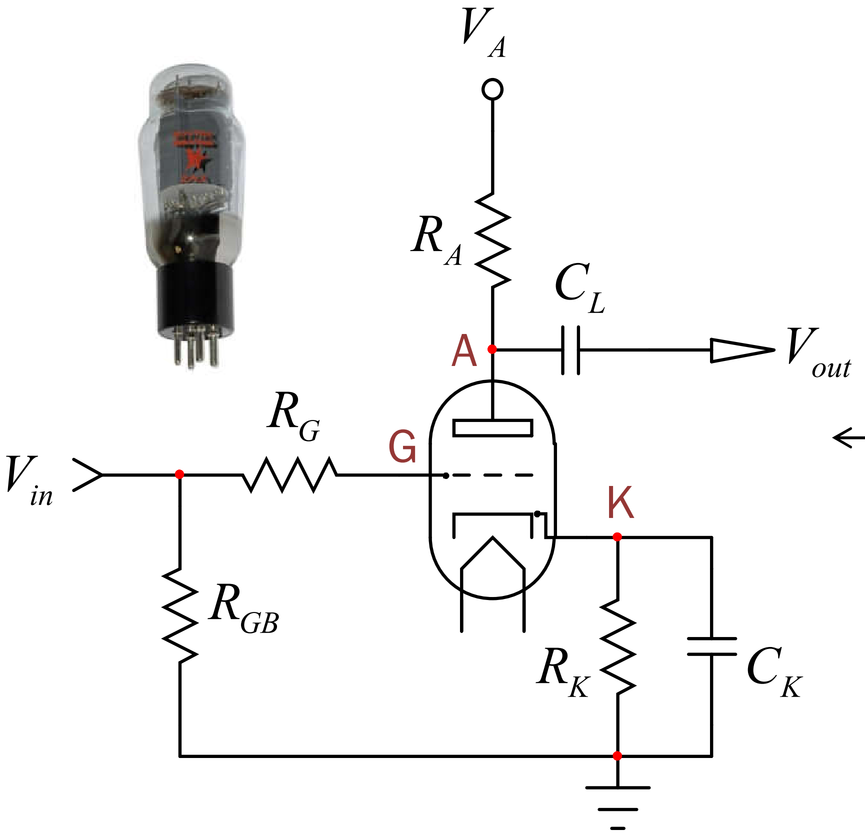
$\mathbf{A}$ is the state coefficient matrix
$\mathbf{B}$ is the source coefficient matrix

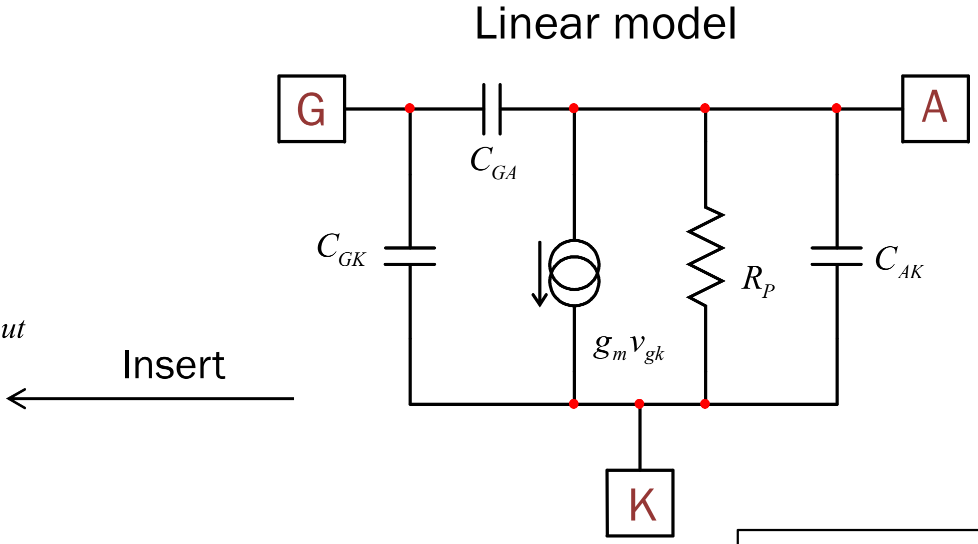
- ☐ This equation is now continuous in time: singularity is gone
- ☐ However, it became a nonlinear equation
- ➢ You need to linearize it by perturbation (or differentiation)
- ➢ If you add a new element, you have to restart from scratch!

S.Ćuk, *Modeling, Analysis and Design of Switching Converters*, Ph. D. Thesis, Caltech November 1976

The Triode Small-Signal Model

- ❑ A triode is a highly non-linear component
- ❑ Replace it by its small-signal model to get the response

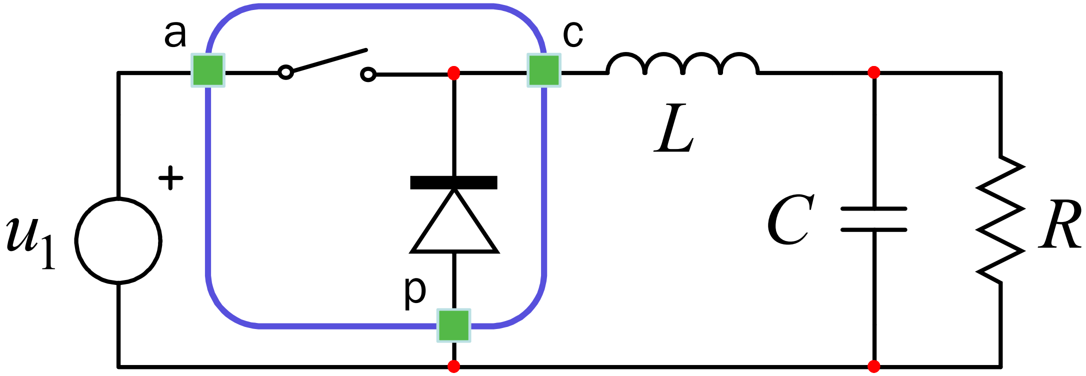




http://web.eecs.umich.edu/~mmccorq/diversions/simulation/index.html

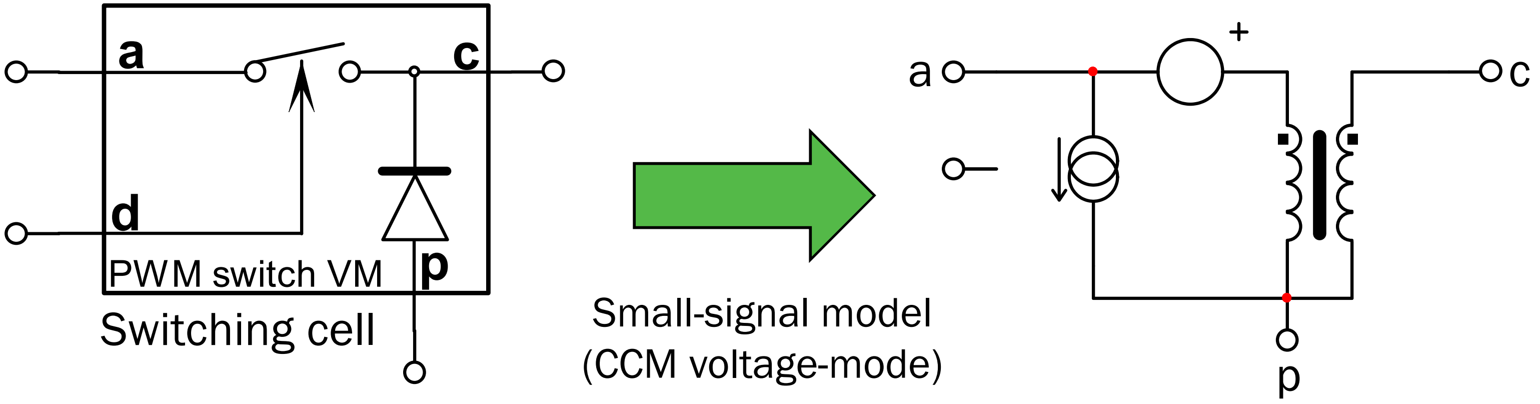
The PWM Switch Model in Voltage Mode

- ❑ We know that non-linearity is brought by the switching cell



a: active
c: common
p: passive

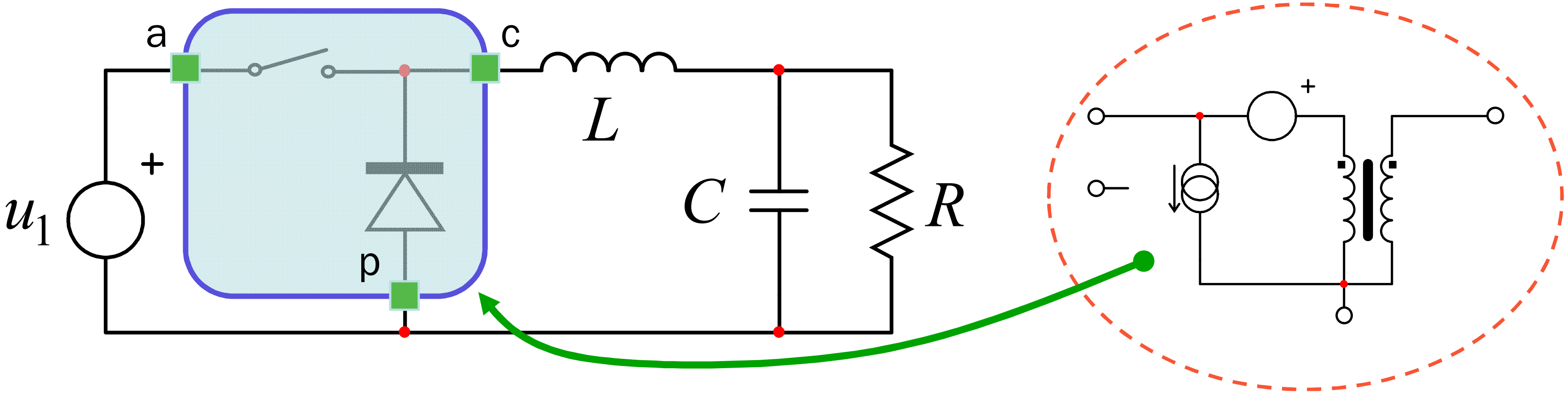
- ❑ Why don't we linearize the cell alone?



V. Vorpérian, *Simplified Analysis of PWM Converters using Model of PWM Switch, parts I and II*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, NO. 3, 1990

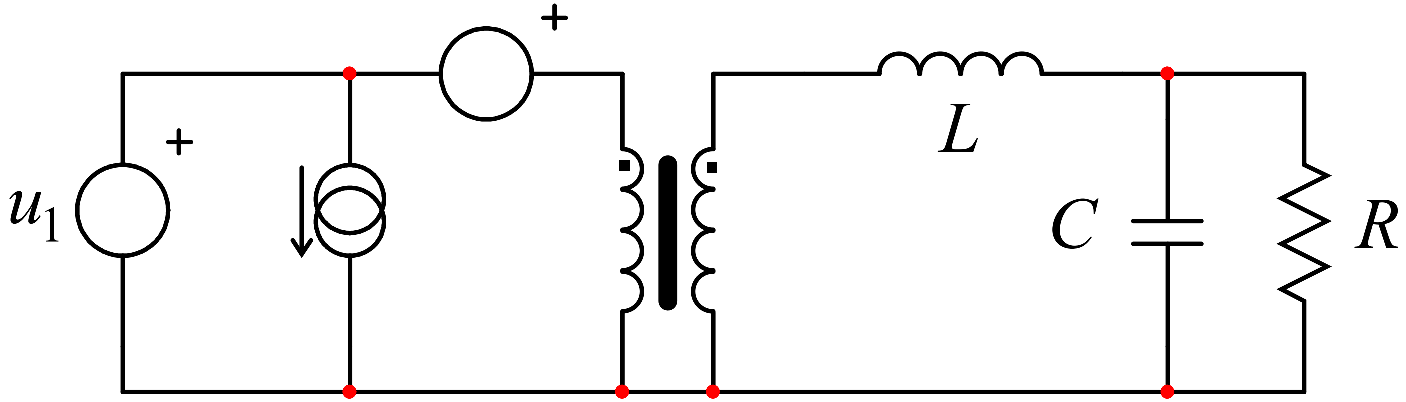
Replace the Switches by the Model

- Like in a bipolar circuit, replace the switching cell…



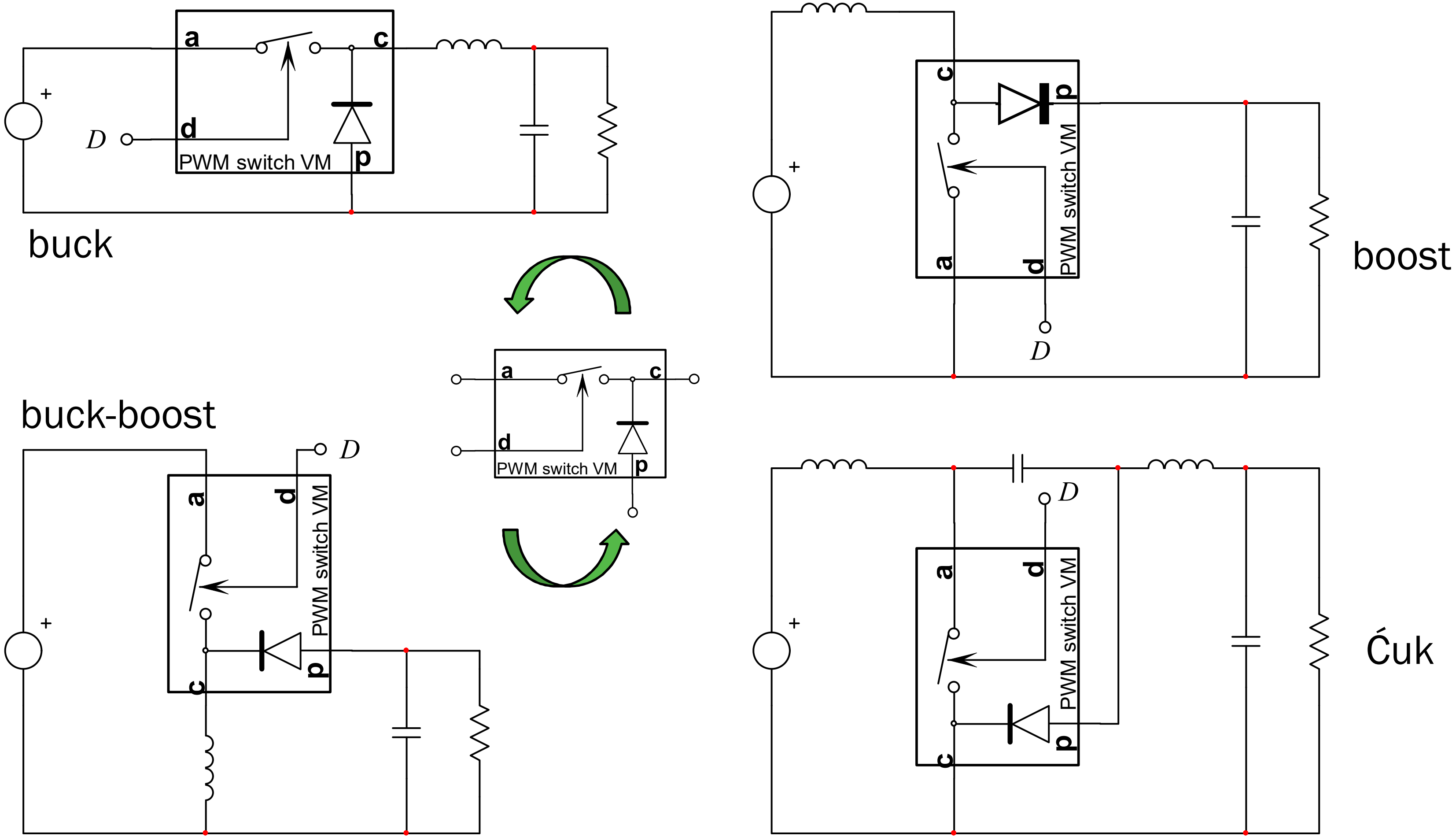
Small-signal model

- …and solve a set of linear equations!



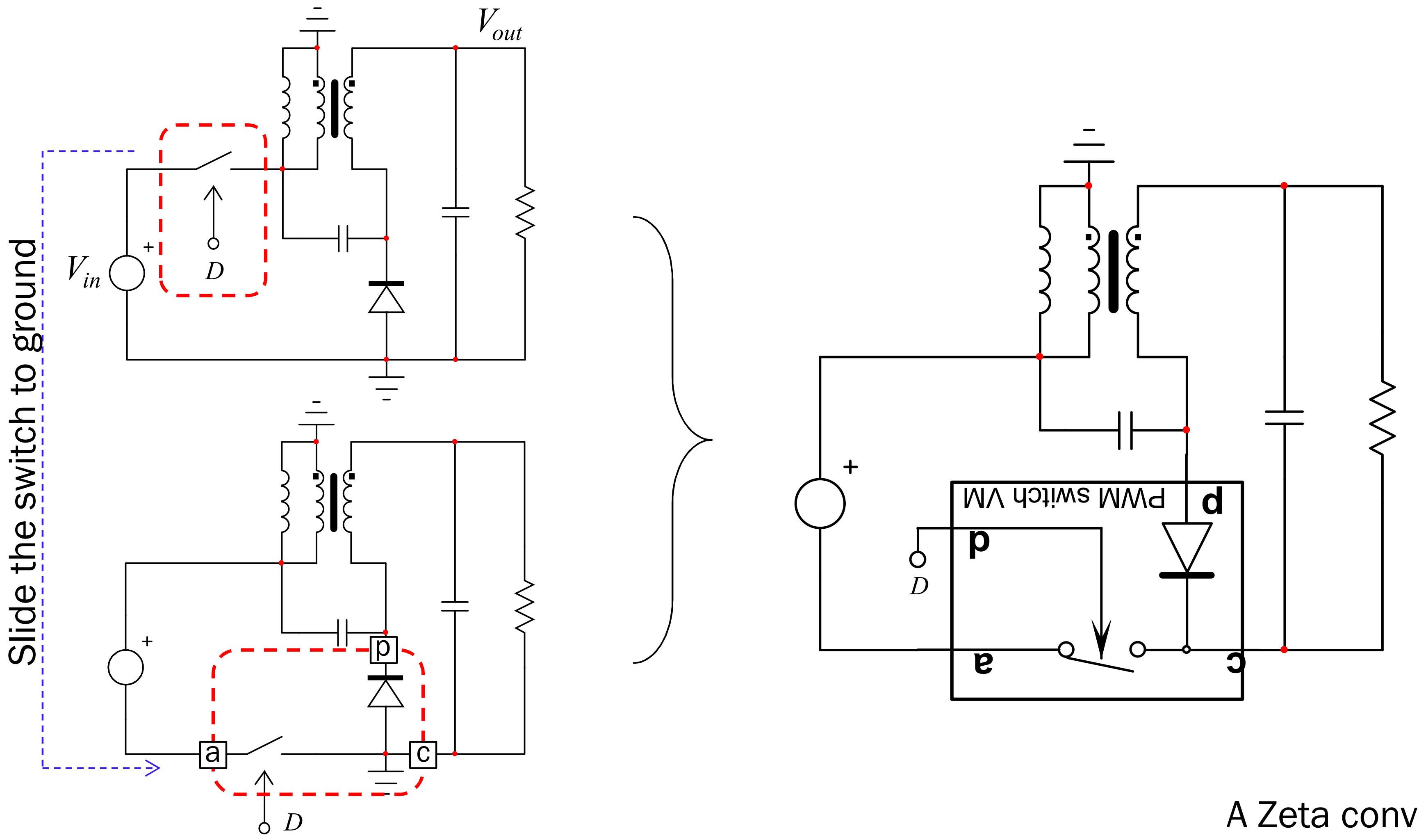
An Invariant Model fits all Structures!

- ❑ The switching cell made of two switches is everywhere!



Rearrange the Schematic

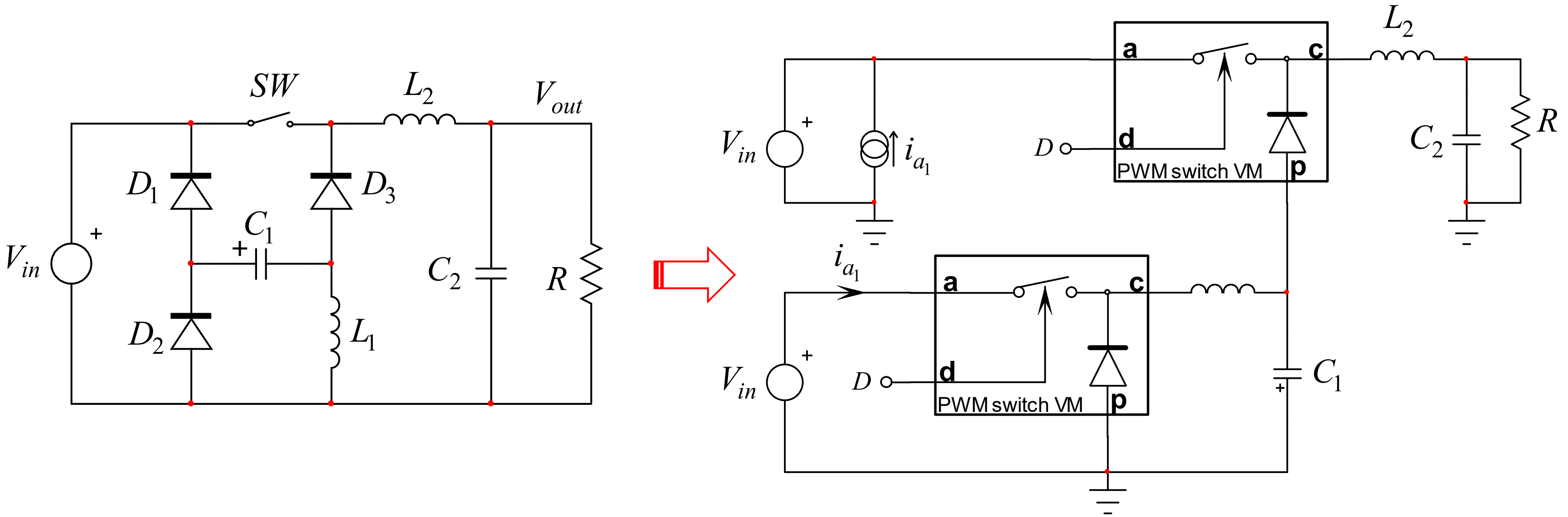
❑ You sometimes need to rework the sketch to reveal the model



A Zeta converter

Combine PWM Switches

- Some complicated structures need more work



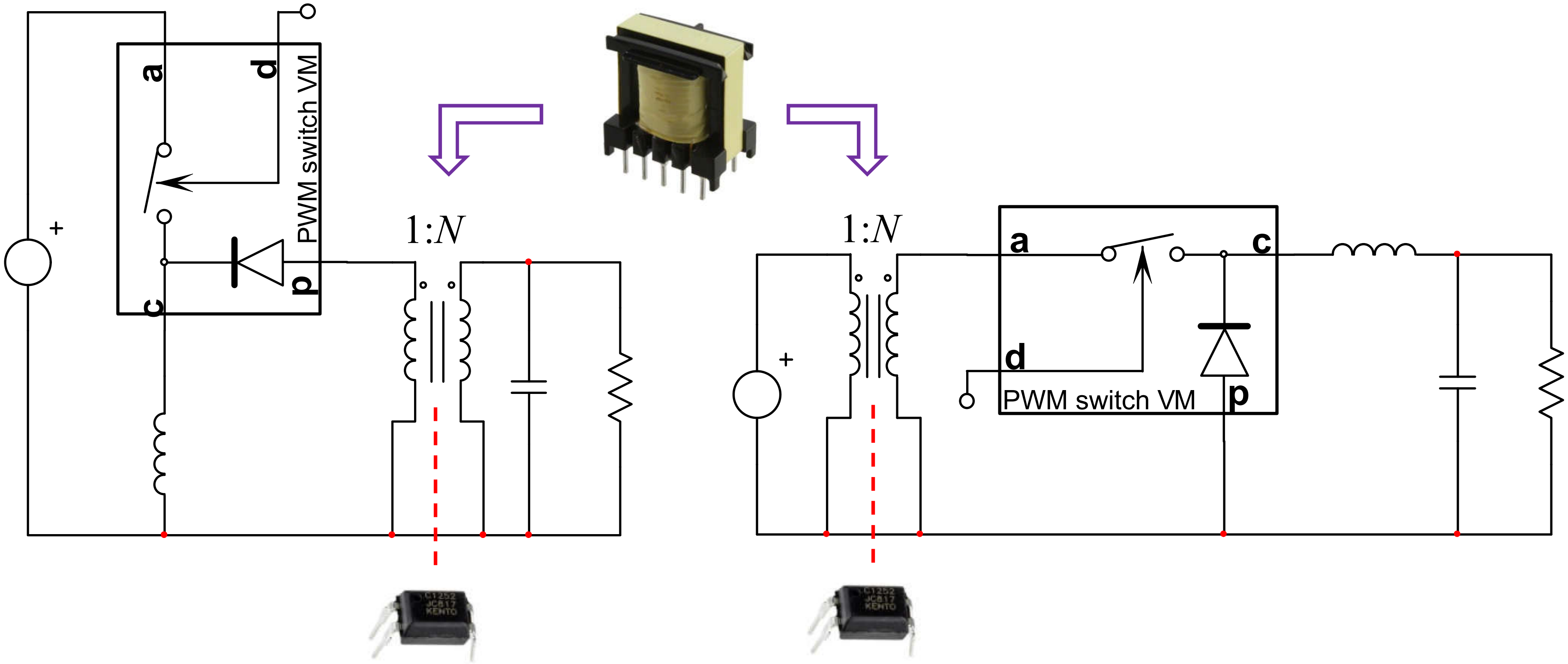
- You need to reveal networks during DT_{sw} and $(1\text{-}D)T_{sw}$
- Combine two PWM switch models at the end
- Run .DC, .AC or .TRAN analyses with this circuit

Oh yeah, and who's gonna do it, uh?

V. Vorpérian, *Fast Analytical Techniques for Electrical and Electronic Circuits*, Cambridge University Press, 978-0-52162-442-8, 2002

It also Works for Isolated Structures

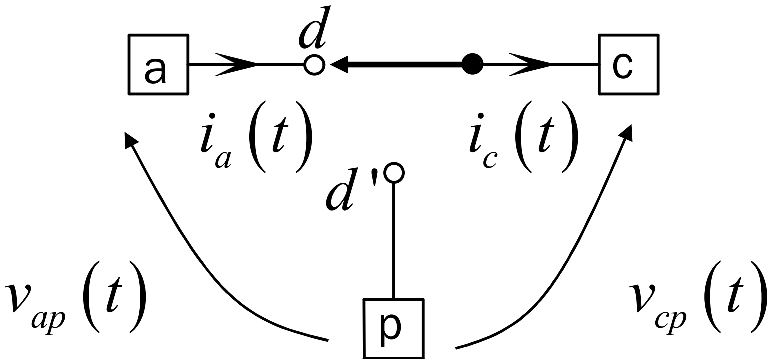
- ☐ Modeling a flyback or a forward converter is very convenient



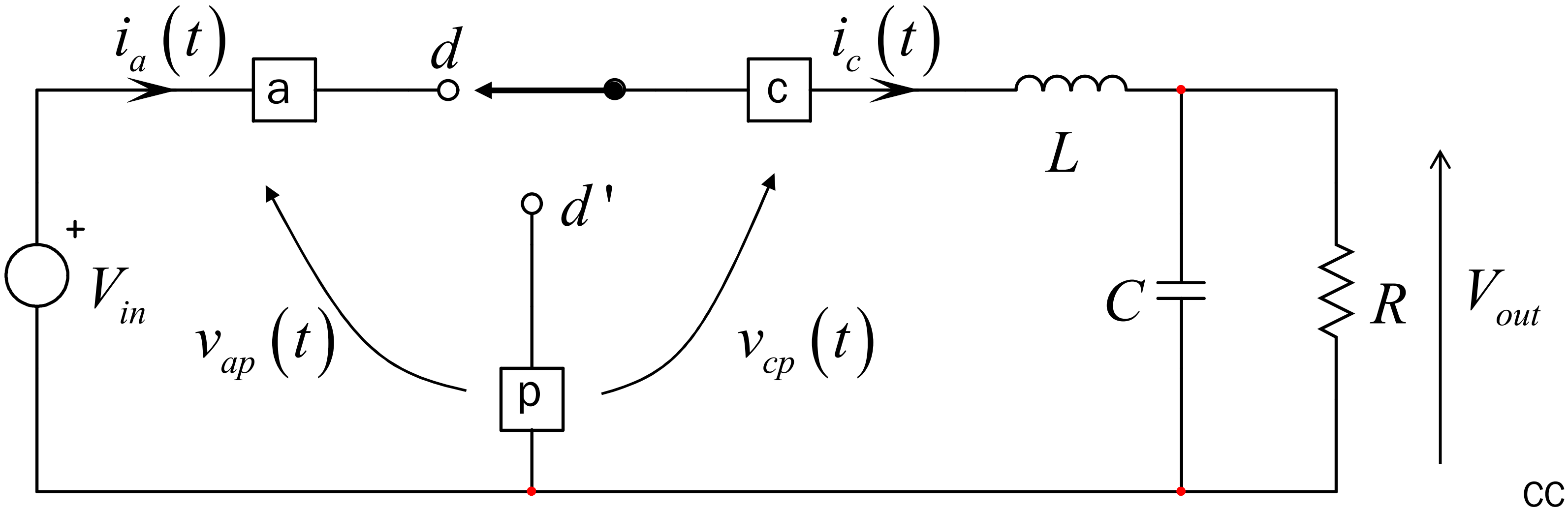
- ☐ You can easily add an optocoupler when studying stability

CCM Common Passive Configuration

- ☐ The PWM switch is a single-pole double-throw model



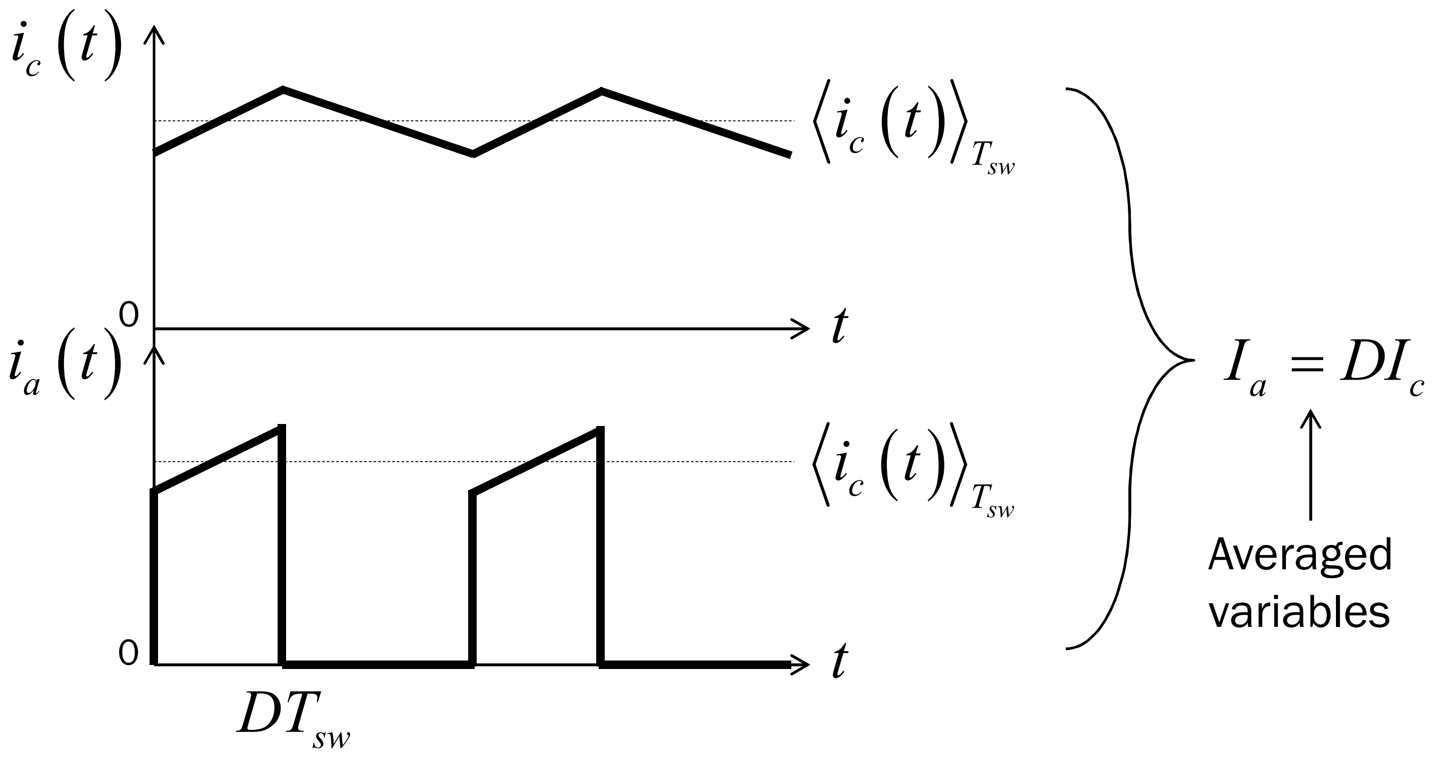
- ☐ Install it in a buck and draw its terminal waveforms



CCM VM

The Common Passive Configuration

- ❑ Average the current waveforms across the PWM switch



$I_a = DI_c$

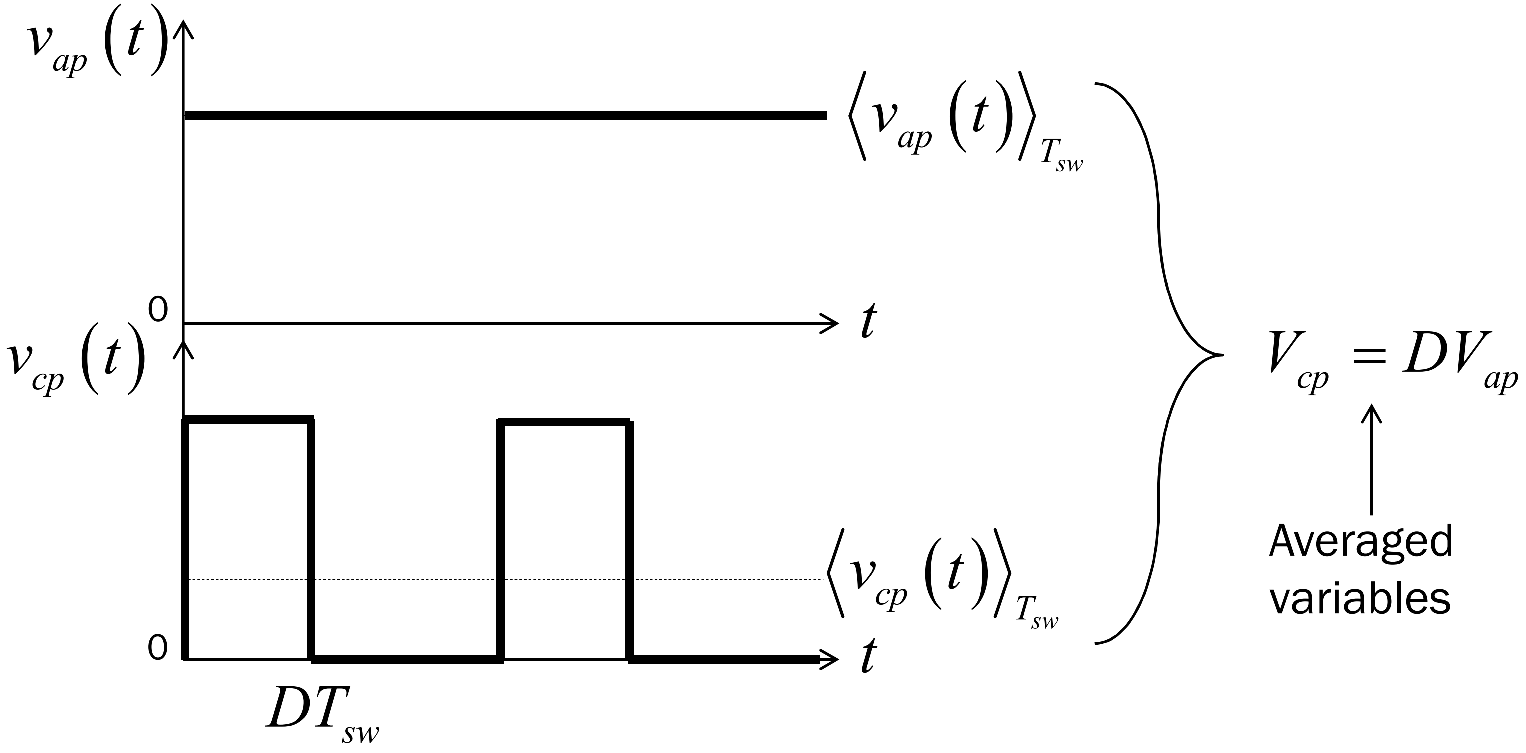
Averaged variables

$$\langle i_a(t)\rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}}\int_0^{DT_{sw}} i_a(t)\,dt = D\langle i_c(t)\rangle_{T_{sw}} = DI_c$$

CCM VM

The Common Passive Configuration

- ☐ Average the voltage waveforms across the PWM switch



$$V_{cp} = DV_{ap}$$

↑

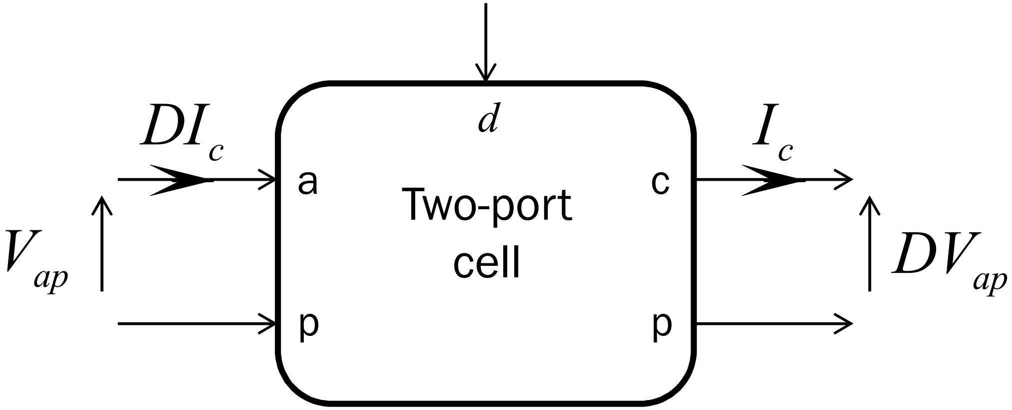
Averaged variables

$$\langle v_{cp}(t)\rangle_{T_{sw}} = V_{cp} = \frac{1}{T_{sw}}\int_{0}^{DT_{sw}} v_{cp}(t)\,dt = D\langle v_{ap}(t)\rangle_{T_{sw}} = DV_{ap}$$

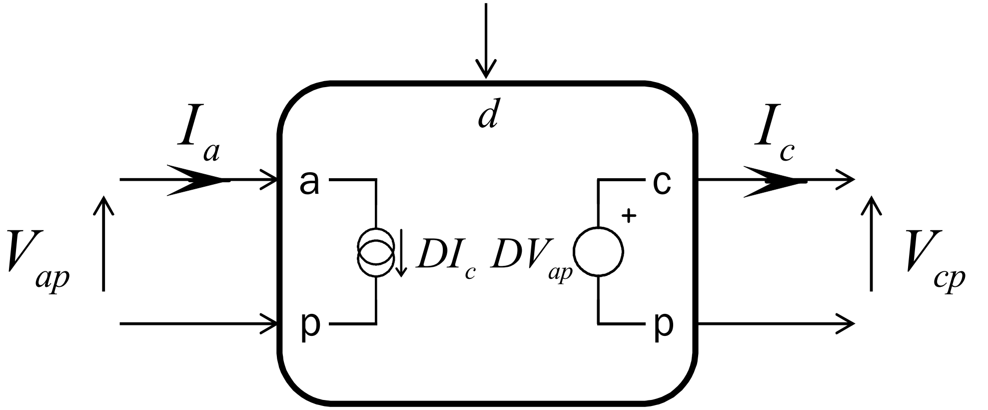
CCM VM

A Two-Port Representation

- ☐ We have a link between input and output variables



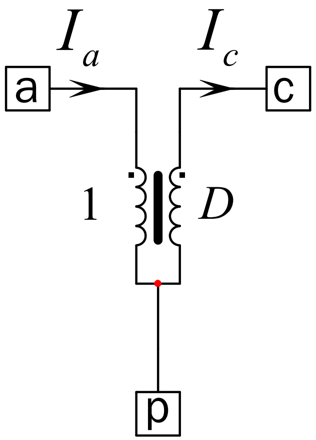
- ☐ We can involve current and voltage sources



CCM VM

A Dc Transformer Model

- ❑ The large-signal model is a dc "transformer"!

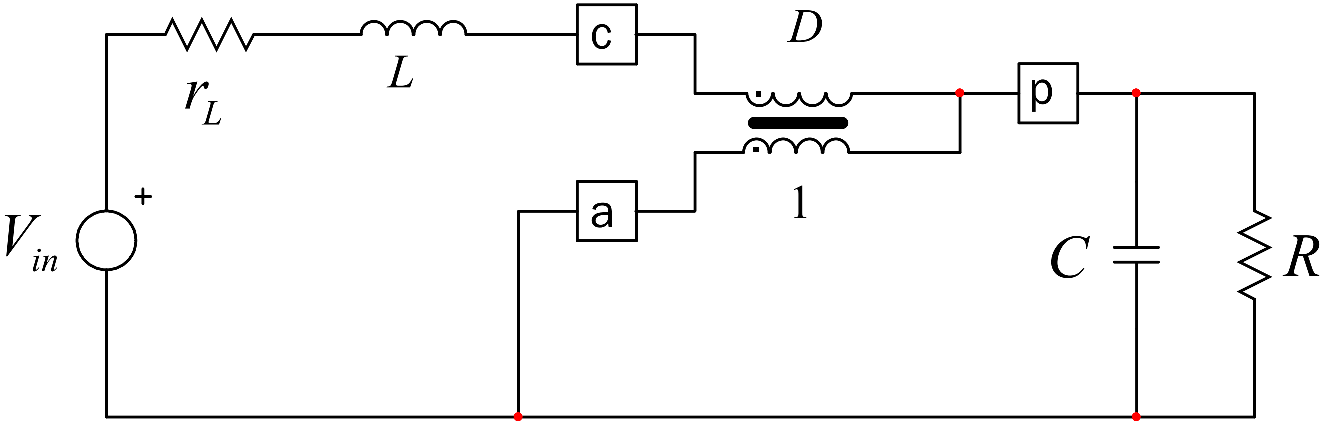


$$I_a = DI_c \qquad I_c = \frac{I_a}{D}$$

$$V_{ap} = \frac{V_{cp}}{D} \qquad V_{cp} = DV_{ap}$$

dc equations!

- ❑ It can be plugged into any 2-switch CCM converter

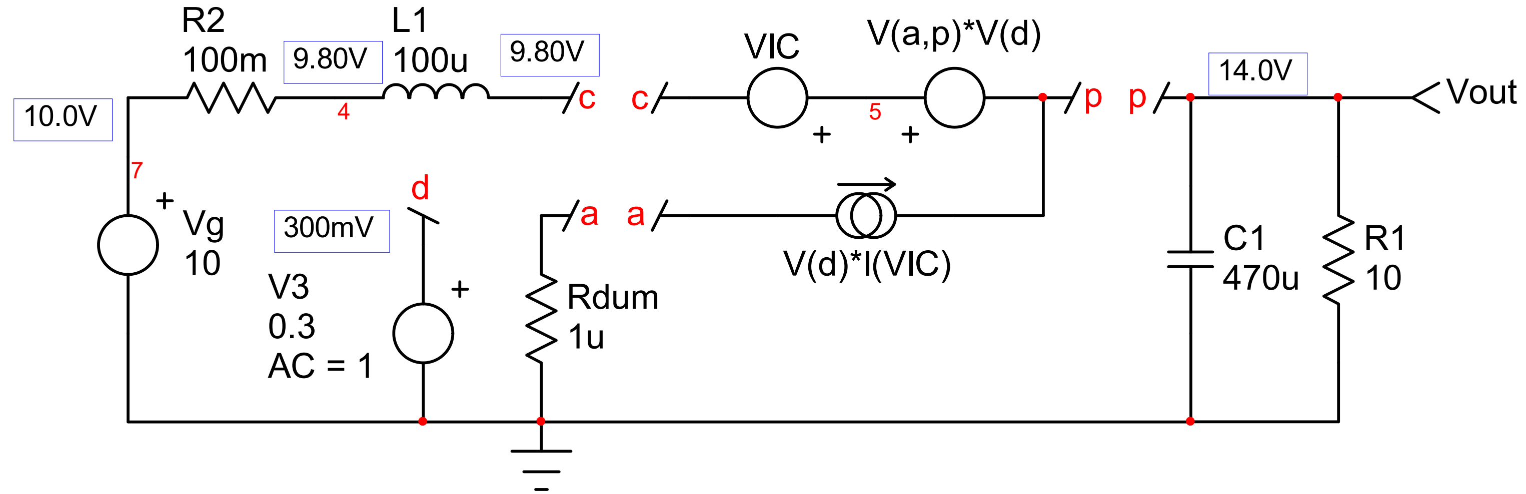


Dc bias point
Ac response

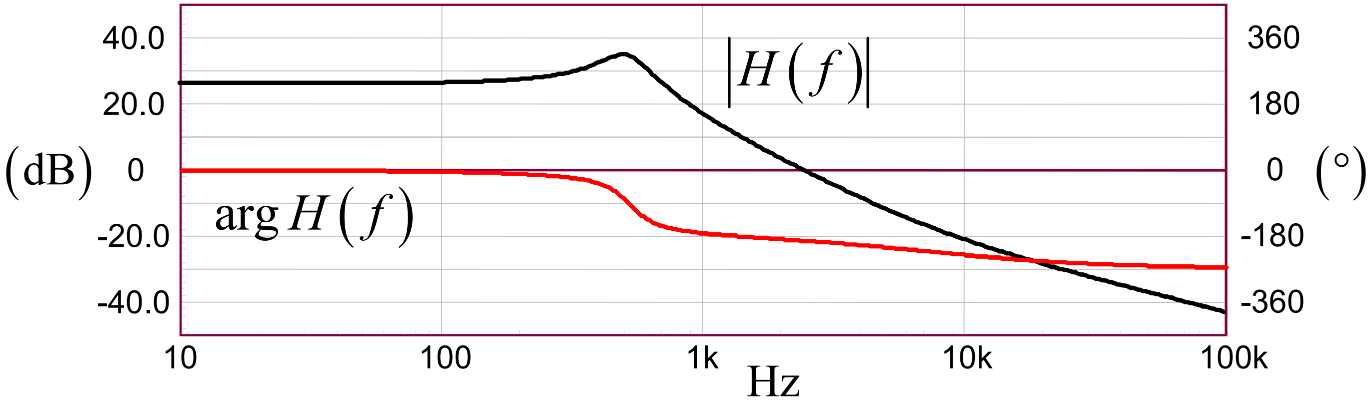
CCM VM

Simulate Immediately

- ❑ SPICE can get you the dc bias point



- ❑ …but also the ac response as it linearizes the circuit

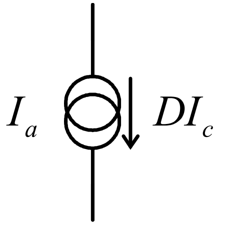


CCM VM

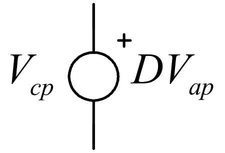
A Small-Signal Model

☐ We need a small-signal version to get the ac response

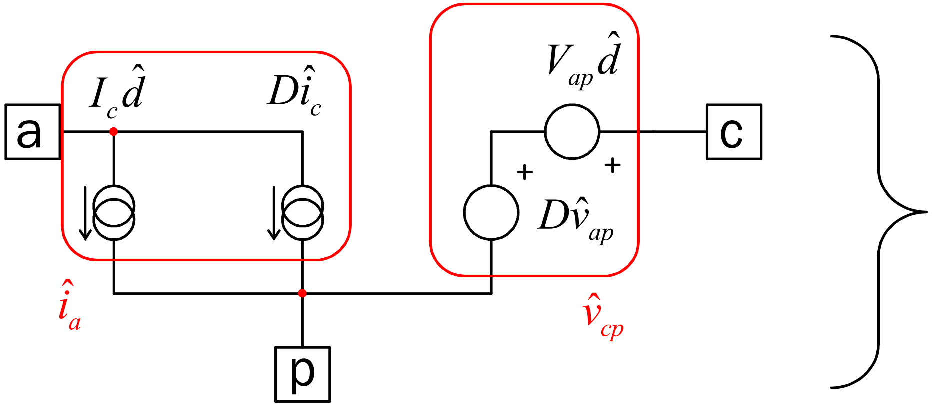
❖ Perturb equations or run partial differentiation

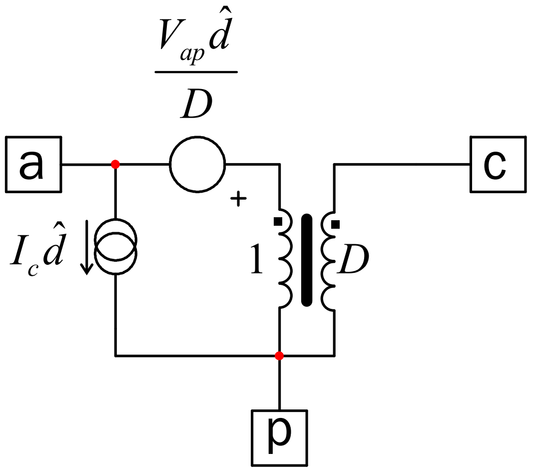


$$\xrightarrow{\text{2 variables}} \quad \hat{i}_a = \frac{\partial f\left(D,I_c\right)}{\partial D}\hat{d} + \frac{\partial f\left(D,I_c\right)}{\partial I_c}\hat{i}_c \quad \Longrightarrow \quad \hat{i}_a = I_c\hat{d} + D\hat{i}_c$$



$$\xrightarrow{\text{2 variables}} \quad \hat{v}_{cp} = \frac{\partial f\left(D,V_{ap}\right)}{\partial D}\hat{d} + \frac{\partial f\left(D,V_{ap}\right)}{\partial V_{ap}}\hat{v}_{ap} \quad \Longrightarrow \quad \hat{v}_{cp} = V_{ap}\hat{d} + D\hat{v}_{ap}$$



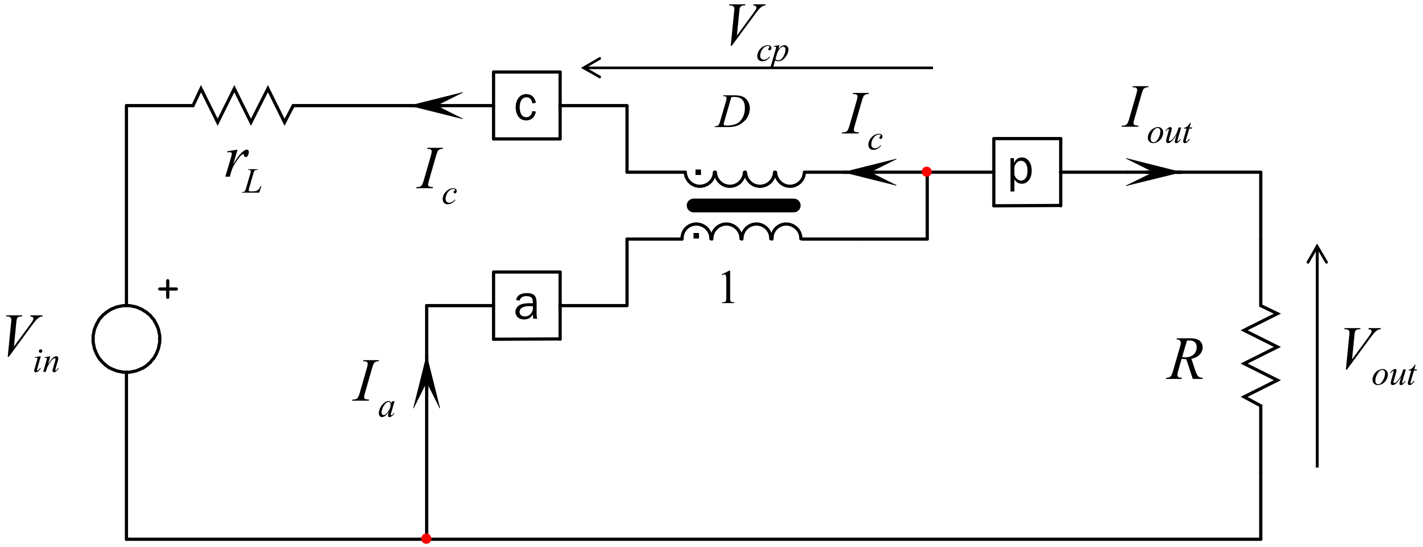


Small-signal model

CCM VM

Use it to Compute the Bias Point

- ☐ Derive the dc transfer function: open caps., short inductors



$$V_{out} = I_{out} R \qquad V_{in} + r_L I_c - V_{cp} = V_{out}$$

$$V_{out} = \left(I_a - I_c\right) R \qquad V_{in} + r_L I_c + D V_{out} = V_{out}$$

$$I_a = D I_c \qquad \boxed{I_c} = \frac{V_{out}\left(1-D\right) - V_{in}}{r_L}$$

$$V_{out} = I_c \left(D-1\right) R$$

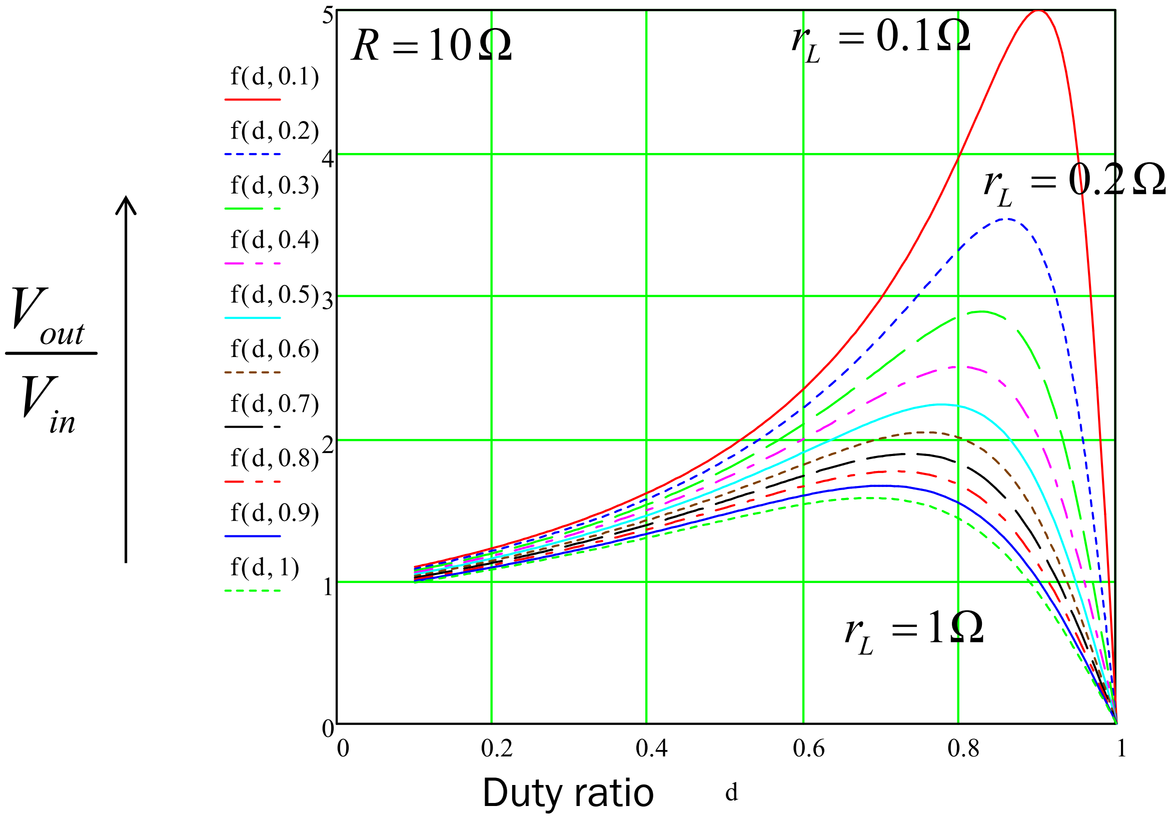
$$\frac{V_{out}}{V_{in}} = \frac{1}{\left(1-D\right) - \frac{r_L}{\left(D-1\right)R}}$$

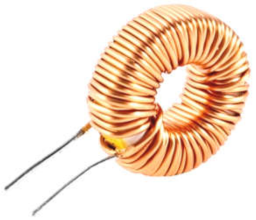
$$\frac{V_{out}}{V_{in}} = \frac{1}{D'} \frac{1}{1 + \frac{r_L}{RD'^2}}$$

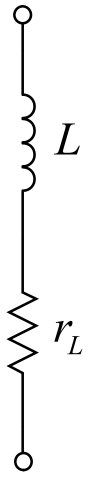
CCM VM

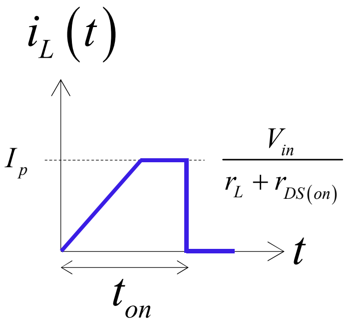
Plotting Transfer Functions

- ☐ Plot the lossy boost transfer function in a snapshot







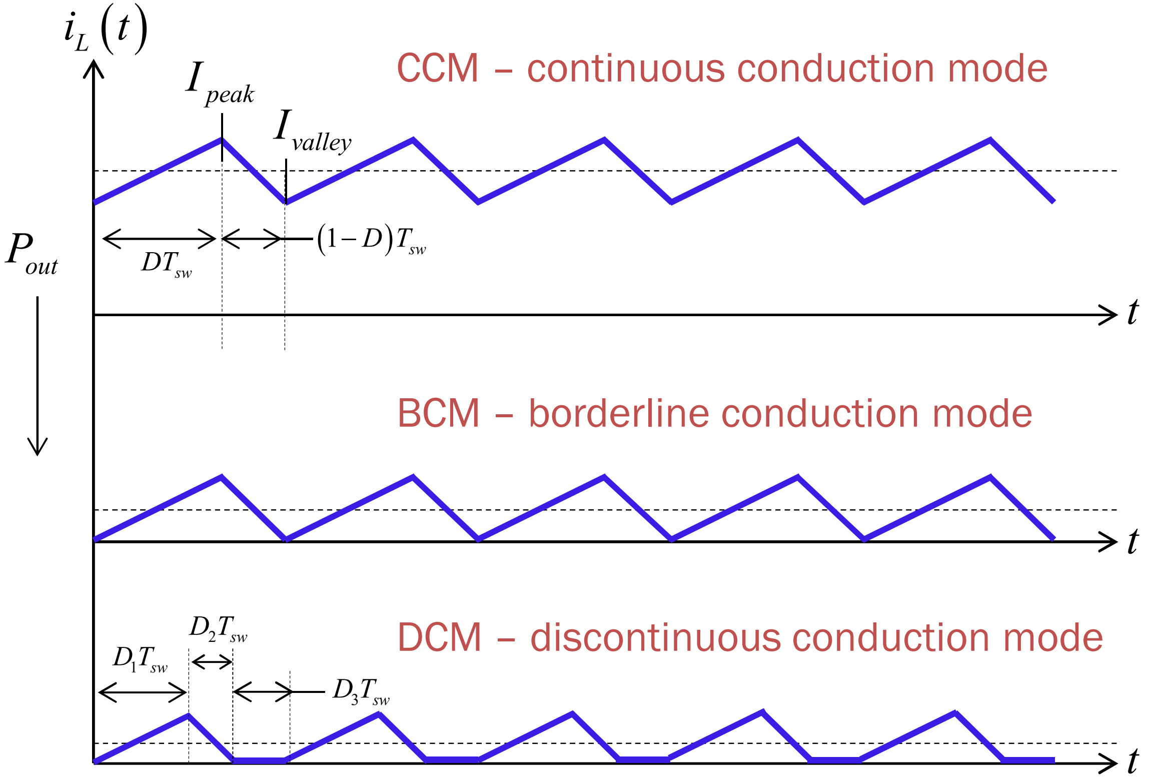


- ☐ Above a certain conversion ratio, latch-up can occur

CCM VM

Changing the Operating Mode

- ☐ The average inductor current reduces as I_{out} gets smaller



$$\langle i_L(t)\rangle = \frac{I_{peak}+I_{valley}}{2} > \frac{I_{peak}}{2}$$

$$\langle i_L(t)\rangle = \frac{I_{peak}}{2}$$

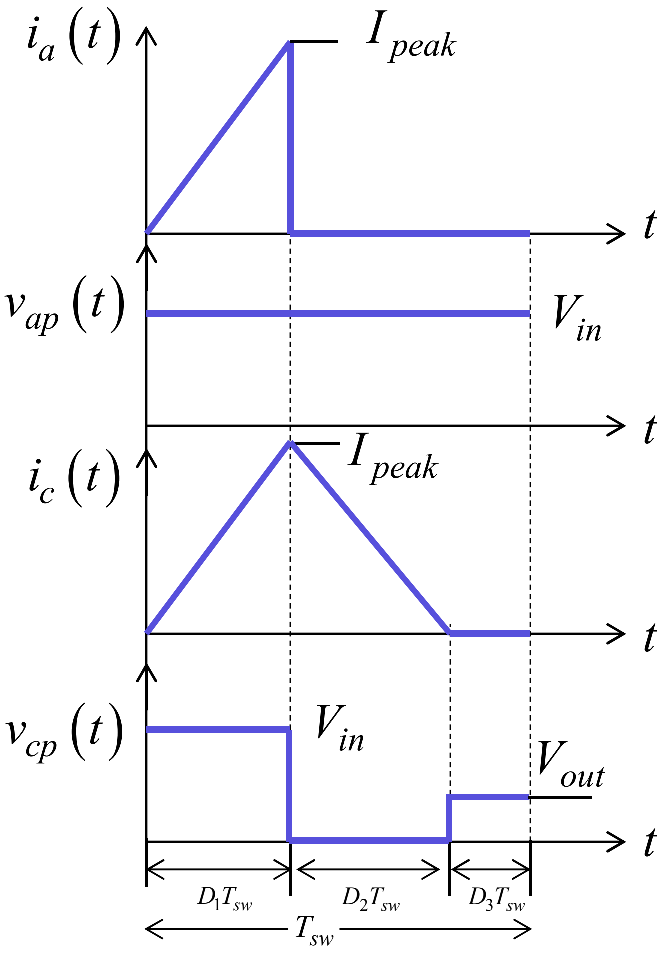
$$\langle i_L(t)\rangle_{T_{sw}} = I_{peak}\left(\frac{D_1+D_2}{2}\right) < \frac{I_{peak}}{2}$$

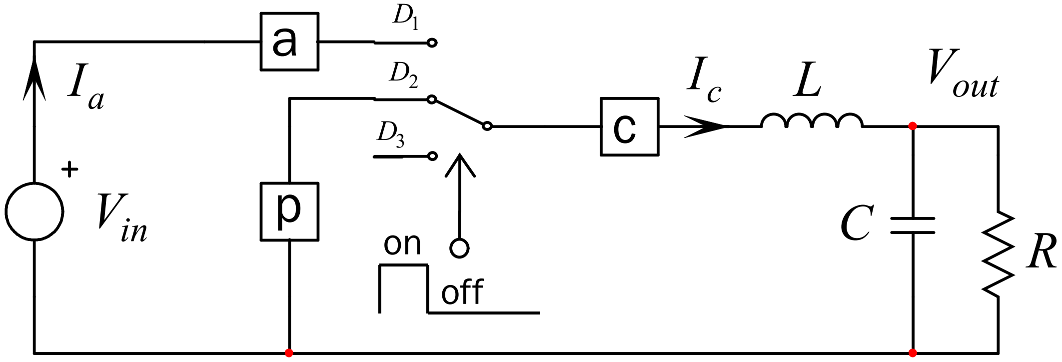
- ☐ When the load becomes lighter, DCM is entered

CCM: continuous conduction mode – DCM: discontinuous conduction mode

A Similar Configuration in DCM

- ❑ Draw the waveforms in the "common passive" configuration





- ❑ Average the waveforms:

$$I_a = \frac{I_{peak}}{2} D_1$$

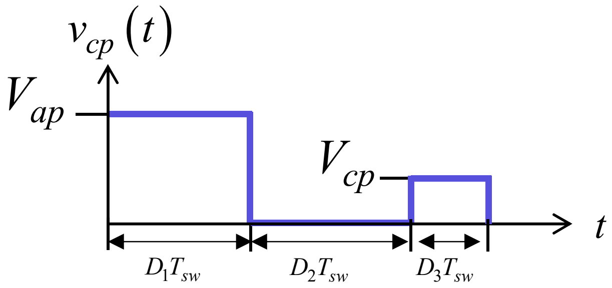
$$I_c = \frac{I_{peak}}{2} D_1 + \frac{I_{peak}}{2} D_2 = \frac{I_{peak}}{2}(D_1 + D_2)$$

$$I_c = \frac{2I_a}{D_1} \frac{D_1 + D_2}{2} = I_a \frac{D_1 + D_2}{D_1}$$

DCM VM

Derive V_{cp} to Unveil the New Model

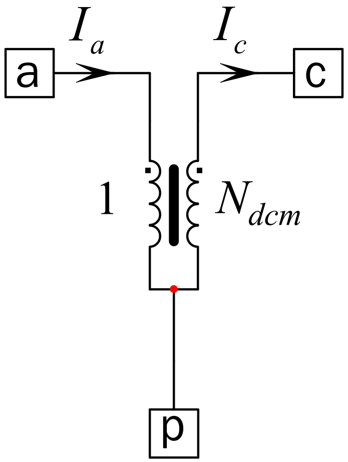
❑ The addition of the third event complicates the equations



$$V_{cp} = V_{ap}D_1 + V_{cp}D_3 \qquad D_1 + D_2 + \boxed{D_3} = 1$$

$$V_{cp} = V_{ap}D_1 + V_{cp}\left(1 - D_1 - D_2\right)$$

$$V_{cp} = V_{ap}\frac{D_1}{D_1 + D_2}$$



$$I_a = N_{dcm}I_c \qquad I_c = \frac{I_a}{N_{dcm}}$$

Control input

$$N_{dcm} = \frac{\boxed{D_1}}{D_1 + D_2} \to f\left(D_1\right)$$

$$V_{ap} = \frac{V_{cp}}{N_{dcm}} \qquad V_{cp} = N_{dcm}V_{ap}$$

DCM VM

Finally, Get the D_2 Value

- ☐ In DCM the inductor average voltage per cycle is always 0

$$\longrightarrow V_{cp} = V_{out}$$

- ☐ What is the averaged inductor peak current?

$$\left.\begin{aligned} I_{peak} &= \frac{\langle v_L(t)\rangle_{D_1T_{sw}}}{L} D_1T_{sw} \\ \langle v_L(t)\rangle_{D_1T_{sw}} &= V_{ac} \end{aligned}\right\} V_{ac} = L\frac{\boxed{I_{peak}}}{D_1T_{sw}}$$

- ☐ The peak current uses a previous expression

$$I_c = \frac{I_{peak}}{2}(D_1 + D_2) \longrightarrow \boxed{I_{peak}} = \frac{2I_c}{D_1 + D_2}$$

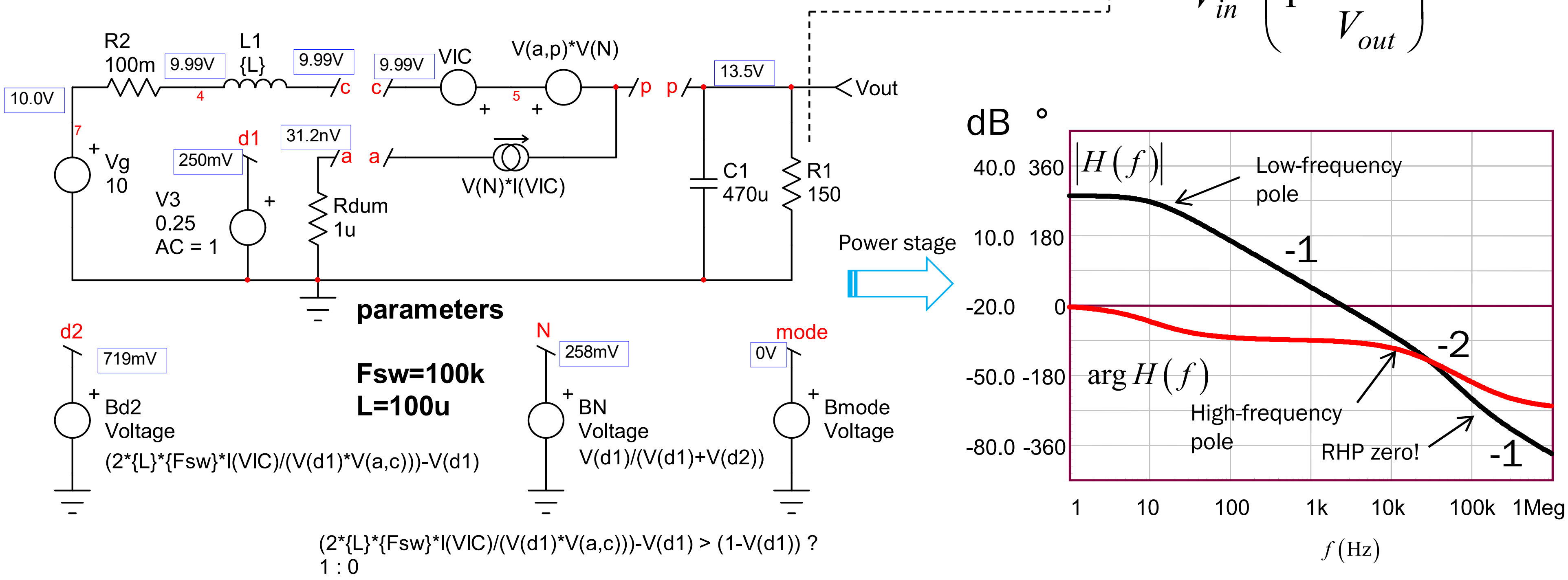
$$\Rightarrow D_2 = \frac{2LF_{sw}}{D_1}\frac{I_c}{V_{ac}} - D_1$$

DCM VM

Simulate with the DCM Model

- ☐ Check the converter is operating in DCM:

$$R_{crit} = \frac{2F_{sw}LV_{out}^{2}}{V_{in}^{2}\left(1-\frac{V_{in}}{V_{out}}\right)}$$



- ☐ In light load, the converter is still a second-order system
  - ❖ There is a high-frequency right-half-plane zero

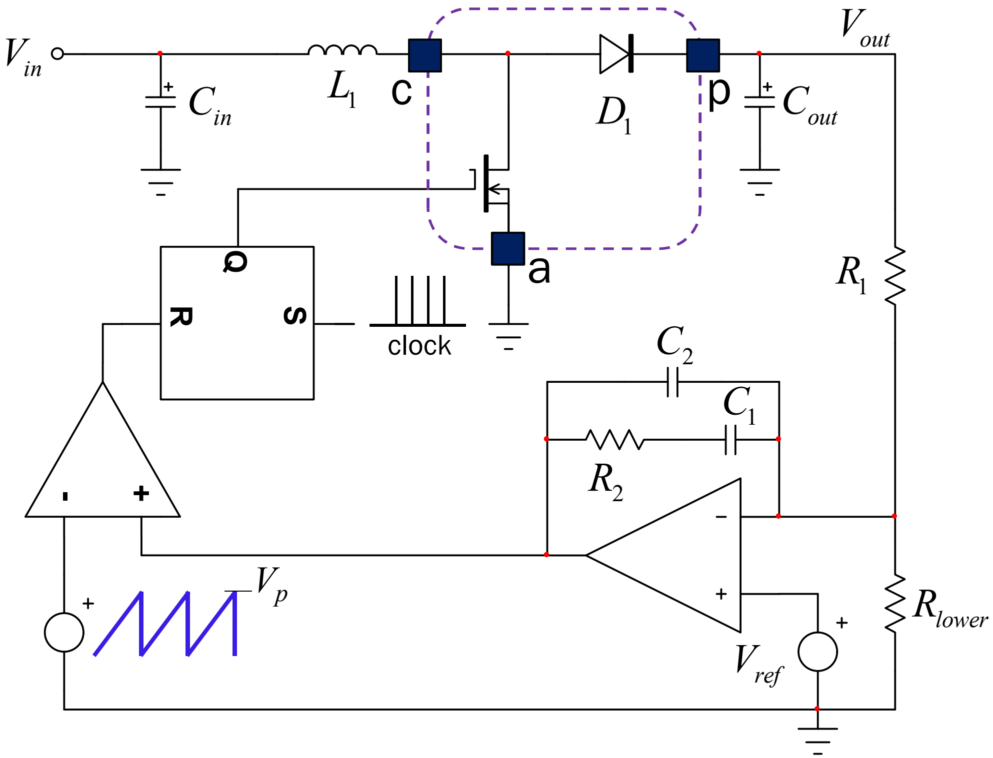
For a comprehensive analysis see APEC 2013 seminar: *Small-Signal Modeling and Analytical Analysis of Power Converter*

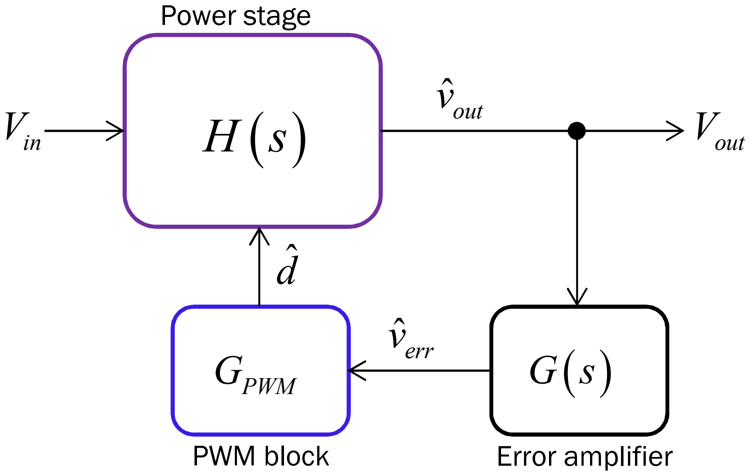
Course Agenda

- ☐ Blocks in a Switching Converter
- ☐ Introduction to Small-Signal Modeling
- ☐ Analytical Analysis of an Output Stage
- ☐ Simulation Models - Averaged or Switched?
- ☐ Crossover Frequency and Phase Margin
- ☐ Compensation Strategy
- ☐ Experiments on Prototypes
- ☐ Conclusion

The Project: A VM Boost Converter

- ☐ We want to build and stabilize a voltage-mode boost converter
- ❖ Input range is 8-12 V, output voltage is 15 V, 1 A





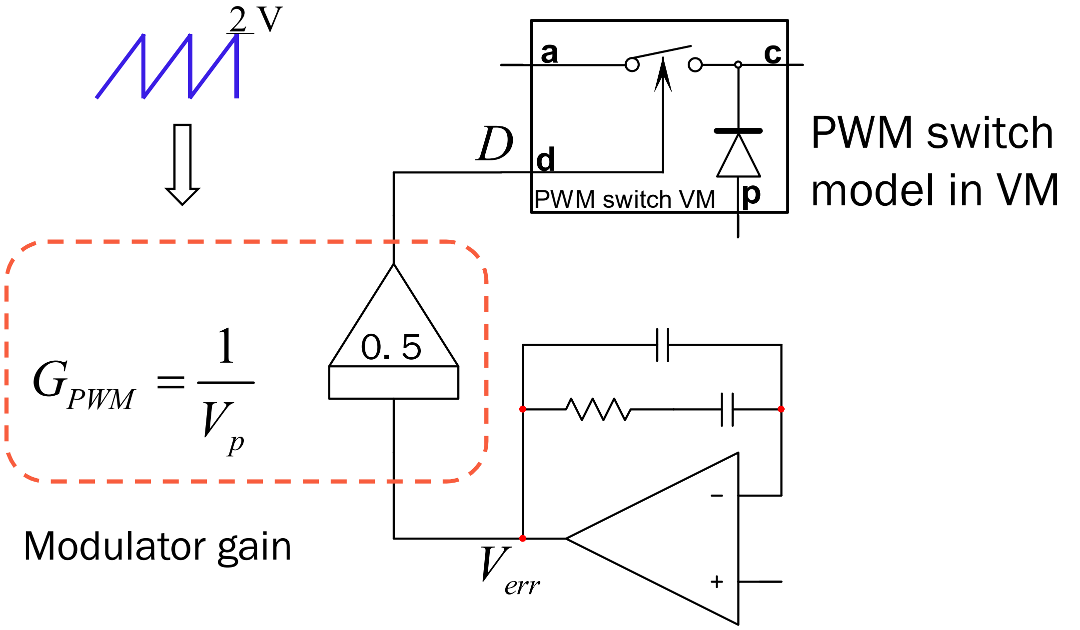
- ☐ Identify the blocks and model them separately

The Naturally-Sampled Modulator

- ☐ It's role is to convert a voltage V_{err} into a duty ratio D

$$d(t) = \frac{v_{err}(t)}{V_p} \xrightarrow[\text{over } T_{sw}]{\text{average}} D(V_{err}) = \frac{V_{err}}{V_p} \longrightarrow G_{PWM} = \frac{dD(V_{err})}{V_{err}} = \frac{1}{V_p}$$

- ☐ It is a simple block inserted before the D input of the model

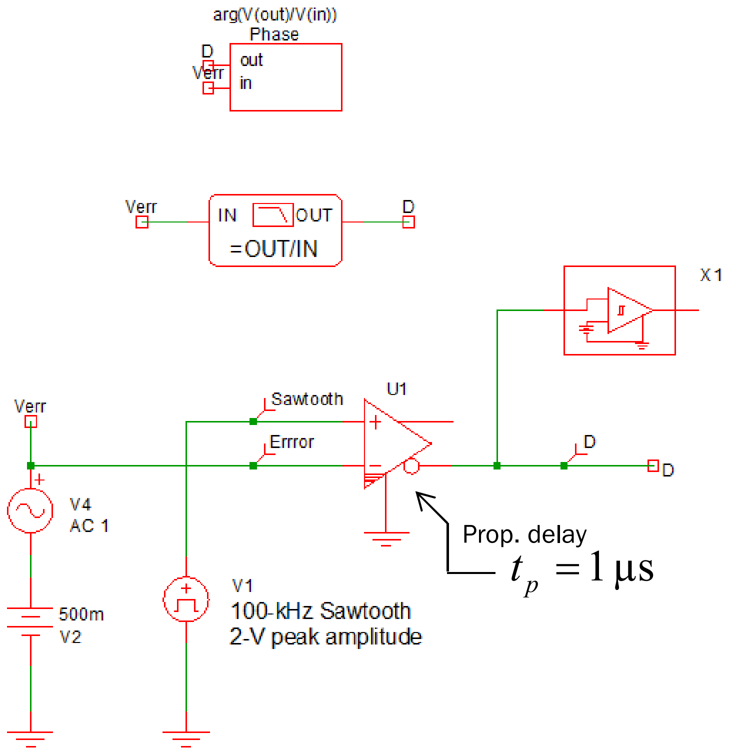


PWM switch
model in VM

Modulator gain

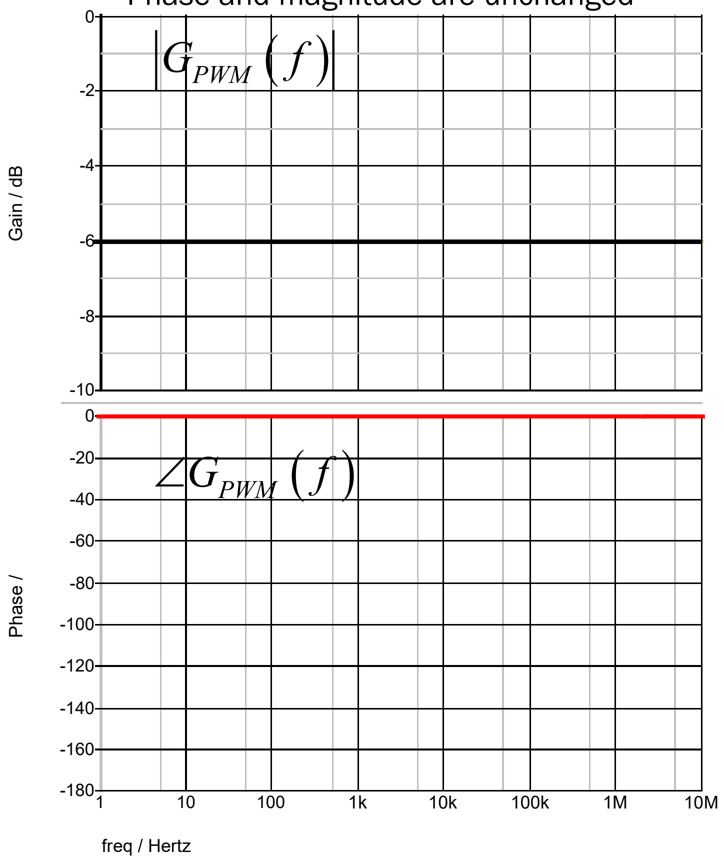
A Flat Frequency Response

- ☐ This modulator can be simulated with Simplis®



$t_p = 1\,\mu\text{s}$

Phase and magnitude are unchanged



$\left|G_{PWM}(f)\right|$

$\angle G_{PWM}(f)$

$$G_{PWM} = \frac{1}{2}$$

$$\downarrow$$

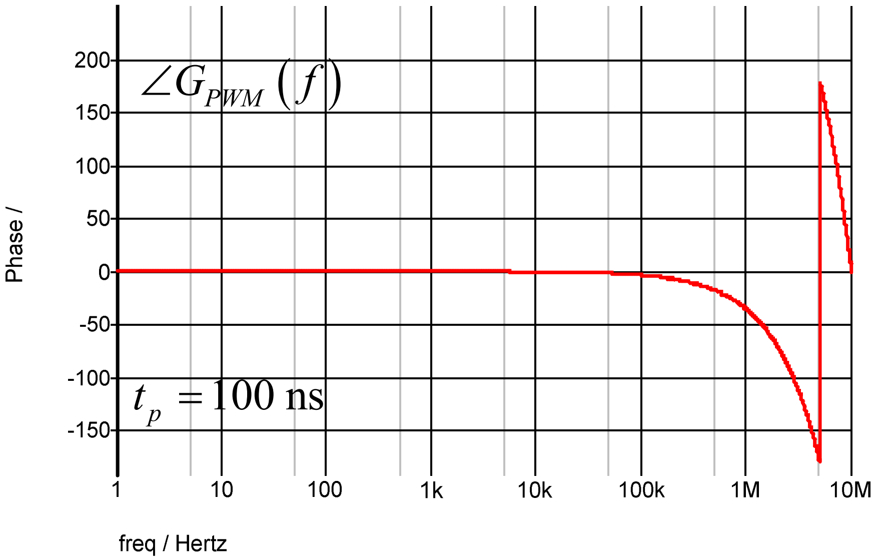
-6 dB

- ☐ With a perfect comparator, no lag in the phase response

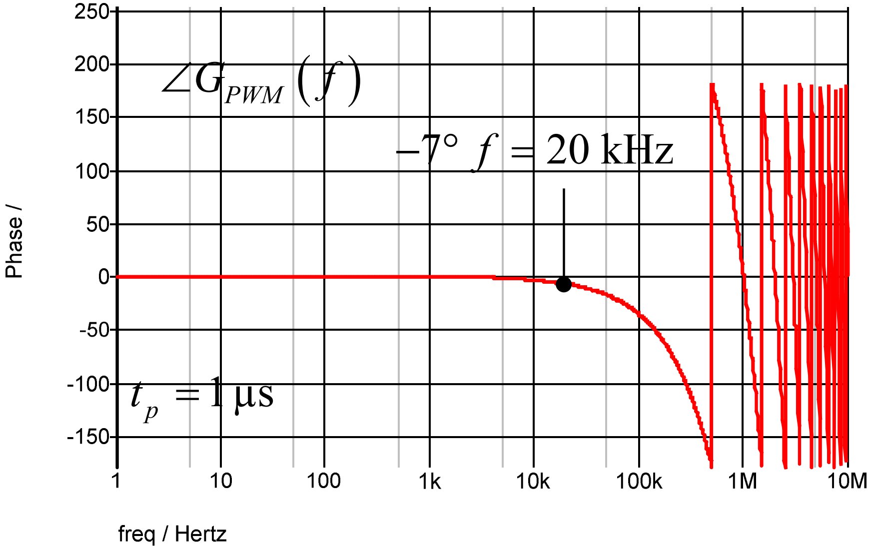
Watch for the Comparator Delay

- The comparator is affected by a propagation delay t_p

Magnitude is unchanged



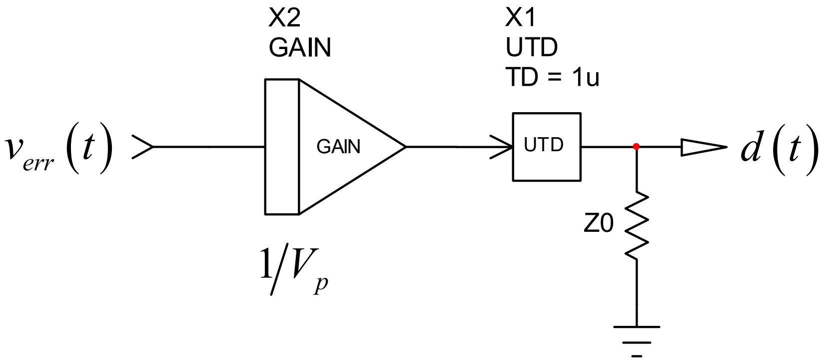
Magnitude is unchanged



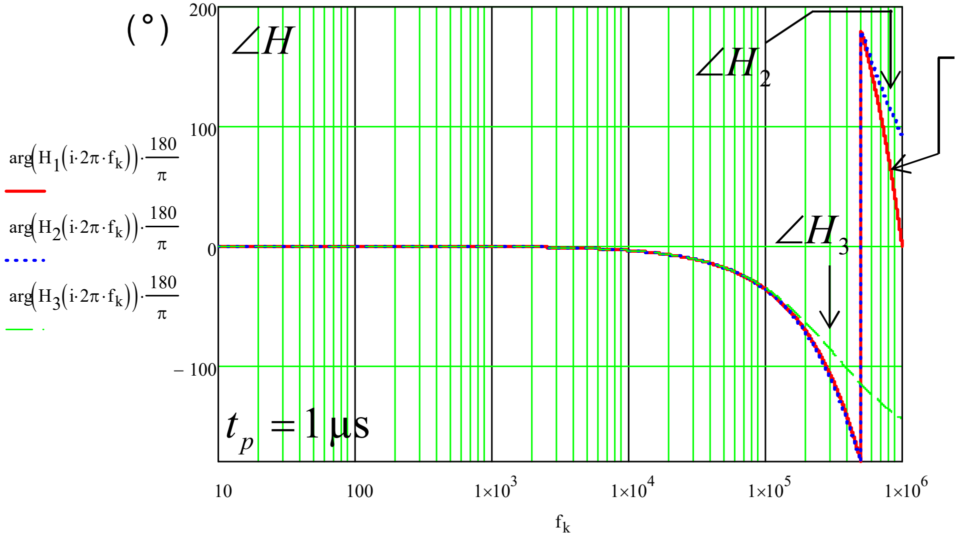
- Assume you reduce bias currents for standby, prop. delay rises!
  - Phase margin can be affected and stability at stake
  - For your high-bandwidth designs, you must include this delay

Model the Delay for High-Speed Designs

- It can be a simple delay line in a SPICE simulation



- First- and second-order approximants can also be used



$$H_1(s)=e^{-s\cdot t_p}$$

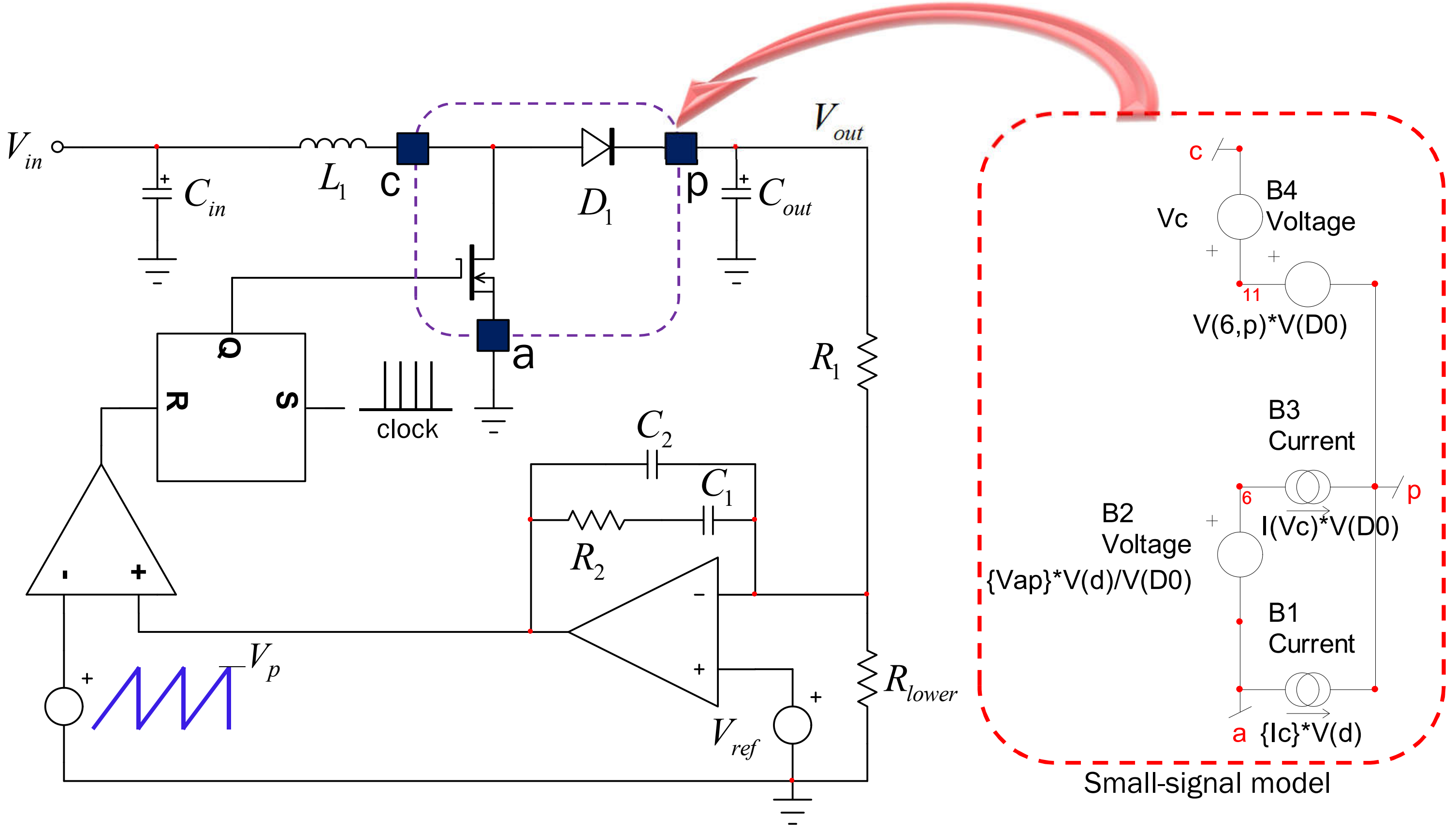
$$H_2(s)=1-\frac{s\cdot t_p}{1+\frac{s}{Q\omega}+\left(\frac{s}{\omega}\right)^2} \qquad Q=\frac{2}{\pi} \quad \omega=\frac{\pi}{t_p}$$

$$H_3(s)=\frac{1-\frac{s\cdot t_p}{2} \quad \longleftarrow \text{RHPZ}}{1+\frac{s\cdot t_p}{2} \quad \longleftarrow \text{LHPP}}$$

H_3 is a 1st-order Padé approximant

Analyzing the CCM Boost Converter

- ☐ Plug the small-signal model of the PWM switch in the schematic



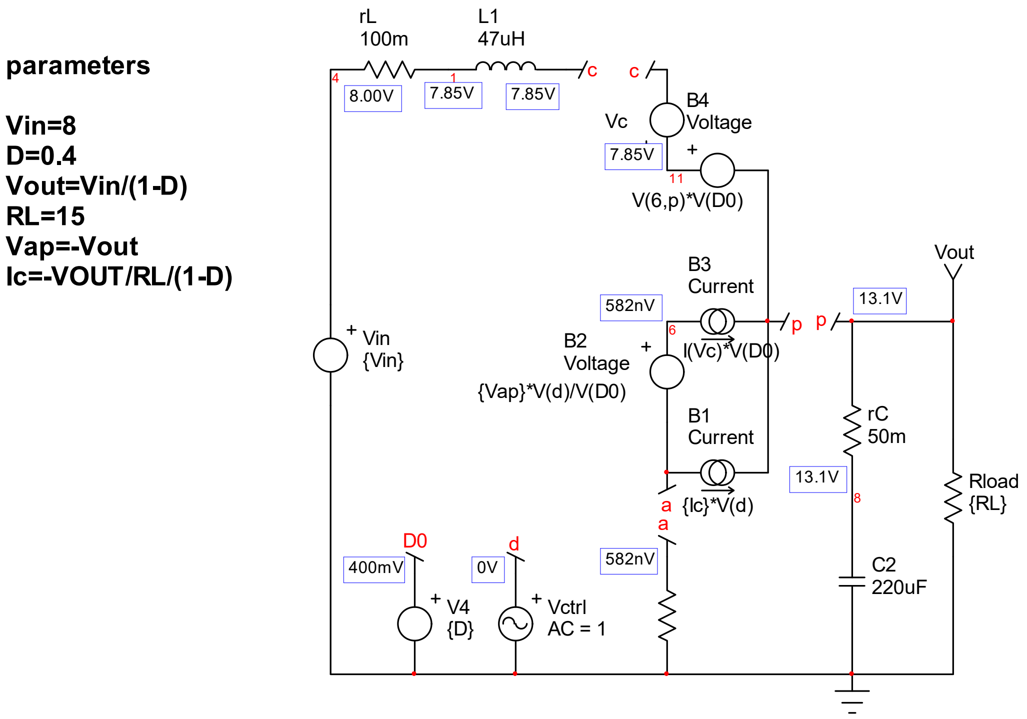
parameters

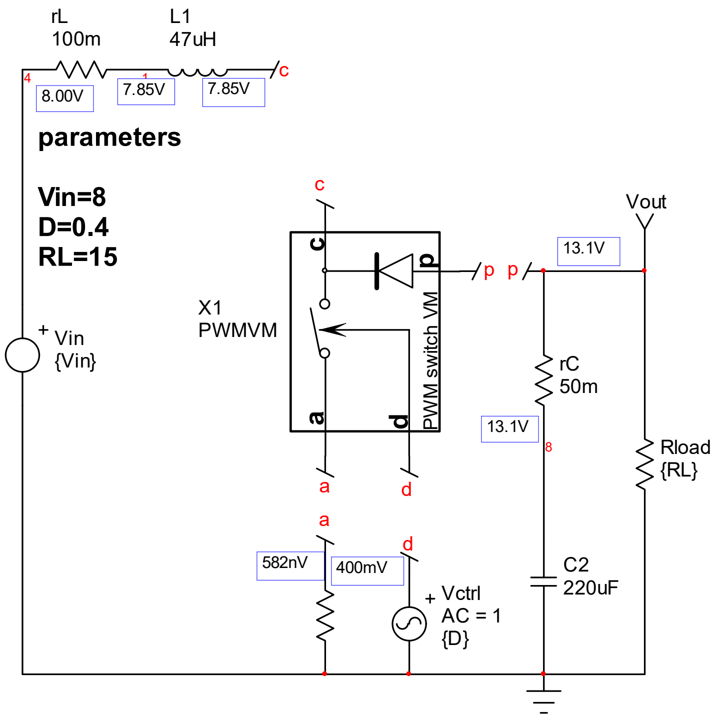
V_{in}=10
D=0.4
$V_{out}=V_{in}/(1-D)$
R_L=10
$V_{ap}=-V_{out}$
$I_c=-V_{out}/R_L/(1-D)$

- ☐ Capture the new schematic featuring the linear sources

Compare Large- and Small-Signal

- ☐ Confirm both approaches deliver the exact same response

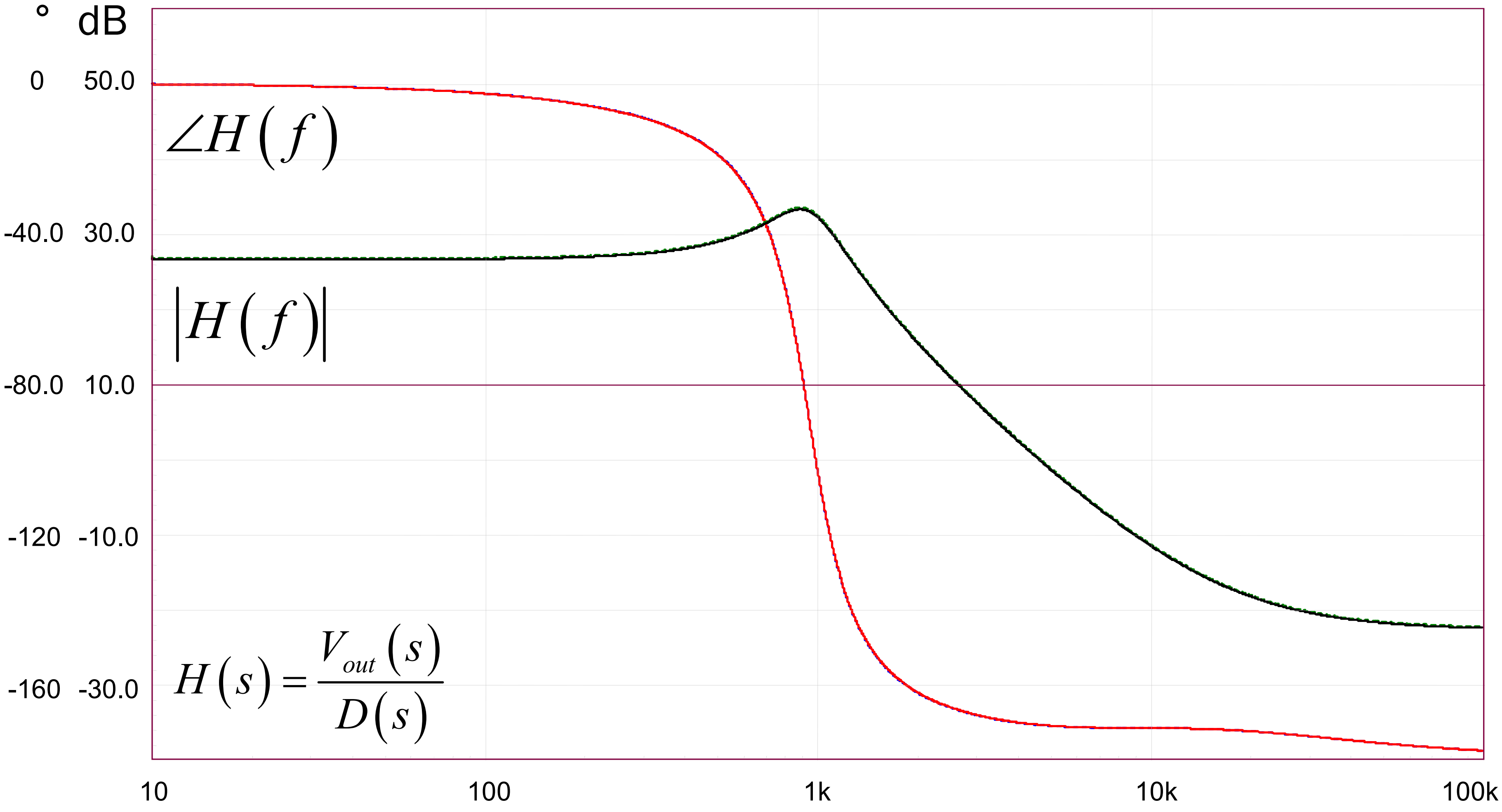




- ☐ Run this sanity check to confirm the linearized model is ok
- ❖ Operating points and dynamic responses must match

We are Good to Go!

- ☐ Refer to these curves after rearranging or simplifying the circuit



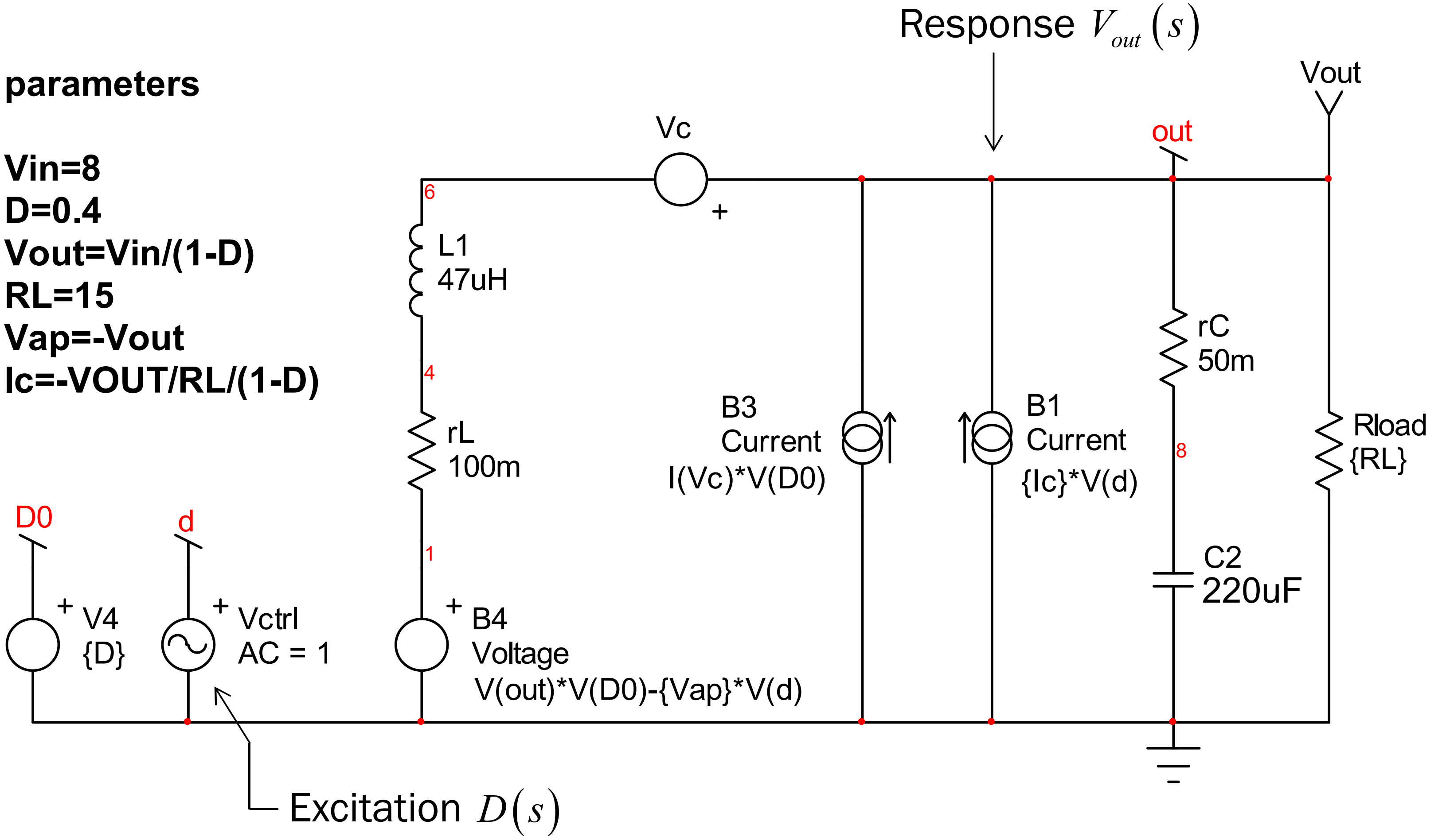
$$H(s) = \frac{V_{out}(s)}{D(s)}$$

Rearrange and Simplify the Sketch

- ☐ We want the control to output transfer function $\frac{V_{out}(s)}{D(s)}$

parameters

Vin=8
D=0.4
Vout=Vin/(1-D)
RL=15
Vap=-Vout
Ic=-VOUT/RL/(1-D)



How to Determine the Transfer Function?

- We can use the Fast Analytical Circuits Techniques

$$\boxed{\frac{V_{out}(s)}{D(s)}} = H_0 \frac{1+a_1 s + a_2 s^2}{1+b_1 s + b_2 s^2} \longrightarrow \left. \begin{matrix} a_1 \text{ and } b_1 \left[\mathrm{s}\right] \\ a_2 \text{ and } b_2 \left[\mathrm{s}^2\right] \end{matrix} \right\}$$

Same dimension

$a_1 = \tau_1 + \tau_2$

Nulled response

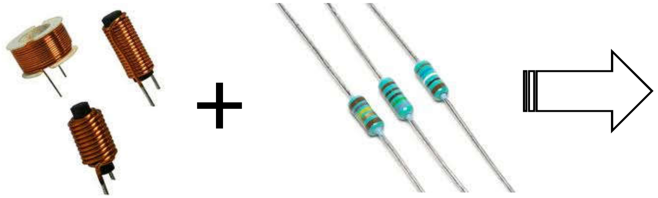
$a_2 = \tau_1 \tau_2^1$

$b_1 = \tau_1 + \tau_2$

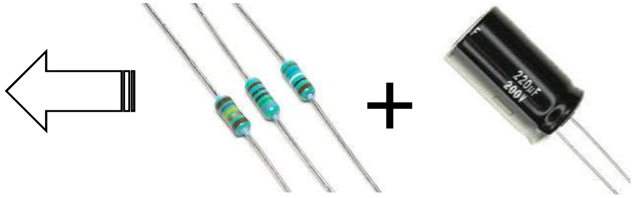
Zeroed excitation

$b_2 = \tau_1 \tau_2^1$

- Energy-storing elements are combined with resistances

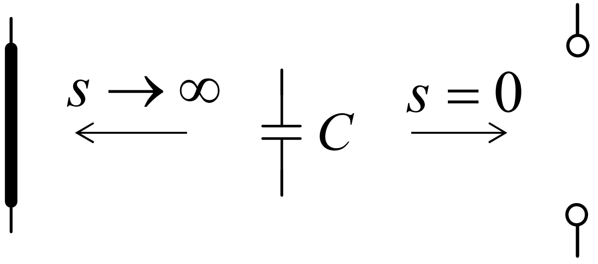


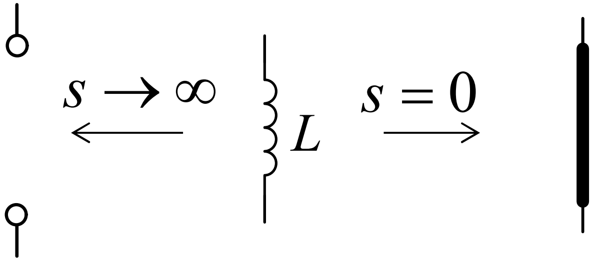
$$\tau = \frac{L}{R} \qquad \tau = RC$$



Time constants

- Capacitors and inductors behave differently for $s = 0$ and $s \to \infty$

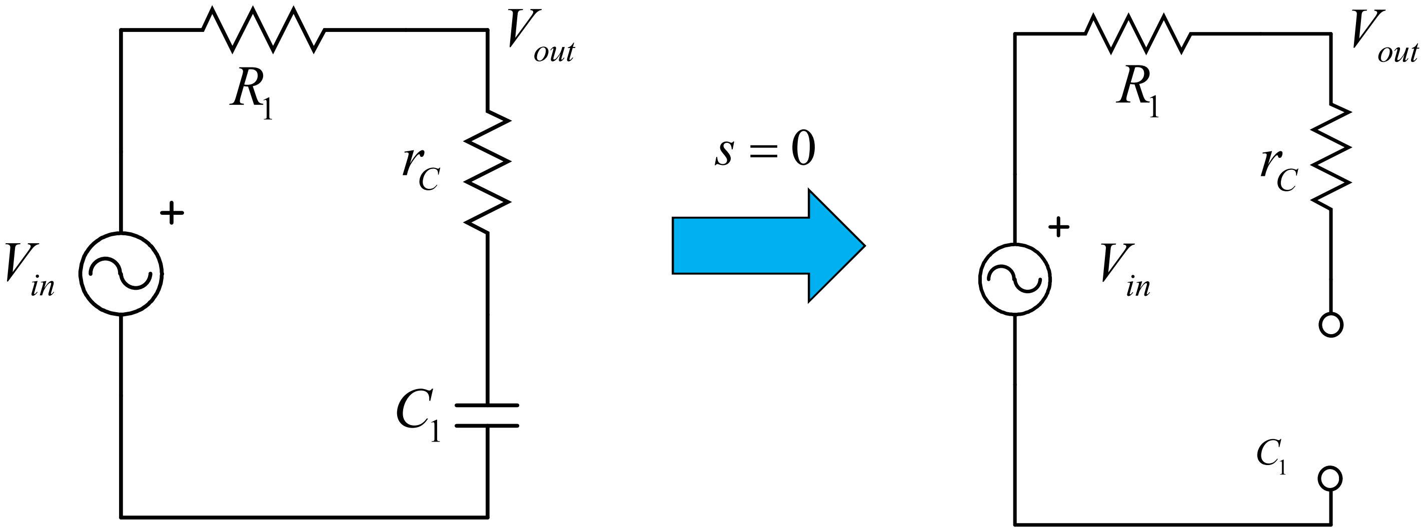




Fast Analytical Techniques at a Glance

- ☐ Look at the circuit for $s = 0$
  - ➢ Capacitor are open circuited
  - ➢ Inductors are short circuited

} SPICE operating point calculation



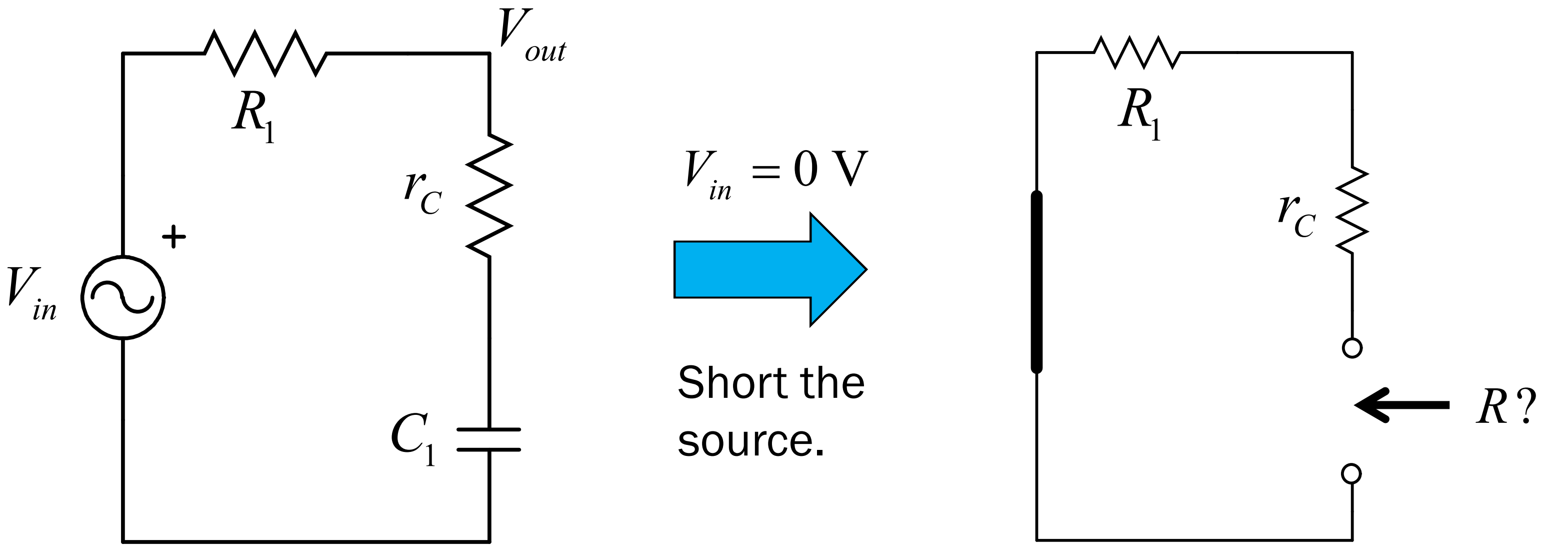
- ☐ Determine the gain in this condition

$$H_0 = V_{out} / V_{in} = 1$$

Fast Analytical Techniques at a Glance

- ❑ Look at the resistance driving the storage element

1. When the excitation is turned off, V_{in} = 0 V



Short the source.

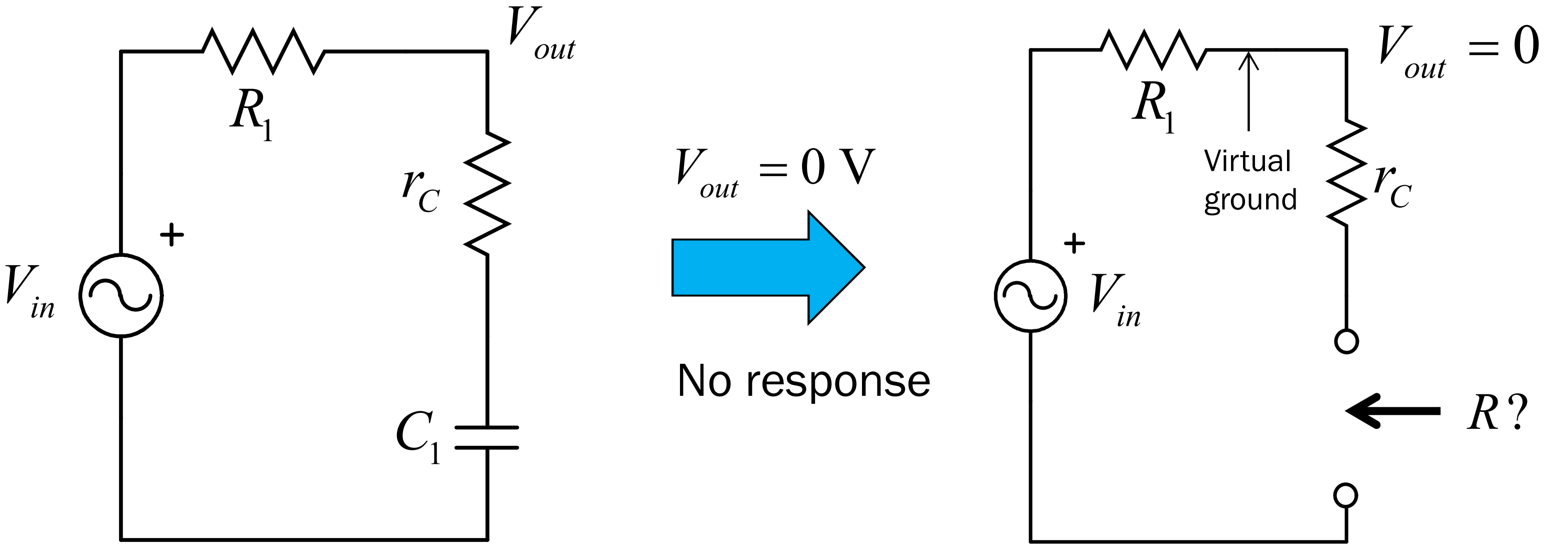
- ❑ Remove the capacitor and look into its terminals

➢ The first time constant is $\tau_1 = \left(r_C + R_1\right)C_1$

Fast Analytical Techniques at a Glance

- ☐ Look at the resistance driving the storage element

2. When the excitation is back but V_{out} = 0 V



- ☐ Remove the capacitor and look into its terminals

➢ The second time constant is $\tau_2 = r_C C_1$

Combining Time Constants

- ☐ By combining time constants, we have

$$H(s) = H_0 \frac{1+s\tau_2}{1+s\tau_1} = \frac{1+sr_C C_1}{1+s(r_C+R_1)C_1}$$

- ☐ Rearrange the equation to unveil a pole and a zero

$$H(s) = H_0 \frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}} \left.\vphantom{\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}}\right\} \quad \omega_z = \frac{1}{r_C C_1} \qquad H_0 = 1$$

$$\omega_p = \frac{1}{(r_C+R_1)C_1}$$

- ☐ This is a *low-entropy* expression

Higher-Order Systems

☐ Set one reactance into its high-frequency state

$$\tau_{2}^{1}$$

$1 \longrightarrow$ Reactance 1 is in its high-frequency state

$2 \longrightarrow$ What resistance drives reactance 2?

$R?$

$$\tau_{1}^{2}$$

$2 \longrightarrow$ Reactance 2 is in its high-frequency state

$1 \longrightarrow$ What resistance drives reactance 1?

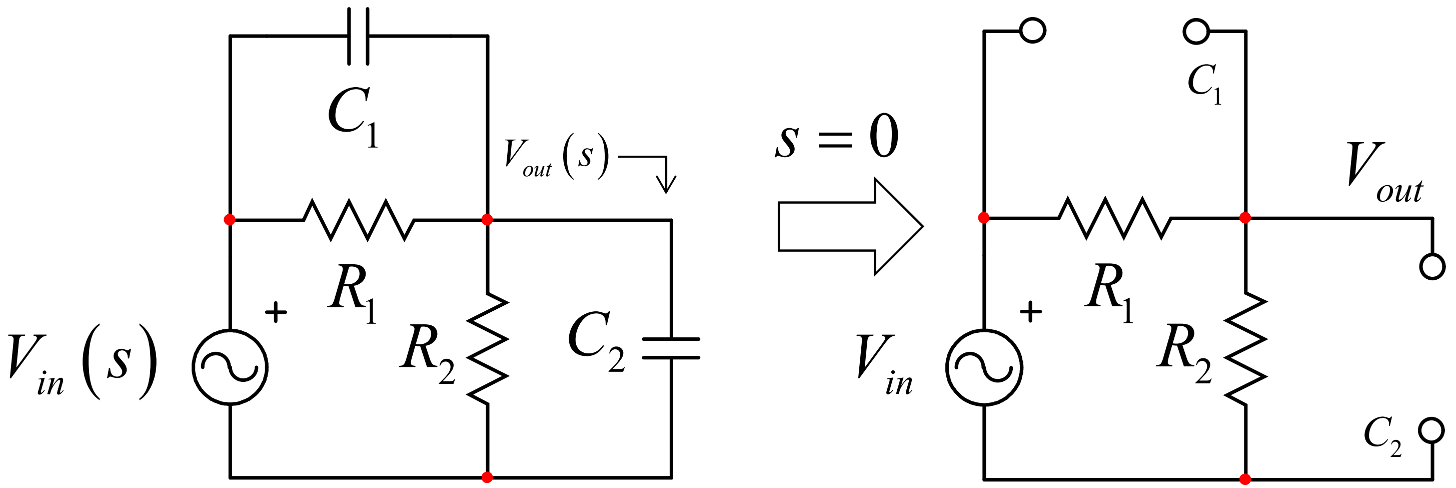
$R?$

☐ There is redundancy: pick the simplest result

$$b_2 = \tau_1 \tau_2^1 \quad \Longleftrightarrow \quad b_2 = \tau_2 \tau_1^2$$

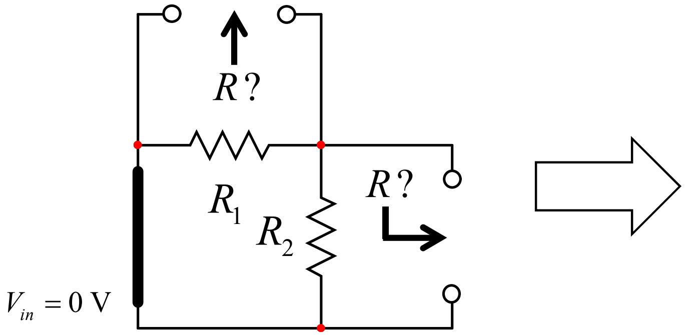
Example with Capacitors

- ☐ Assume the following 2-capacitor circuit



$$H_0 = \frac{R_2}{R_1 + R_2}$$

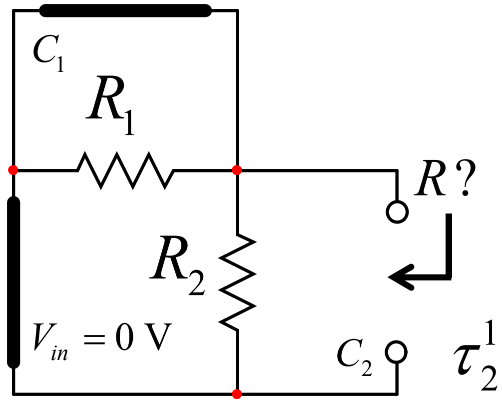
- ☐ Determine the two time constants while V_{in} is 0 V



$$\left.\begin{aligned}\tau_1 &= C_1\left(R_1 \,||\, R_2\right)\\ \tau_2 &= C_2\left(R_1 \,||\, R_2\right)\end{aligned}\right\} \quad b_1 = \left(C_1 + C_2\right)\left(R_1 \,||\, R_2\right)$$

Determining the Higher-Order Term

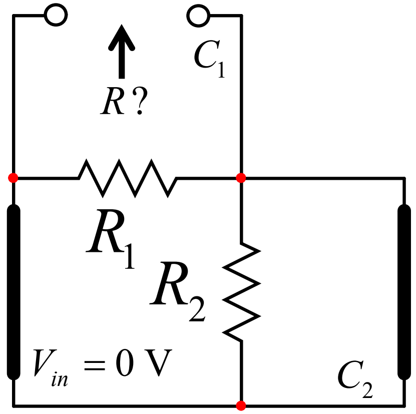
- ☐ Place C_1 in its high-frequency and look into C_2



$$\tau_2^1 = 0 \cdot C_2$$

$$b_2 = \tau_1 \tau_2^1 = C_1 \left(R_1 \,||\, R_2 \right) \cdot 0 \cdot C_2 = 0$$

- ☐ Place C_2 in its high-frequency and look into C_1



$$\tau_1^2 = 0 \cdot C_1$$

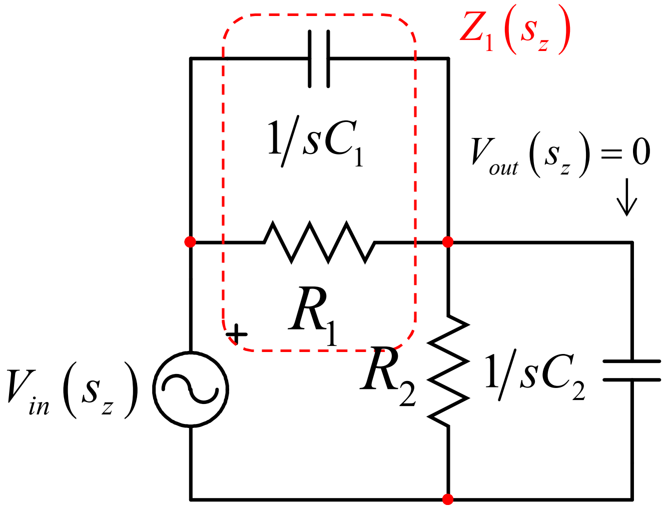
$$b_2 = \tau_2 \tau_1^2 = C_2 \left(R_1 \,||\, R_2 \right) \cdot 0 \cdot C_2 = 0$$

Denominator is Completed

- ☐ The denominator can be assembled

$$D(s)=1+b_1s+b_2s^2=1+(C_1+C_2)(R_1\,||\,R_2)s+0\cdot s^2$$

- ☐ Is there a zero in this network?



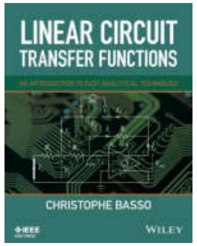
$$Z_1(s_z)\to\infty$$

$$\frac{N(s)}{\boxed{1+sR_1C_1}}\to\infty$$

$$1+sR_1C_1 \to =0$$

$$s_z=-\frac{1}{R_1C_1}$$

$$\omega_z=\frac{1}{R_1C_1}$$



$$H(s)=\frac{R_2}{R_1+R_2}\frac{1+sr_CC_1}{1+s(C_1+C_2)(R_1\,||\,R_2)}=H_0\frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}$$

C. Basso, *Linear Circuits Transfer Functions: An Introduction to Fast Analytical Techniques*, Wiley 2016

No algebra!

Check with Mathcad®

- ☐ It is easy to check results versus a raw expression

$R_1 := 50k\Omega \quad R_2 := 10k\Omega \quad C_1 := 100nF \quad C_2 := 10nF \quad \|(x,y) := \frac{x \cdot y}{x+y}$

$$H_0 := \frac{R_2}{R_1 + R_2}$$

$\tau_1 := C_1 \cdot (R_1 \| R_2) = 833.333\mu s \quad \tau_2 := C_2 \cdot (R_1 \| R_2) = 83.333\mu s \quad b_1 := \tau_1 + \tau_2 = 916.667\mu s$

$\tau_{12} := C_2 \cdot 0 = 0 \cdot \mu s \quad \tau_{21} := C_1 \cdot 0 \quad b_2 := \tau_2 \cdot \tau_{21} = 0 \cdot \mu s^2$

$N_1(s) := 1 + s \cdot R_1 \cdot C_1 \quad D_1(s) := 1 + b_1 \cdot s + b_2 \cdot s^2 \quad H_1(s) := H_0 \cdot \frac{N_1(s)}{D_1(s)}$

$\omega_z := \frac{1}{R_1 \cdot C_1} \quad \omega_p := \frac{1}{b_1}$

Low-entropy formula

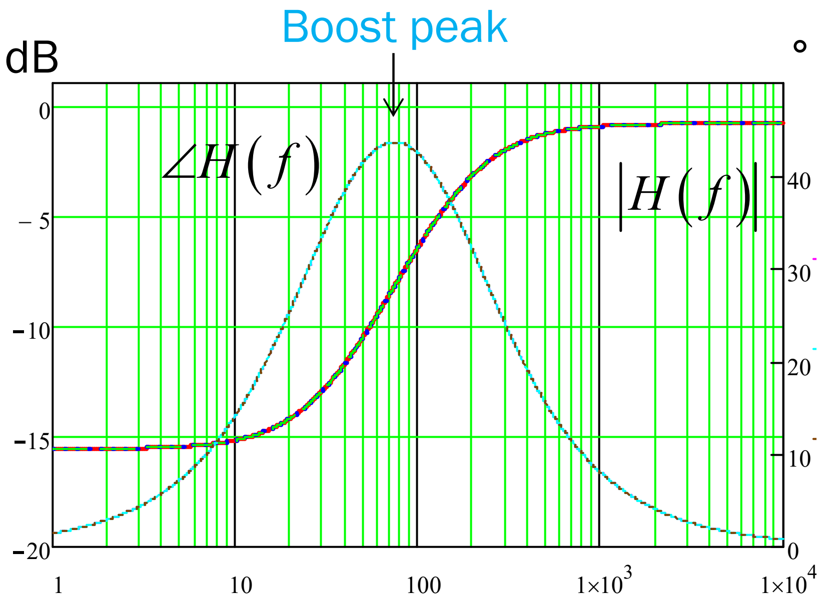
$$\boxed{H_3(s) := H_0 \cdot \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}}$$

$f_z := \frac{\omega_z}{2 \cdot \pi} = 31.831Hz \quad f_p := \frac{\omega_p}{2 \cdot \pi} = 173.624Hz \quad \sqrt{f_z \cdot f_p} = 74.341Hz$

$Z_1(s) := (R_1) \| \left(\frac{1}{s \cdot C_1}\right) \quad Z_2(s) := (R_2) \| \left(\frac{1}{s \cdot C_2}\right)$

$$\boxed{H_2(s) := \frac{Z_2(s)}{Z_2(s) + Z_1(s)}}$$

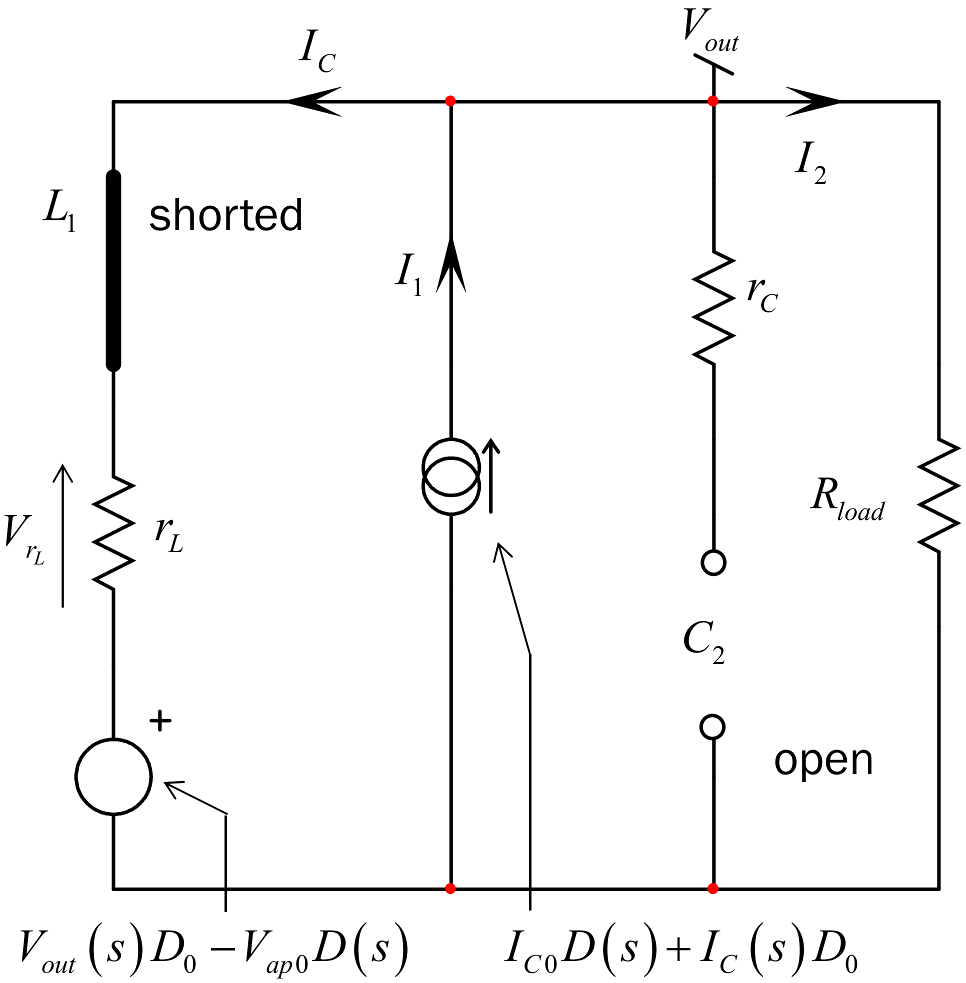
Raw formula



Comparison between raw and low-entropy formula: perfect match

Dc Analysis – Short L_1 and Open C_2

- ☐ Apply KCL on a simple circuit without reactances, $s = 0$



$$I_C(s) = \frac{V_{out}(s) - V_{out}(s)D_0 + V_{ap0}D(s)}{r_L}$$

$$I_2(s) = I_{C0}D(s) - I_C(s)(1 - D_0)$$

$$V_{out}(s) = I_2(s)R_{load} \qquad V_{ap0} = -V_{out}$$

Substitute
rearrange

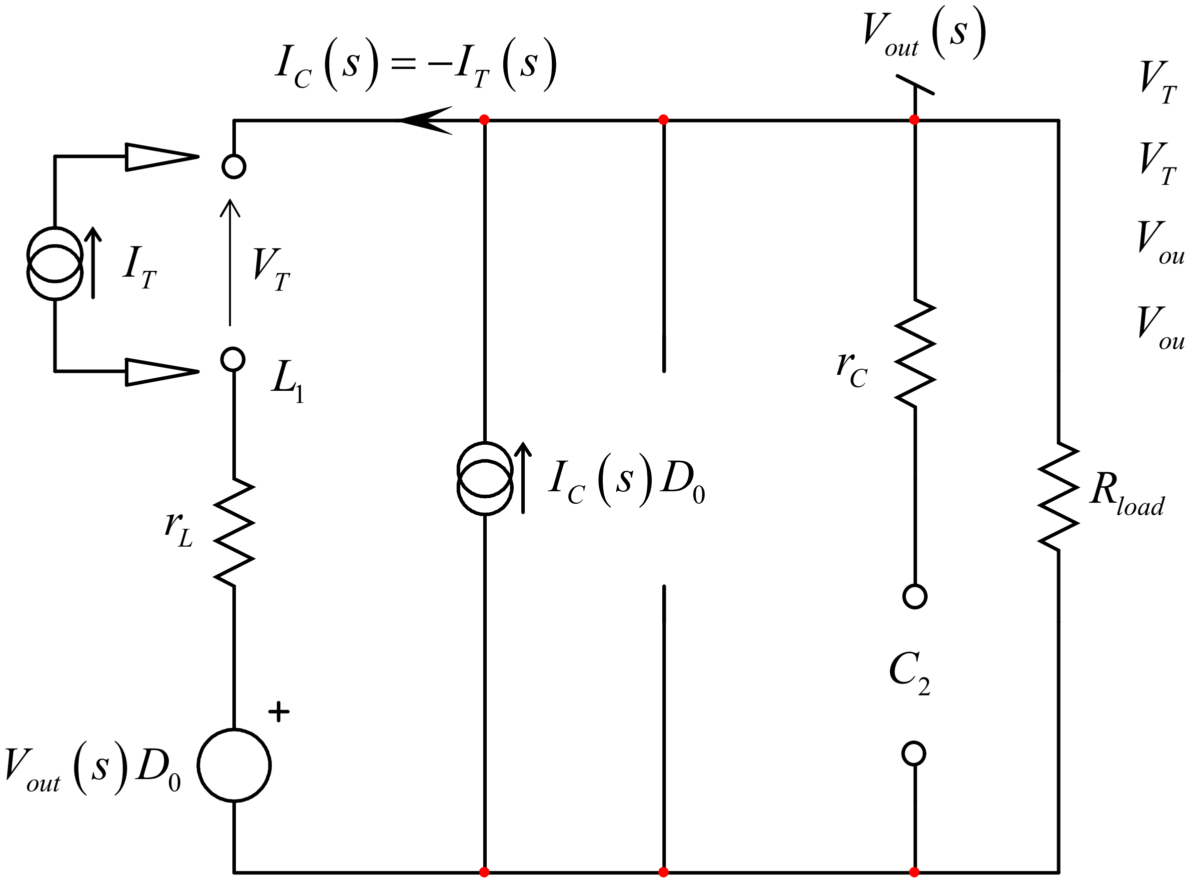
$$H_0 = V_{in}R_L \frac{\left[(1-D_0)^2 R_L - r_L\right]}{\left[(1-D_0)^2 R_L + r_L\right]^2}$$

$$r_L \to 0, V_{out} = V_{in}\frac{1}{1-D_0}$$

$$H_0 \approx \frac{V_{in}}{(1-D_0)^2}$$

Excitation is Turned off - τ_1

- ❑ All expressions featuring $D(s)$ are set to 0



$$V_T(s) = V_{out}(s) + I_T(s)r_L - V_{out}(s)D_0$$

$$V_T(s) = I_T(s)r_L + V_{out}(s)(1-D_0)$$

$$V_{out}(s) = \left[I_C(s)D_0 - I_C(s)\right]R_{load}$$

$$V_{out}(s) = I_C(s)(D_0 - 1)R_{load}$$

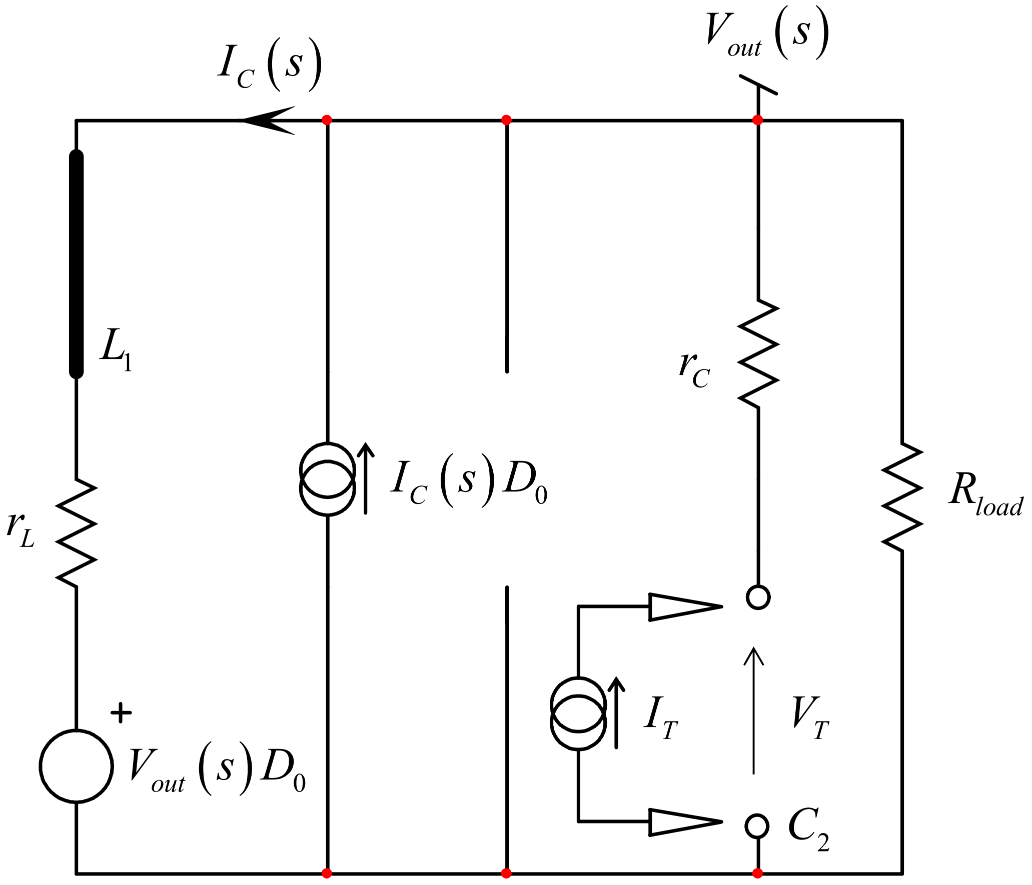
$$= I_T(s)(1-D_0)R_{load}$$

Substitute
rearrange

$$\tau_1 = \frac{L_1}{r_L + R_{load}(1-D_0)^2}$$

Excitation is Turned off - τ_2

- ☐ Inductor L_1 is replaced by a short circuit



$$I_C(s) = \frac{V_{out}(s) - V_{out}(s)D_0}{r_L}$$

$$I_C(s) = \frac{V_{out}(s)(1-D_0)}{r_L}$$

$$V_{out}(s) = R_{load}\left[I_C(s)(D_0 - 1) + I_T(s)\right]$$

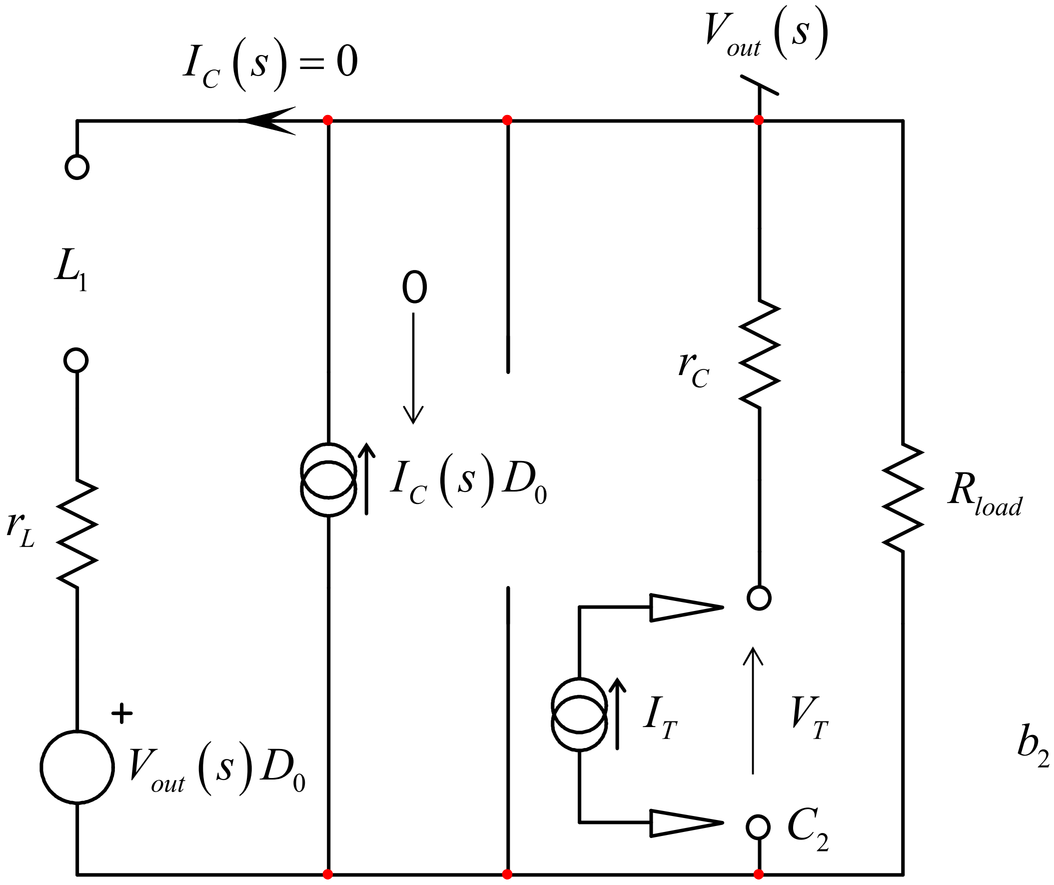
$$V_{out}(s) = V_T(s) - I_T(s)r_C$$

Substitute rearrange

$$\tau_2 = \left[r_C + \frac{r_L}{(1-D_0)^2 + \frac{r_L}{R_{load}}}\right]C_2$$

Excitation is Turned off - τ_2^1

- ❑ Inductor L_1 is replaced by an open circuit



$$I_C(s)=0$$

$$\Downarrow$$

$$\tau_2^1=(r_C+R_{load})C_2$$

$$\Downarrow$$

$$b_2=\tau_1\tau_2^1=\frac{L_1}{r_L+R_{load}(1-D_0)^2}(r_C+R_{load})C_2$$

Denominator Expression

- ☐ The 2nd-order denominator can be formed

$$b_1 = \tau_1 + \tau_2 = \frac{L_1}{r_L + R_{load}\left(1 - D_0\right)^2} + \left[r_C + \frac{r_L}{\left(1 - D_0\right)^2 + \frac{r_L}{R_{load}}} \right] C_2$$

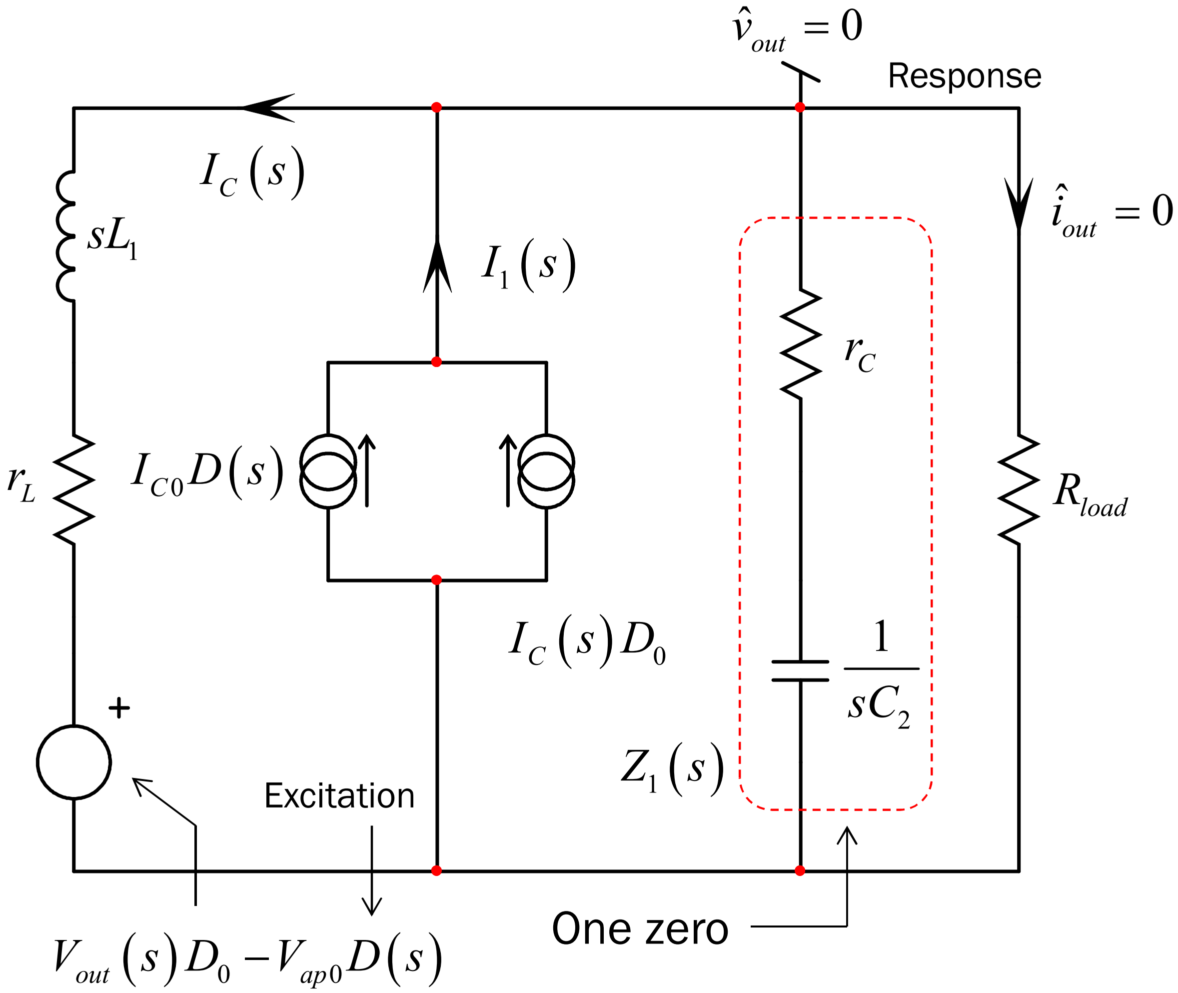
$$b_2 = \tau_1 \tau_2^1 = \frac{L_1}{r_L + R_{load}\left(1 - D_0\right)^2} \left(r_C + R_{load}\right) C_2$$

$$D\left(s\right) = 1 + \left[\frac{L_1}{r_L + \left(1 - D_0\right)^2 R_{load}} + C_2 \left(r_C + \frac{R_{load} r_L}{R_{load}\left(1 - D_0\right)^2 + r_L} \right) \right] s + \left[L_1 C_2 \frac{r_C + R_{load}}{r_L + \left(1 - D_0\right)^2 R_{load}} \right] s^2$$

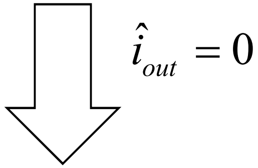
$$\omega_0 = \frac{1}{\sqrt{L_1 C_2}} \sqrt{\frac{r_L + R_{load} D'^2}{r_C + R_{load}}} \qquad Q \approx \frac{\omega_0}{\frac{r_L}{L_1} + \frac{1}{C\left(r_C + R_{load}\right)}}$$

Determining the Numerator

- ☐ To determine zeros, bring the excitation back



"What conditions in the transformed circuit null the response?"



$$Z_1(s) = r_C + \frac{1}{sC_2} = 0$$

$$I_1 = I_C$$

What if L_1 and C_2 are in HF state?

First Zero is Easy

- ☐ The equivalent series resistance brings the first zero

$$Z_1(s_z) = r_C + \frac{1}{sC_2} = 0 \longrightarrow Z_1(s_z) = \frac{1 + sr_C C_2}{sC_2} = 0$$

- ☐ The negative (LHP) root is simply

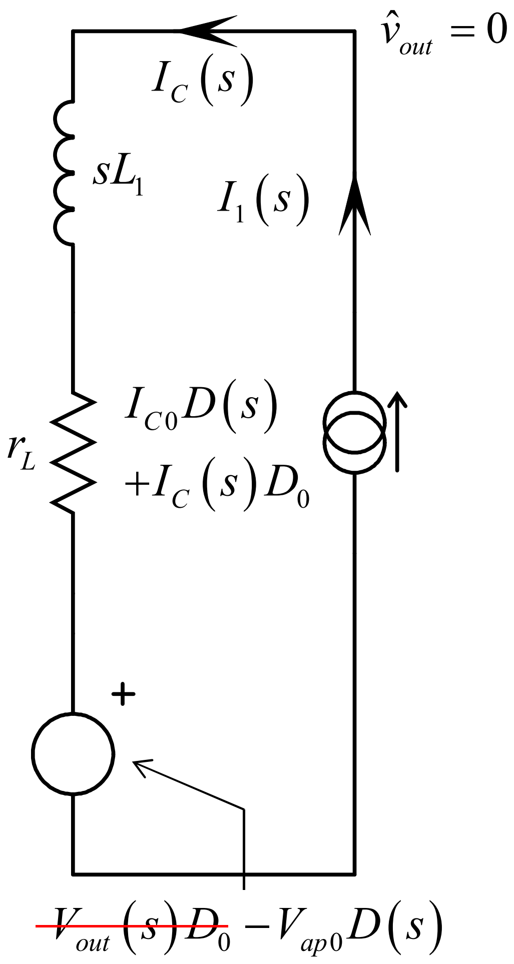
$$s_{z_1} = -\frac{1}{r_C C_2} \longrightarrow \omega_{z_1} = \frac{1}{r_C C_2}$$

- ☐ Almost there...

$$H(s) = H_0 \frac{\left(1 + \frac{s}{\omega_{z_1}}\right)(\ldots)}{1 + b_1 s + b_2 s^2}$$

Equate Current Expressions

- ☐ The output null implies that $\hat{v}_{out} = 0$



$$\underset{\uparrow}{I_1} - \underset{\uparrow}{I_C} = 0 \qquad I_C(s) = \frac{V_{ap0}D(s)}{sL_1 + r_L}$$

$$I_1(s) = I_{C0}D(s) + I_C(s)D_0$$

Substitute I_C in I_1

$$I_{C0}D(s) + \frac{V_{ap0}D(s)}{r_L + sL_1}D_0 - \frac{V_{ap0}D(s)}{r_L + sL_1} = 0$$

Solve for the root

$$s_{z_2} = \frac{(1-D_0)^2 R_{load} - r_L}{L_1} \xrightarrow[r_L << R_{load}]{} \omega_{z_2} \approx \frac{(1-D_0)^2 R_{load}}{L_1}$$

Positive root, RHPZ!

The CCM Boost Transfer Function

- ☐ Assemble pieces to express the transfer function

$$\frac{V_{out}(s)}{D(s)} = H_0 \frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1-\frac{s}{\omega_{z_2}}\right)}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2} \qquad \omega_{z_1} = \frac{1}{r_C C_2} \qquad \omega_{z_2} \approx \frac{(1-D_0)^2 R_{load}}{L_1}$$

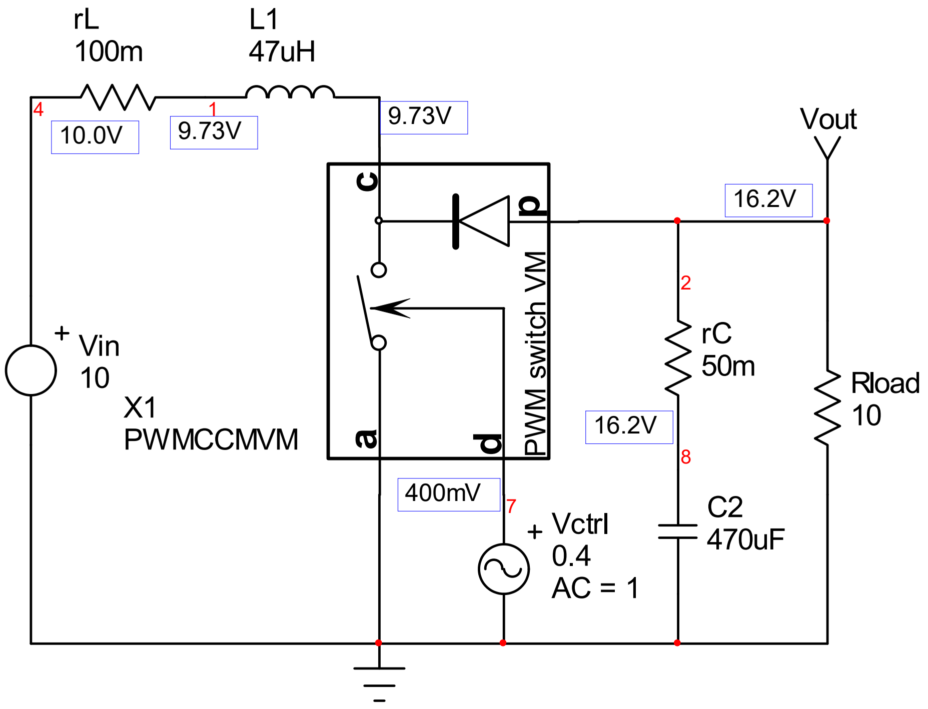
$$\omega_0 = \frac{1}{\sqrt{L_1 C_2}}\sqrt{\frac{r_L + R_{load} D'^2}{r_C + R_{load}}} \qquad Q \approx \frac{\omega_0}{\frac{r_L}{L_1}+\frac{1}{C_2(r_C+R)}} \qquad H_0 \approx \frac{V_{in}}{(1-D_0)^2}$$

$$\omega_0 \approx \frac{1-D_0}{\sqrt{L_1 C_2}} \qquad Q \approx (1-D_0) R_{load}\sqrt{\frac{C_2}{L_1}}$$

- ☐ Time to check the dynamic response

Checking Ac Responses

- ☐ Capture a simple boost converter schematic



$r_L := 0.1\Omega \qquad r_C := 0.05\Omega \qquad C_2 := 470\mu F \qquad L_1 := 47\mu H$

$D_0 := 40\% \qquad V_{in} := 10V \qquad R_L := 10\Omega \qquad \|(x,y) := \frac{x \cdot y}{x+y}$

$$V_{out} := \frac{R_L \cdot V_{in} \cdot (1 - D_0)}{r_L \cdot \left[\frac{R_L \cdot (1-D_0)^2}{r_L} + 1\right]} = 16.216V \qquad V_{ap} := -V_{out}$$

$$I_{C0} := \frac{V_{out} - (V_{in} + D_0 \cdot V_{out})}{r_L} = -2.703A$$

$$H_0 := \frac{D_0 \cdot R_L \cdot V_{ap} - R_L \cdot V_{ap} + I_{C0} \cdot R_L \cdot r_L}{R_L \cdot D_0^{\,2} - 2 \cdot R_L \cdot D_0 + R_L + r_L} = 25.566V \qquad 20 \cdot \log\left(\frac{|H_0|}{1V}\right) = 28.153$$

$$R_{tau1} := r_L + (1 - D_0)^2 \cdot R_L = 3.7\Omega \qquad \tau_1 := \frac{L_1}{R_{tau1}} = 12.703\mu s$$

$$R_{tau2} := r_C + \frac{r_L}{(1-D_0)^2 + \frac{r_L}{R_L}} = 0.32\Omega \qquad \tau_2 := C_2 \cdot R_{tau2} = 150.527\mu s$$

$$b_1 := \tau_1 + \tau_2 = 163.23\mu s \qquad \tau_{12} := C_2 \cdot (r_C + R_L) = 4.724 \times 10^3 \cdot \mu s \qquad b_2 := \tau_1 \cdot \tau_{12} = 6 \times 10^4 \cdot \mu s^2$$

$$\omega_{z1} := \frac{1}{r_C \cdot C_2} \qquad f_{z1} := \frac{\omega_{z1}}{2\pi} = 6.773kHz \qquad D(s) := 1 + b_1 \cdot s + b_2 \cdot s^2$$

$$\omega_{z2} := -\frac{D_0 \cdot V_{ap} - V_{ap} + I_{C0} \cdot r_L}{I_{C0} \cdot L_1} \qquad f_{z2} := \frac{\omega_{z2}}{2\pi} = 11.852kHz$$

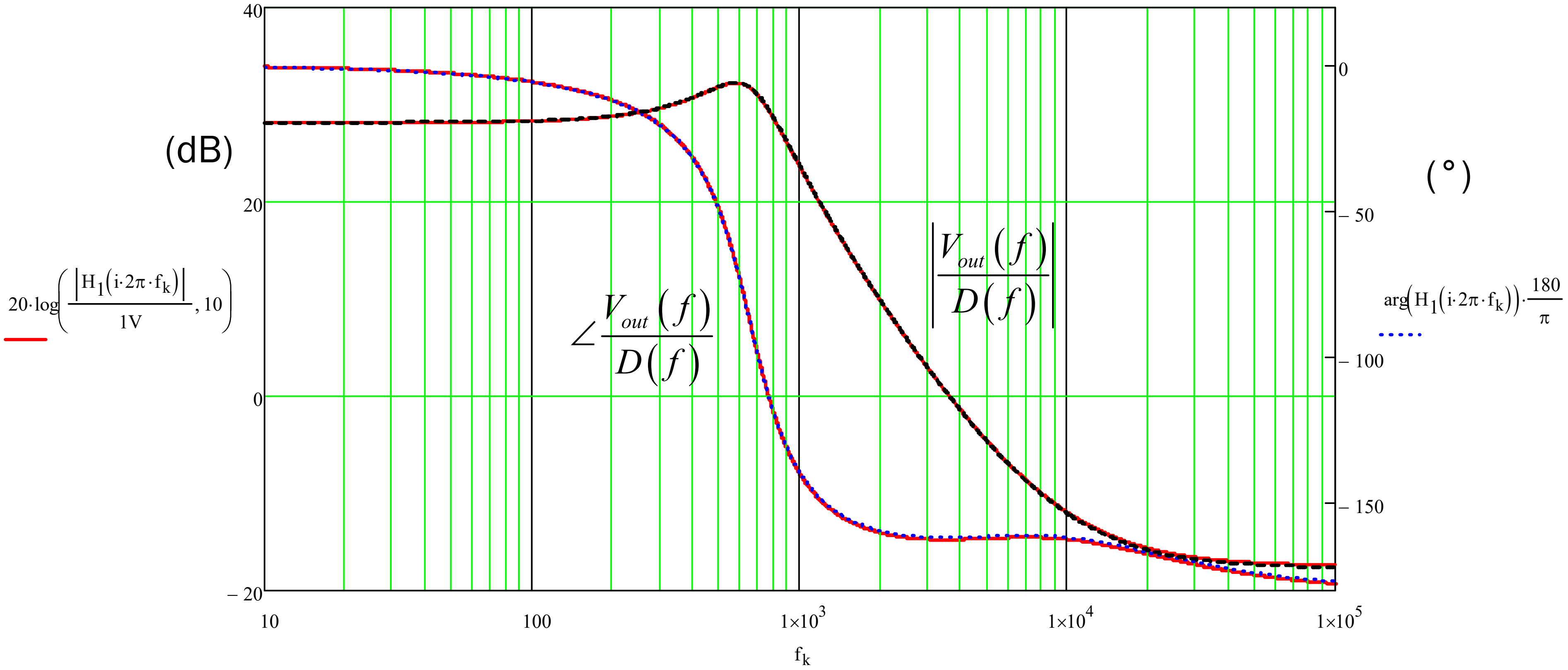
$$\omega_0 := \frac{1}{\sqrt{b_2}} = 4.082 \times 10^3 \frac{1}{s} \qquad f_0 := \frac{\omega_0}{2\pi} = 649.741Hz$$

$$Q := \frac{\sqrt{b_2}}{b_1} = 1.501 \qquad H_1(s) := H_0 \cdot \frac{\left(1 + \frac{s}{\omega_{z1}}\right) \cdot \left(1 - \frac{s}{\omega_{z2}}\right)}{D(s)}$$

- ☐ Check versus the Mathcad® plots

SPICE and Mathcad® Plots

- ❑ Curves superimpose: transfer function is correct!



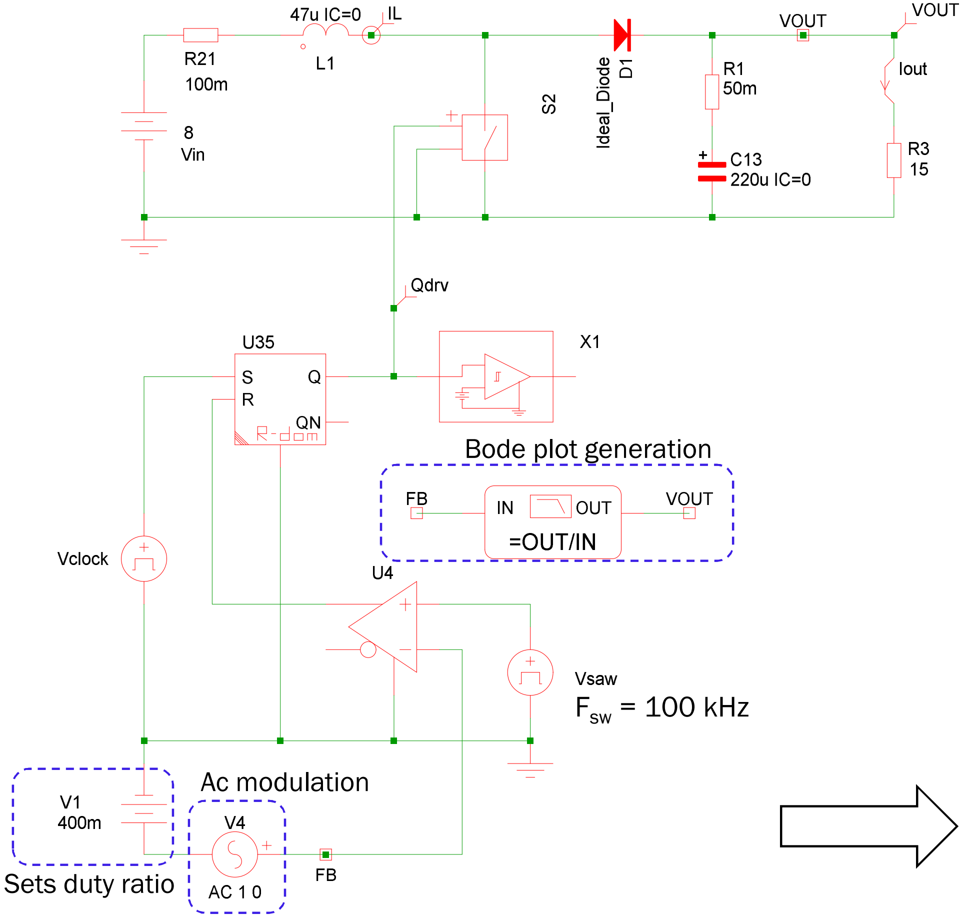
- ❑ A deviation would indicate a flaw in the analysis

Course Agenda

- ☐ Blocks in a Switching Converter
- ☐ Introduction to Small-Signal Modeling
- ☐ Analytical Analysis of an Output Stage
- ☐ **Simulation Models - Averaged or Switched?**
- ☐ Crossover Frequency and Phase Margin
- ☐ Compensation Strategy
- ☐ Experiments on Prototypes
- ☐ Conclusion

Simulation with a Switching Circuit

- ☐ PWL simulators let you extract the dynamic response



Bode plot generation

=OUT/IN

F_{sw} = 100 kHz

Ac modulation

Sets duty ratio

- Use component models

- Observe 2nd-order effects

- Cross-conduction in CCM
- Leakage inductance damping

- Works if no average model exists

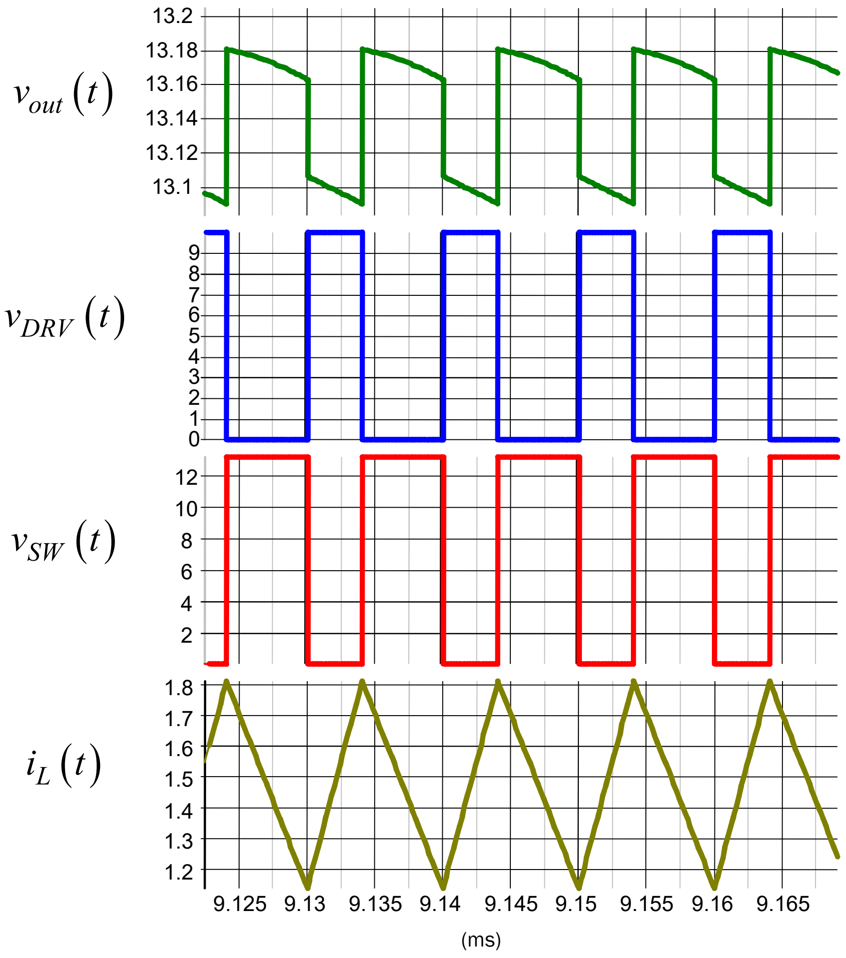
- Switching circuit simulates quickly

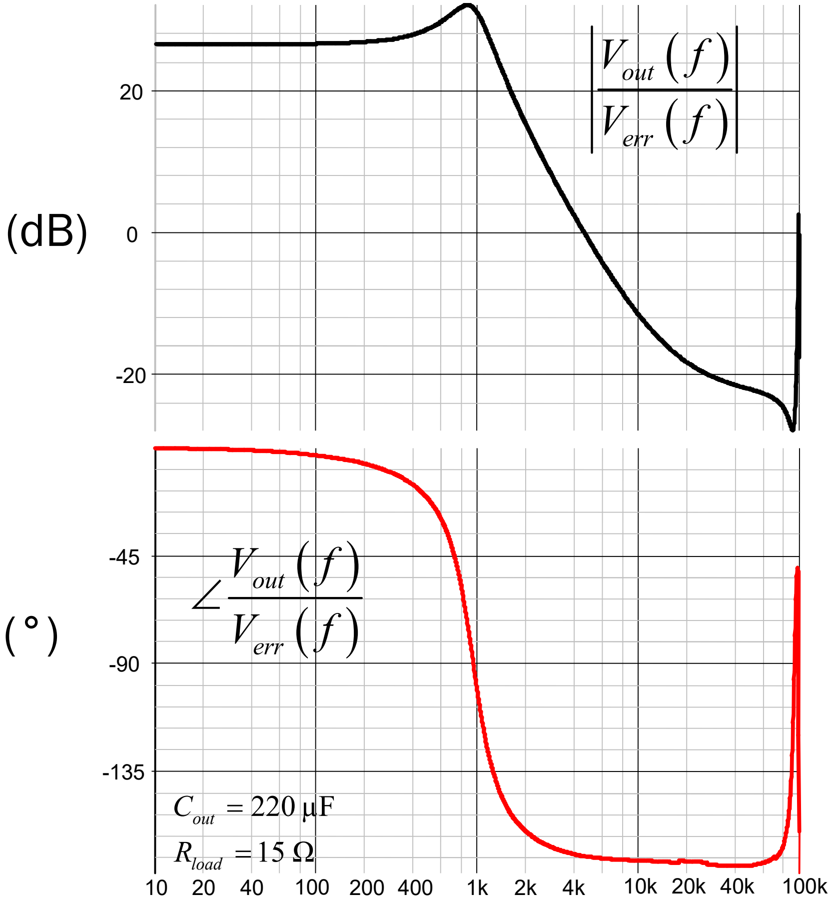
⇨ All examples work with the demo version!

http://www.simplistechnologies.com/product/elements

Switching and Dynamic Response

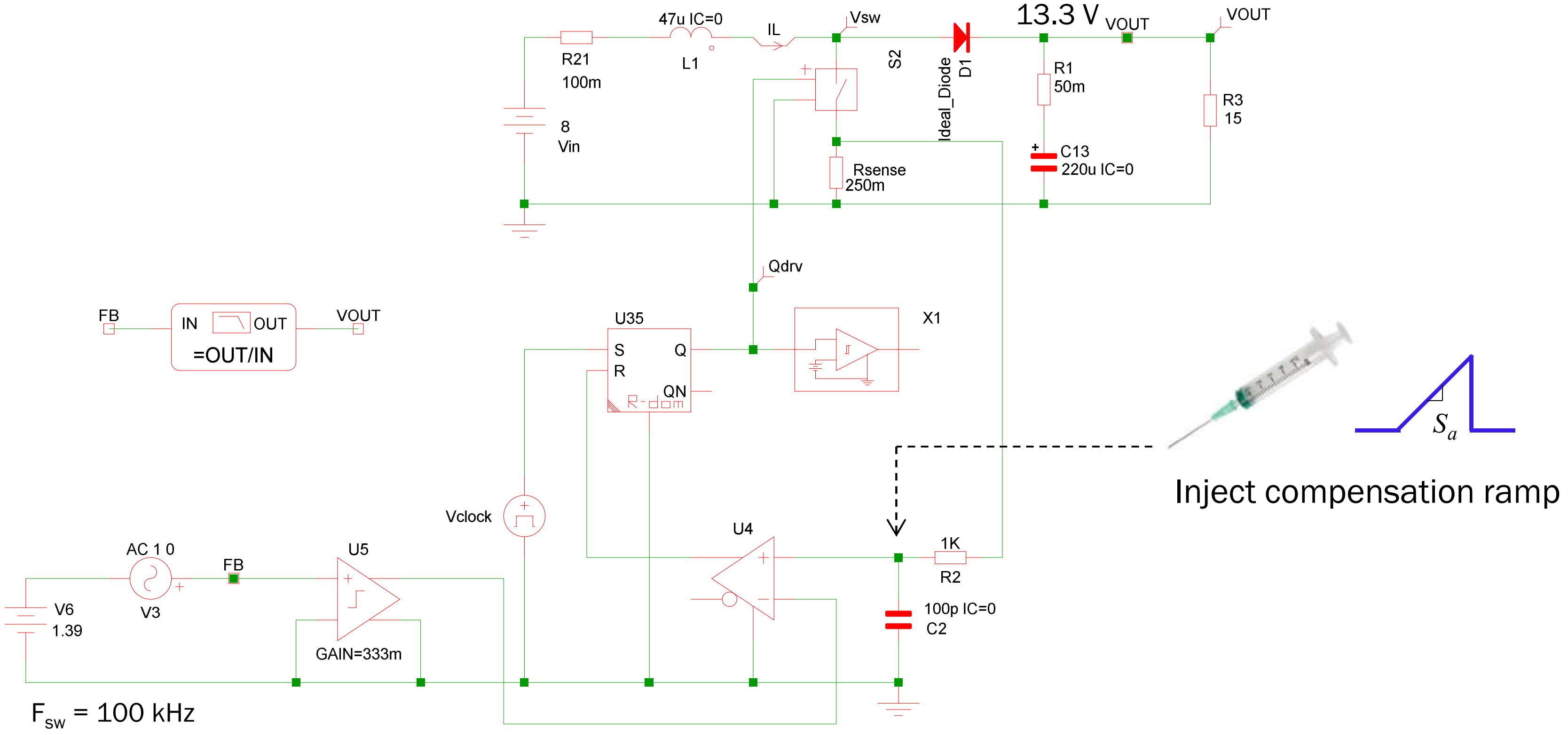
- ☐ You have cycle-by-cycle and dynamic analyses in one shot





Current Mode and Subharmonic Poles

- ☐ It is easy to implement current mode control



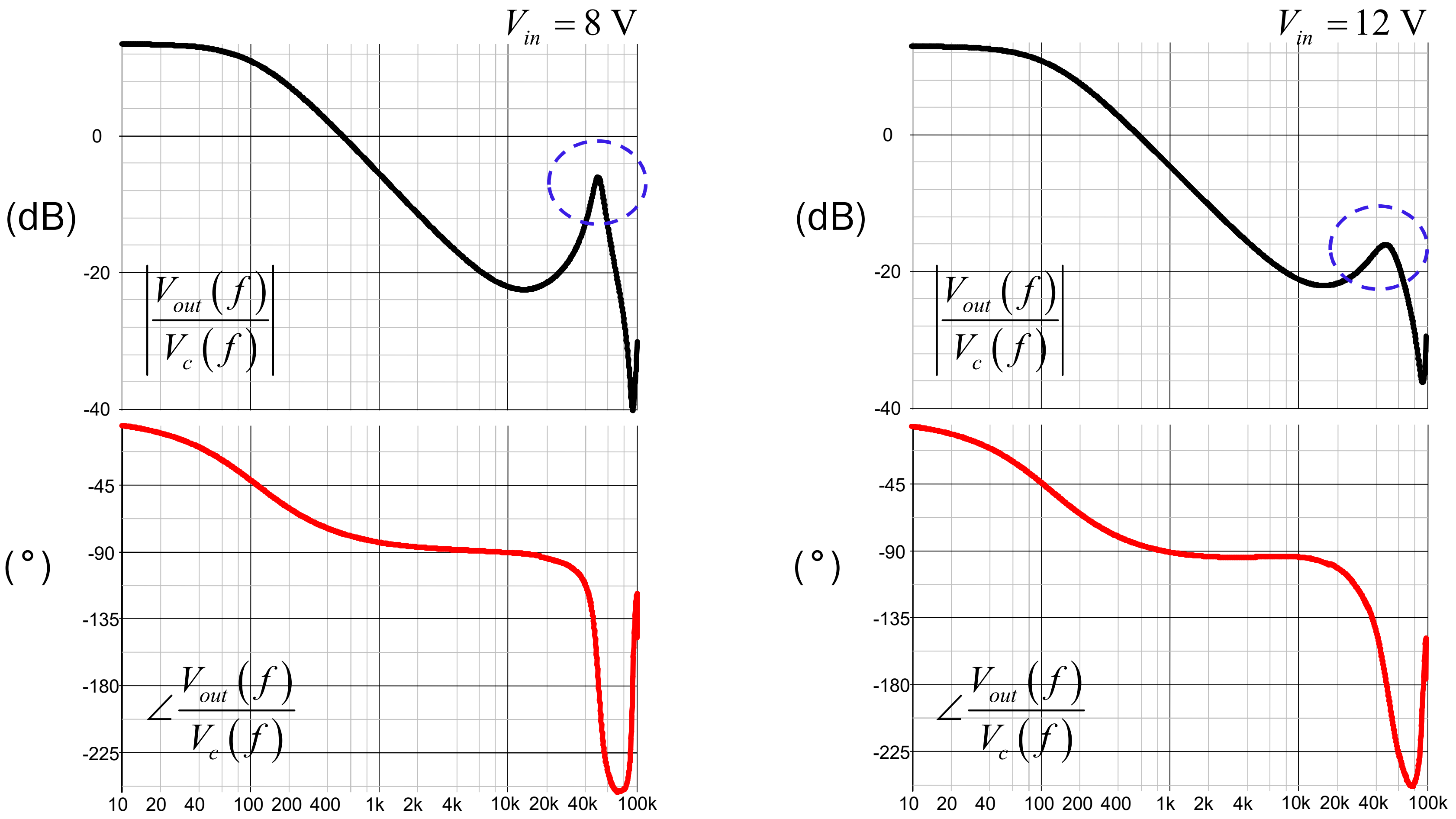
Inject compensation ramp

F_{sw} = 100 kHz

- ☐ Inject external ramp to damp the subharmonic poles at $F_{sw}/2$

Dynamic Responses Reveal Peaking

- ☐ Subharmonic poles need to be damped



How Much Ramp Should you Inject?

- ☐ Damp the double poles located at $F_{sw}/2$

$$Q = \frac{1}{\pi\left(m_c D' - 0.5\right)} \qquad m_c = 1 + \frac{S_a}{S'_n}$$

S_a ← External ramp slope [V]/[s] → To be determined

S'_n ← On-time slope [V]/[s] → $\frac{V_{in}}{L} R_{sense}$

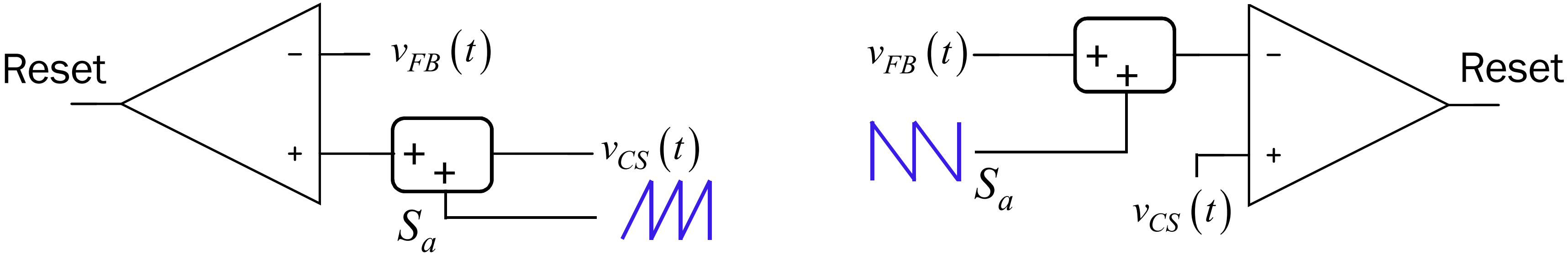
- ☐ Adjust m_c to reduce Q to 1

$$m_c = \frac{\frac{1}{\pi} + 0.5}{1 - D} = \frac{0.818}{1 - 0.42} \approx 1.41 \longrightarrow S_a = S'_n\left(m_c - 1\right) = \frac{8}{47u} \times 0.25 \times 0.41 \approx 28\,\text{kV/s}$$

From simulation (0.42)

Amount of external ramp to damp Q

- ☐ Add a positive ramp to $v_{CS}(t)$ or a negative ramp to $v_{FB}(t)$

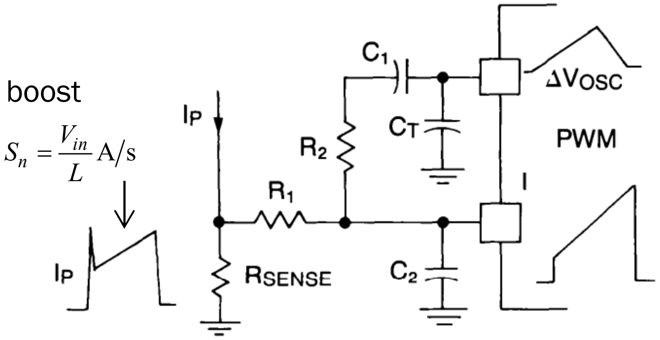


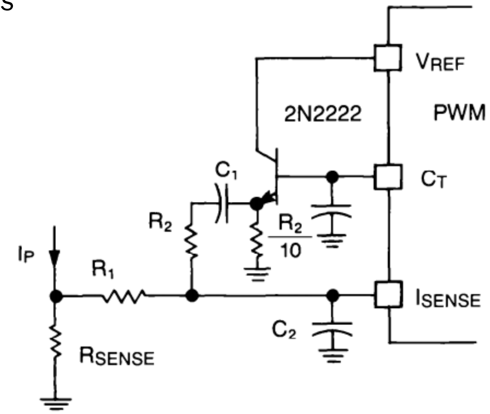
R. Ridley, *A New Small-Signal Model for Current Mode Control*, Ph. D. dissertation, Virginia Polytechnic Institute and State University, 1990

How to Generate a Compensation Ramp?

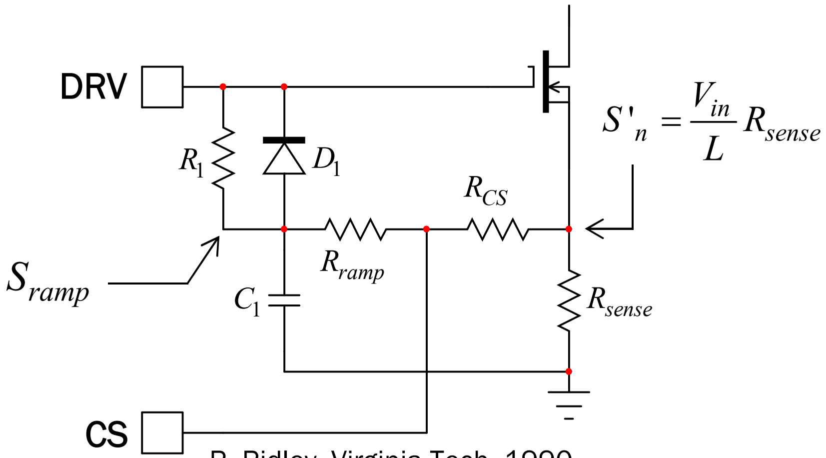
- ☐ Buffer the oscillation ramp and inject on the CS pin

Unitrode U-111, *Practical Considerations in current mode power supplies*



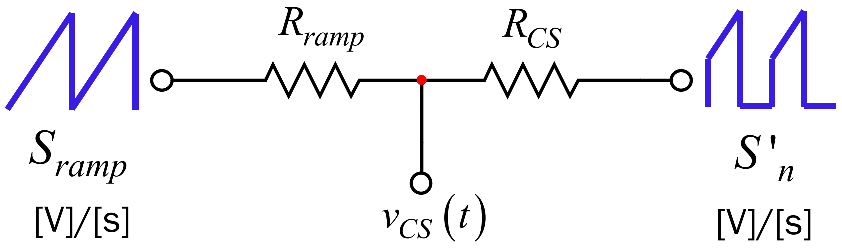


- ☐ Risks exist to disturb oscillator – use a low-impedance pin



$$S'_n = \frac{V_{in}}{L} R_{sense}$$

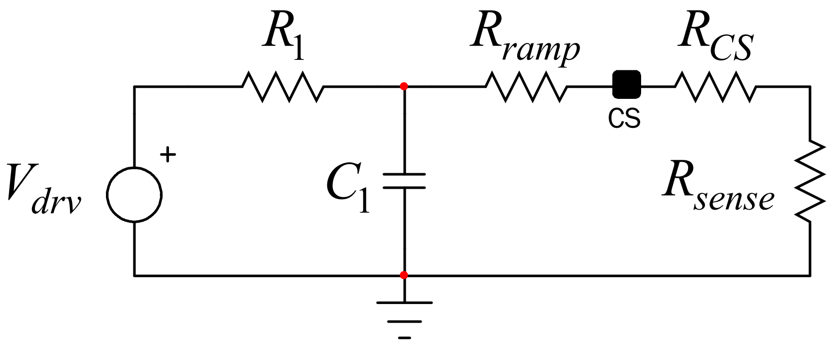
R. Ridley, Virginia Tech, 1990

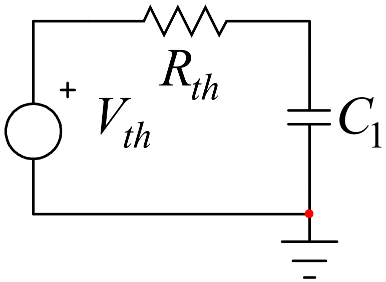


$$R_{ramp} = \frac{S_{ramp}}{S_a} R_{CS}$$

Size Ramp to Meet Compensation Needs

- ☐ Simplify the charging scheme by neglecting R_{sense}





$$V_{th} = V_{drv}\frac{R_{ramp}+R_{CS}}{R_{ramp}+R_{CS}+R_1}$$

$$R_{th} = R_1 \parallel \left(R_{ramp}+R_{CS}\right)$$

- ☐ The slope of this ramp is obtained by differentiation

$$v_{C_1}(t) = V_{th}\left(1-e^{-\frac{t}{\tau}}\right) \qquad d\frac{v_{C_1}(t)}{dt} = V_{th}\frac{e^{-\frac{t}{\tau}}}{\tau} \longrightarrow S_{ramp} \approx \frac{V_{th}\left(1-\frac{t_{on}}{\tau}\right)}{\tau}$$

At $t = t_{on}$

Considering $R_{CS} << R_{ramp}$: $S_{ramp} \approx \dfrac{V_{drv}\dfrac{R_{ramp}}{R_{ramp}+R_1}\left(1-\dfrac{t_{on}}{C_1\left(R_1 \parallel R_{ramp}\right)}\right)}{C_1\left(R_1 \parallel R_{ramp}\right)}$

Build a Low-Impedance Ramp

- ☐ Adopt a low charging current (1 mA) to spare the driver:

$$R_1 = \frac{V_{drv}}{I_{chg}} = \frac{12}{1m} = 12\ \text{k}\Omega$$

Choose: $S_{ramp} = 300\ \text{mV}/\mu\text{s}$

$R_{ramp} = 19\ \text{k}\Omega$

$t_{on} = 5\ \mu\text{s}$

$$C_1 = V_{drv}\frac{1+\sqrt{1-\frac{4S_a t_{on}\left(R_1+R_{ramp}\right)}{R_{ramp}V_{drv}}}}{2R_1S_a} = 2.6\ \text{nF}$$

$\downarrow$

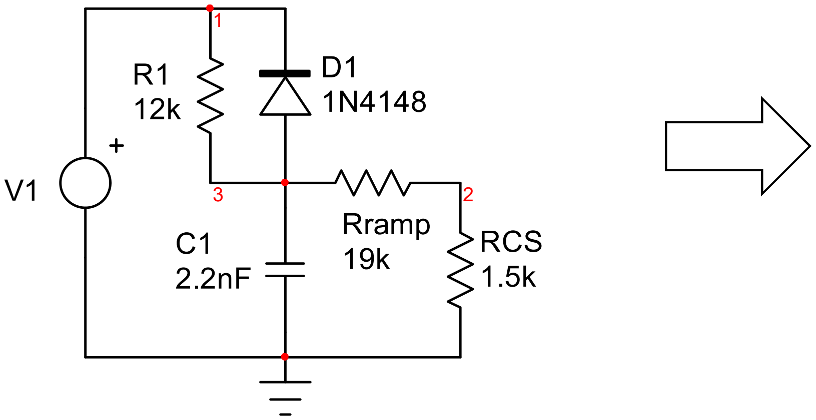
2.2 nF

$\downarrow$

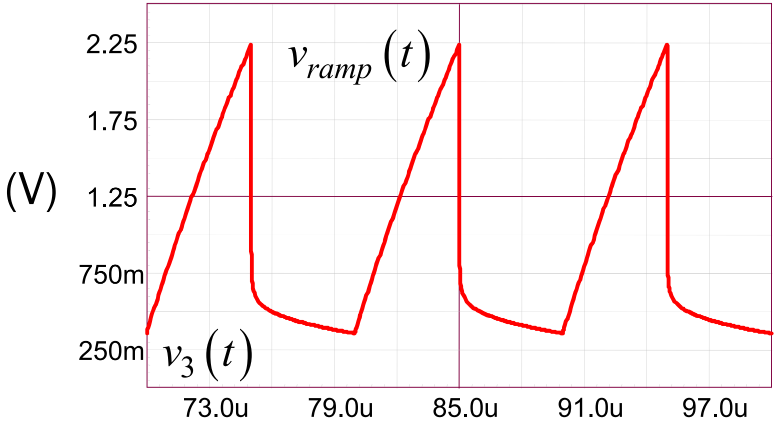
$0.34\ \text{V}/\mu\text{s}$

$$R_{CS} = R_{ramp}\frac{S_a}{S_{ramp}} \approx 1.5\ \text{k}\Omega$$

- ☐ Check simulation results are ok

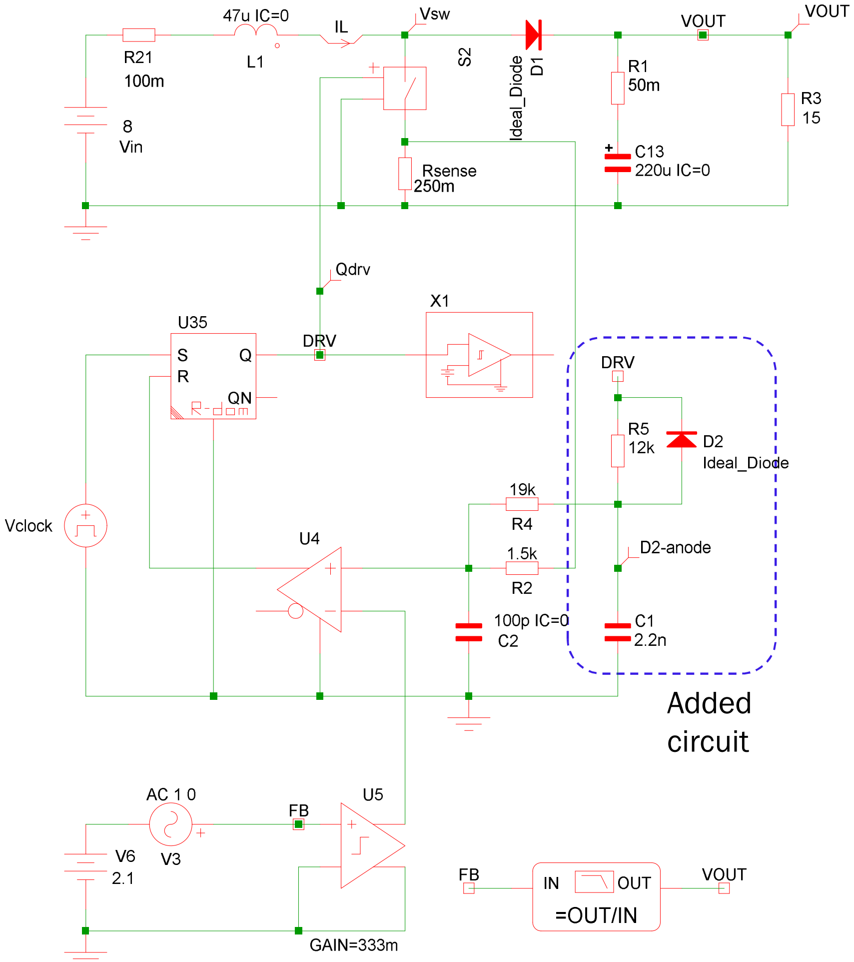


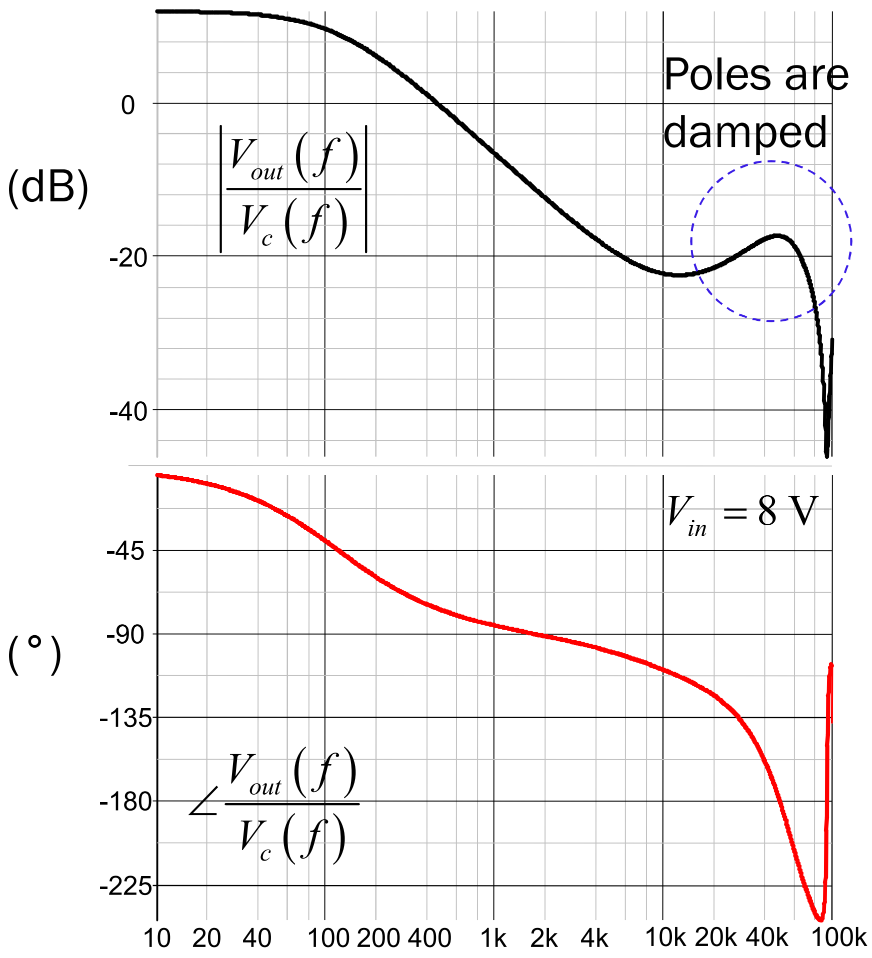
$S_{ramp} \approx 0.38\ \text{V}/\mu\text{s}$



Implement the Circuit in the Converter

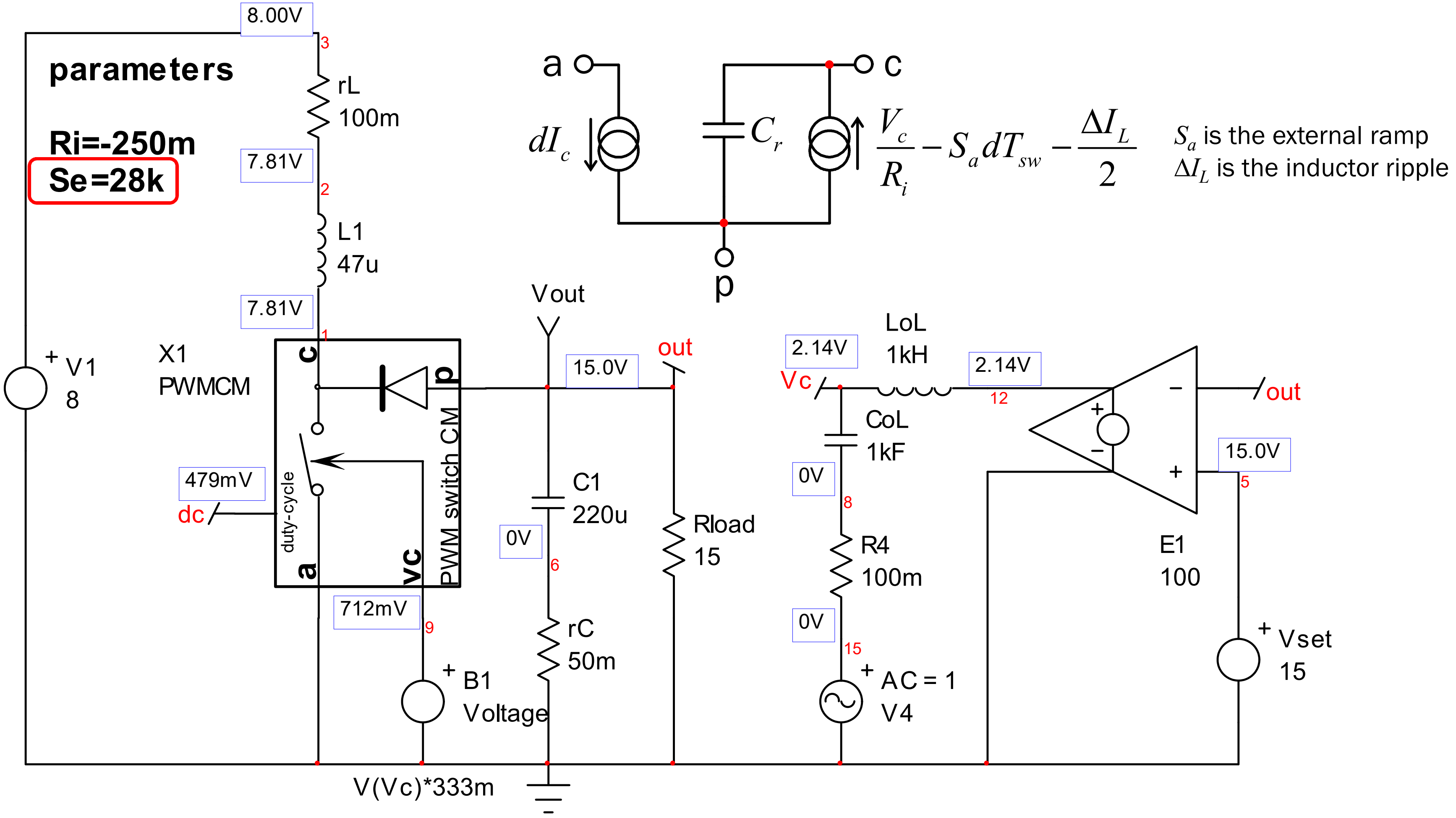
- ☐ You can immediately check the impact on the response





The PWM Switch Model in Current Mode

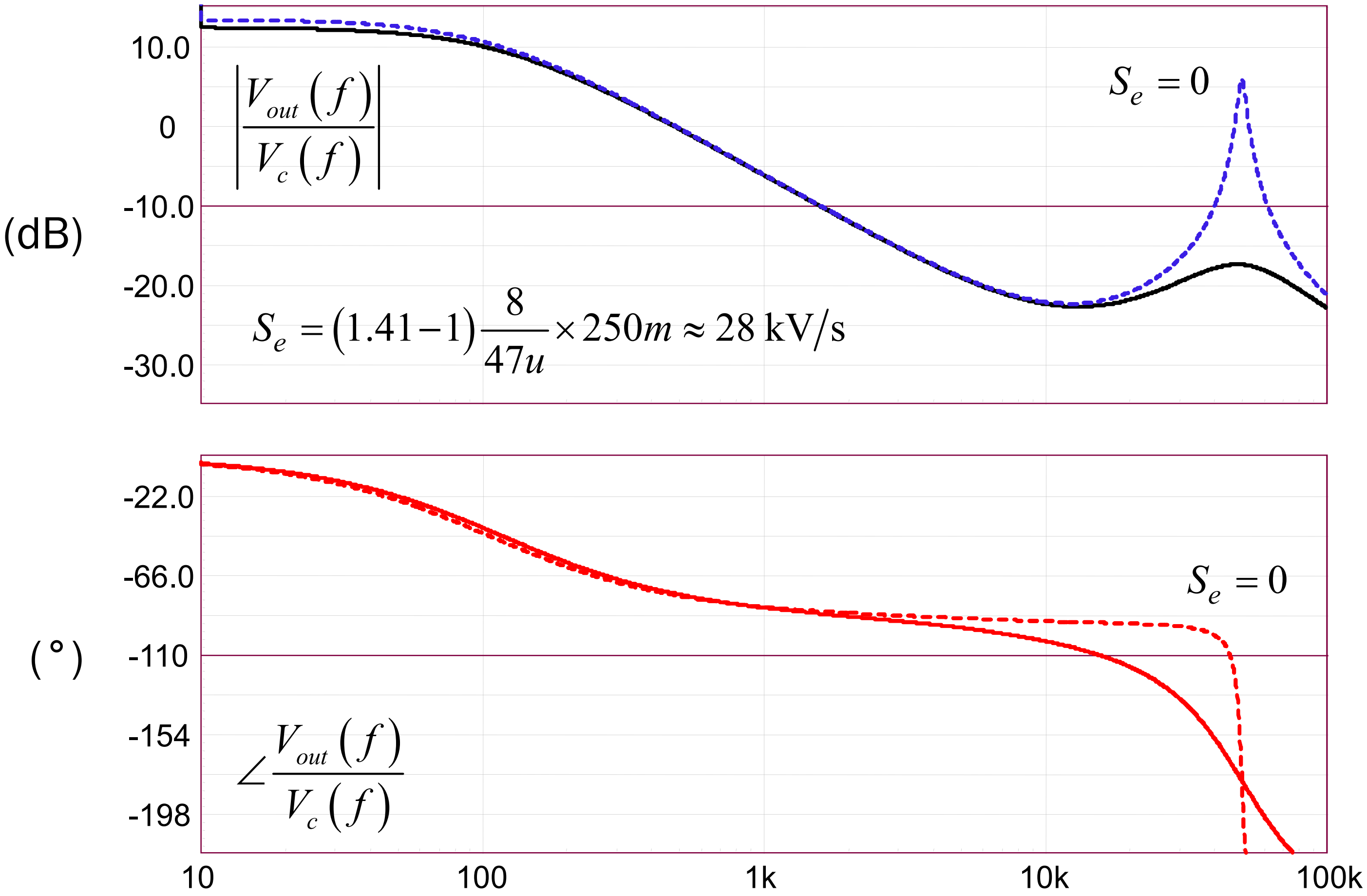
- ☐ A current mode version predicts the subharmonic poles



V. Vorpérian, "Analysis of current-controlled PWM converters using the model of the current-controlled PWM switch", PCIM conference proceedings, 1990, pp 183-195

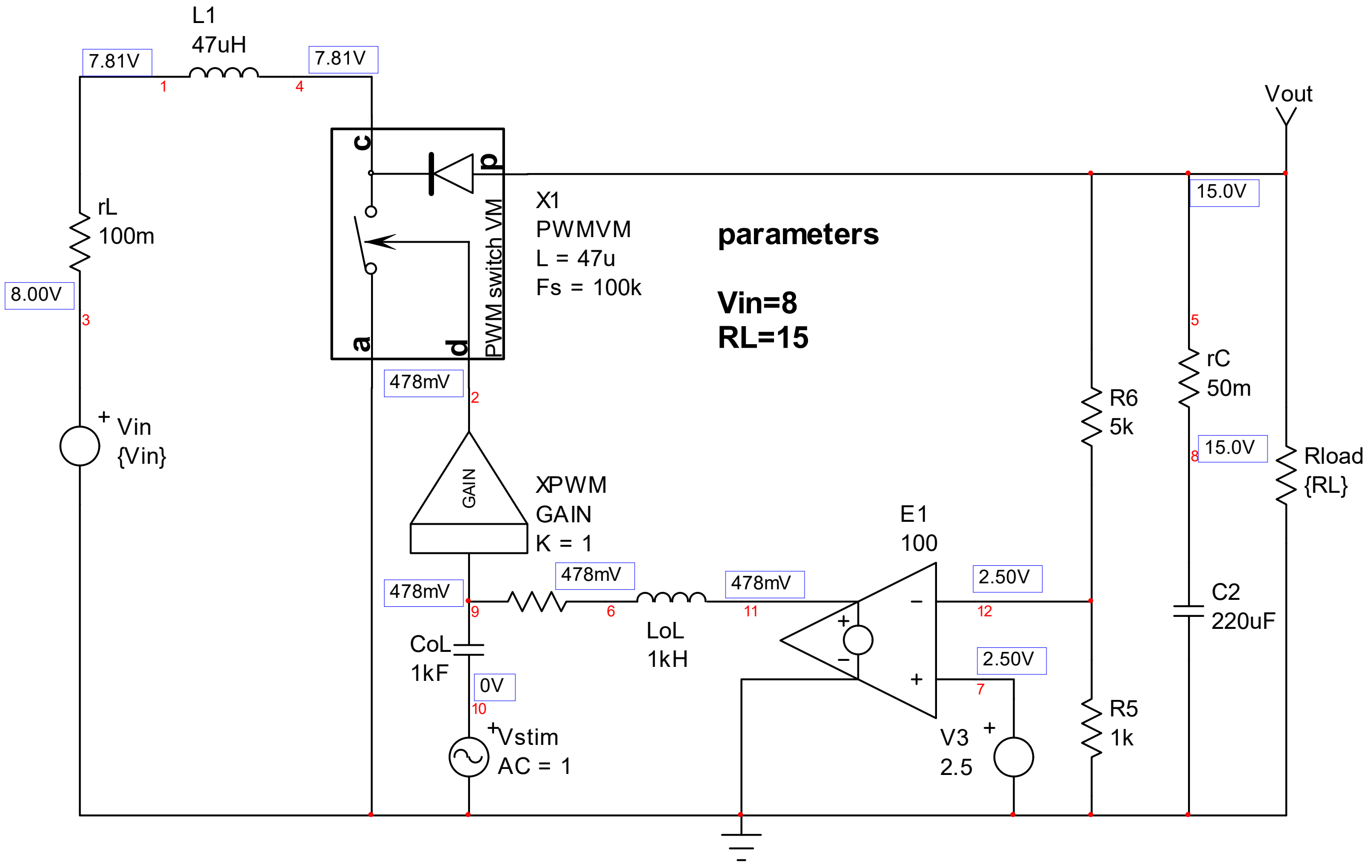
Check Slope Compensation is Effective

- ☐ The compensation ramp S_e or S_a is passed as a parameter



Voltage-Mode Control is Easy

- ☐ Choose the auto-toggling DCM-CCM PWM switch



$$R_{crit} = 2F_{sw}L\frac{M^3}{M-1}$$

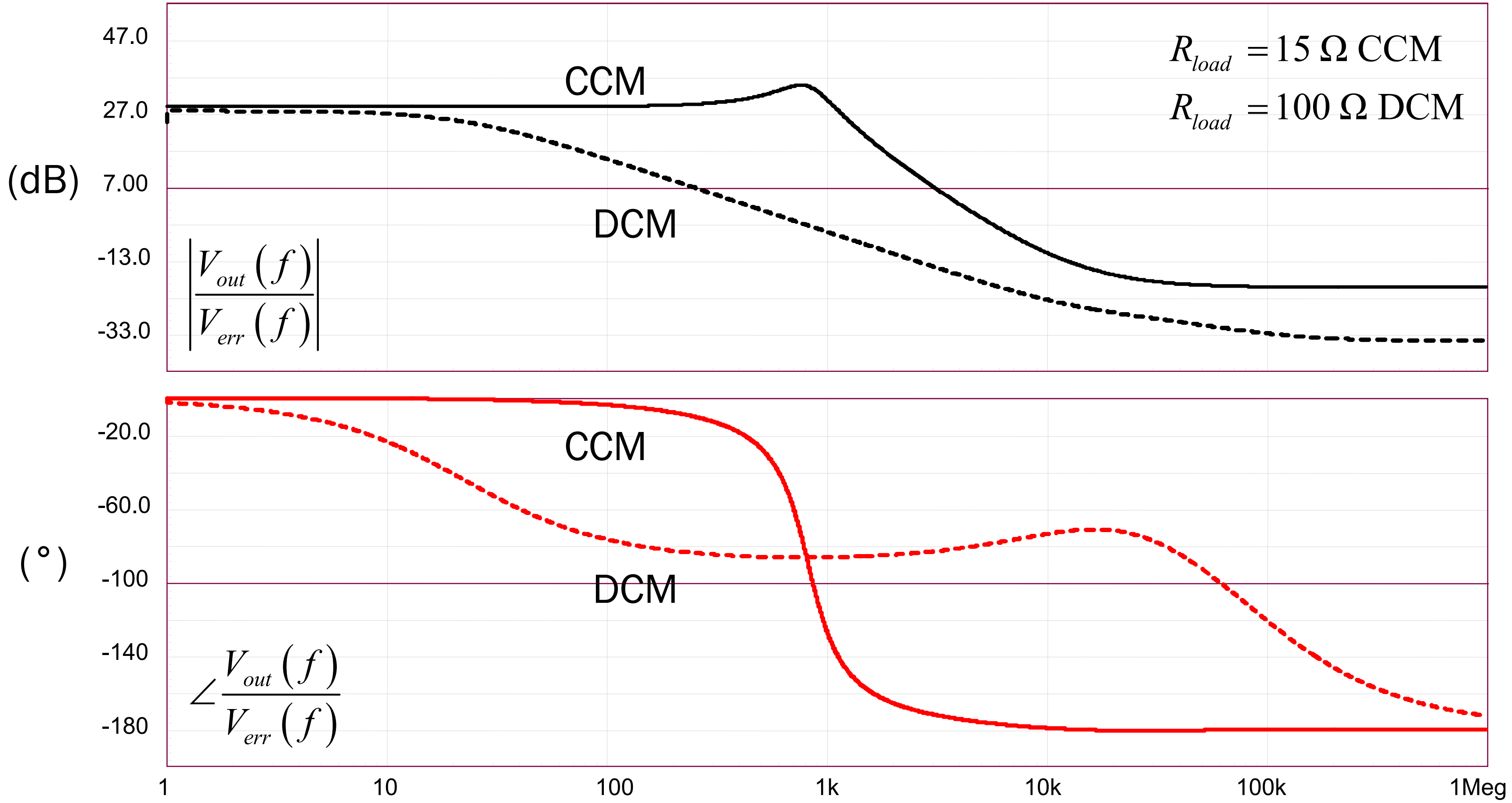
$$\downarrow \quad M = \frac{V_{out}}{V_{in}}$$

$$R_{crit} \approx 71\,\Omega$$

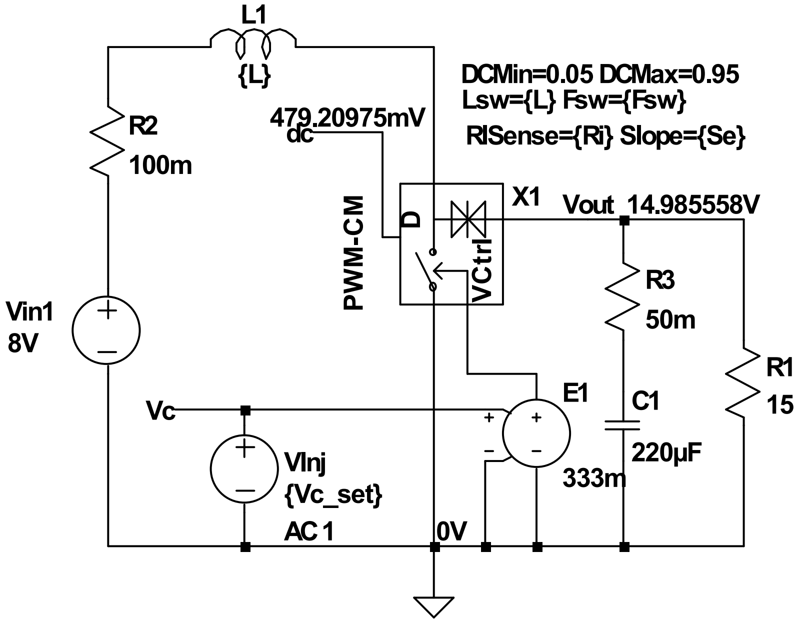
- ☐ Reduce the output current to automatically enter DCM

Dynamic Response is Immediate

- ☐ The DCM response is still of second order with a RHPZ

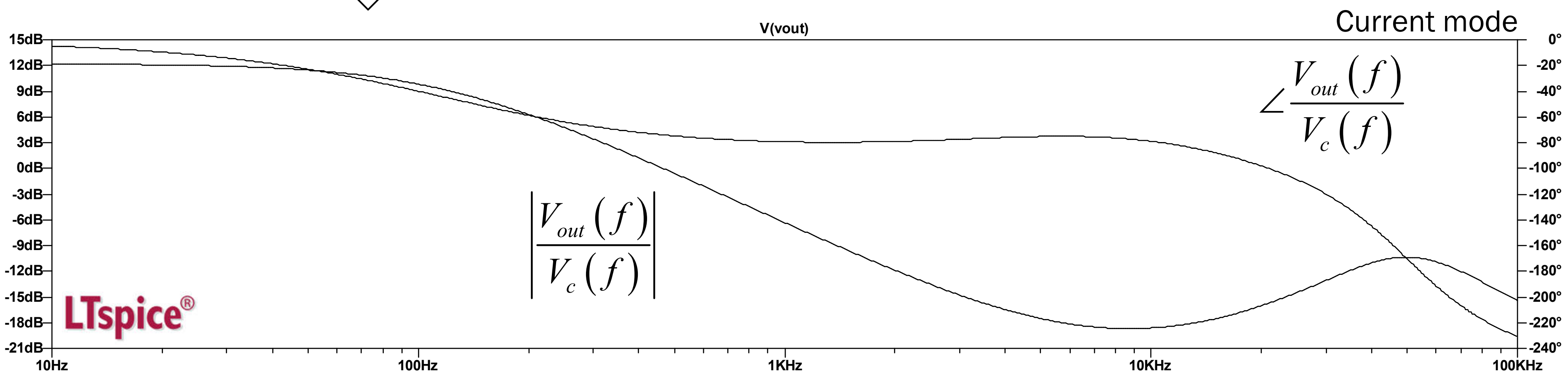


Port the Models to Other Platforms



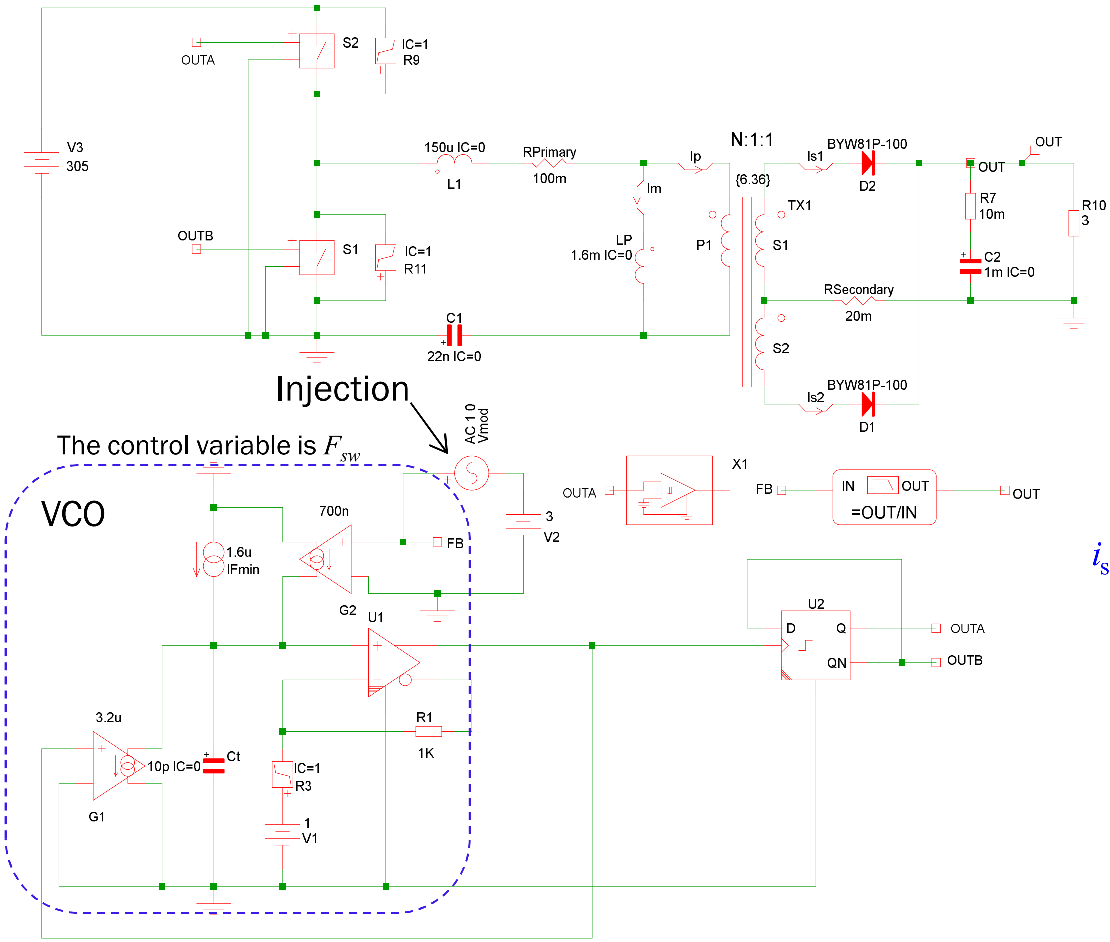
- ☐ LTspice lends itself well to this type of simulations.

.param L=47uH
.ac dec 1000 10 100kHz
.param Vc_set=2.09
.param Fsw=100k
.param Ri=-250m
.param Se=24.4k



No Simple Average Model for LLC

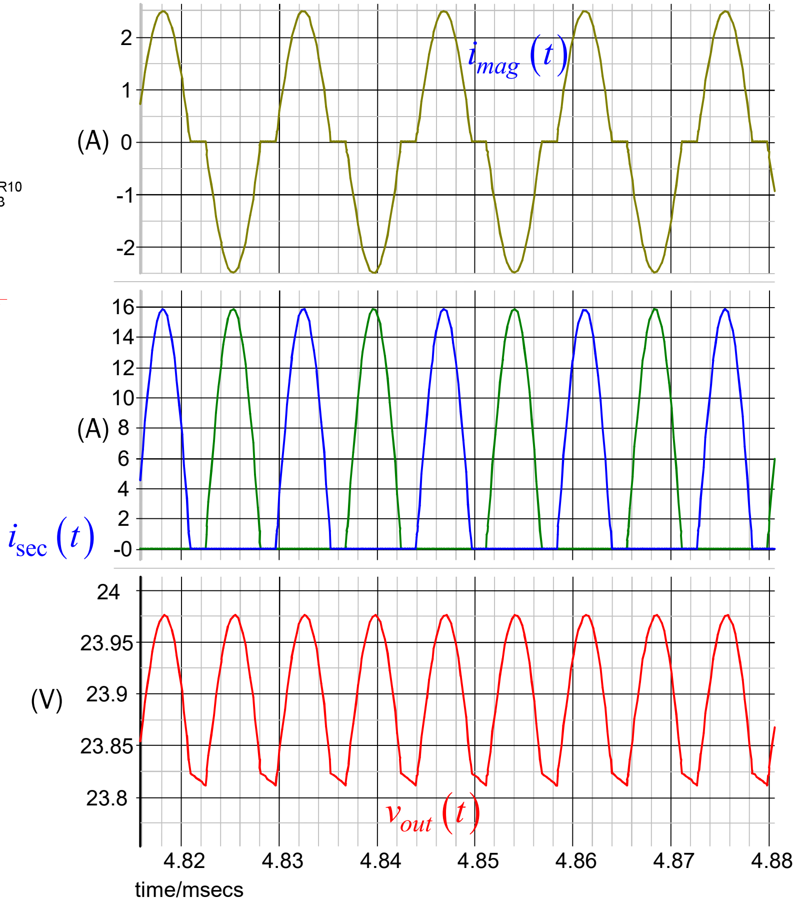
- ☐ PWL simulation is perfectly suited for a LLC converter



Injection

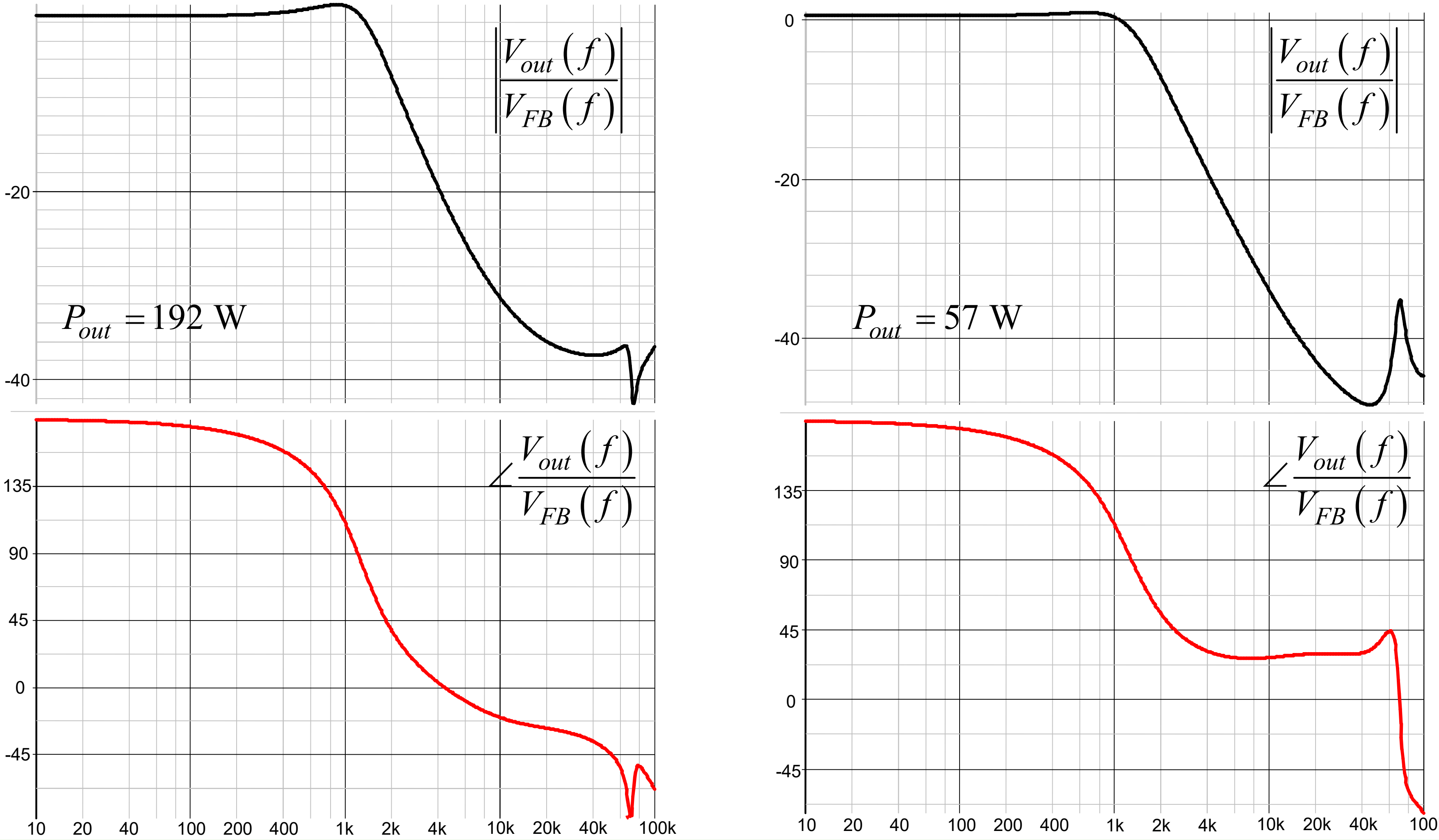
The control variable is F_{sw}

VCO



Dynamic Response of a LLC

- ☐ The Bode plot for two different loads is obtained in 10 s

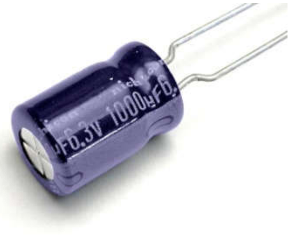


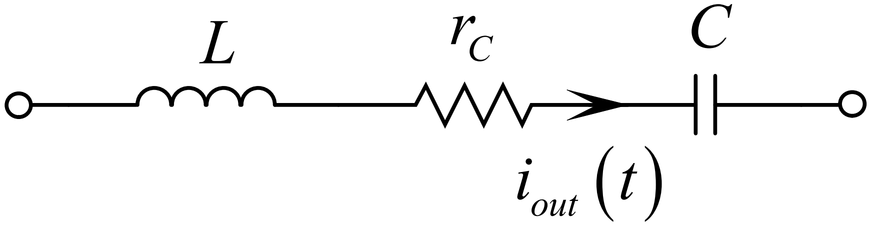
Course Agenda

- ☐ Blocks in a Switching Converter
- ☐ Introduction to Small-Signal Modeling
- ☐ Analytical Analysis of an Output Stage
- ☐ Simulation Models - Averaged or Switched?
- ☐ **Crossover Frequency and Phase Margin**
- ☐ Compensation Strategy
- ☐ Experiments on Prototypes
- ☐ Conclusion

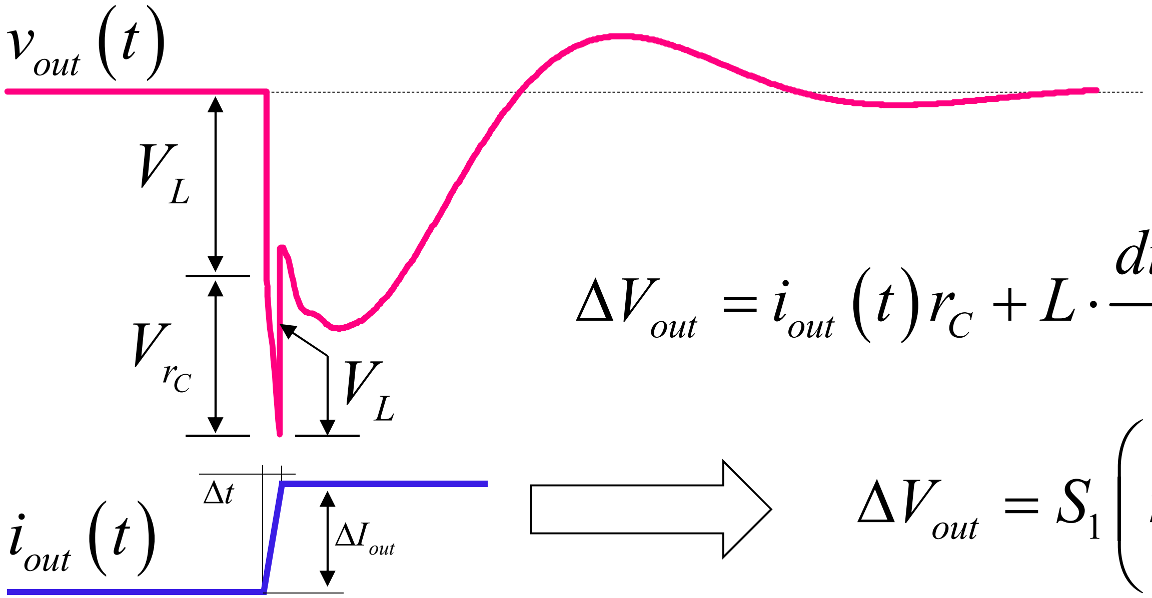
What Type of Transient Response?

☐ Parasitics affect the response to a load step





☐ The drop is a combination of three voltages

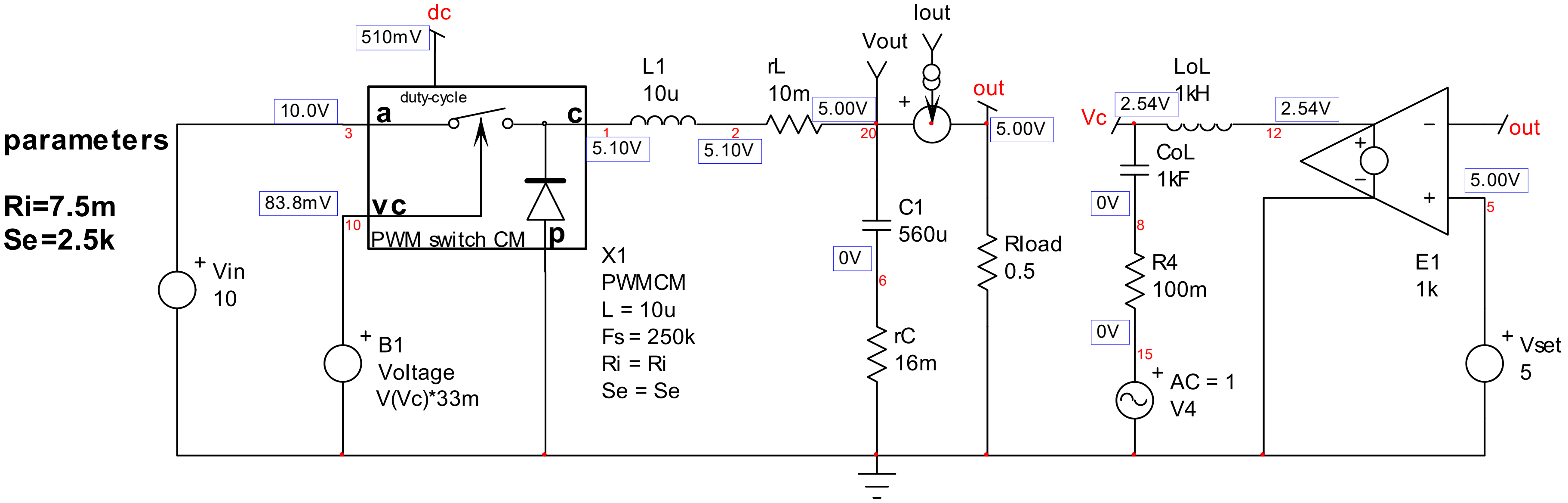


$$\Delta V_{out} = i_{out}(t) r_C + L \cdot \frac{di_{out}(t)}{dt} + \frac{1}{C}\int_0^{\Delta t} i_{out}(t)\,dt$$

$$\Delta V_{out} = S_1\left(r_C \Delta t + L + \frac{\Delta t^2}{2C} \right)$$

How to Study the Compensation Effects?

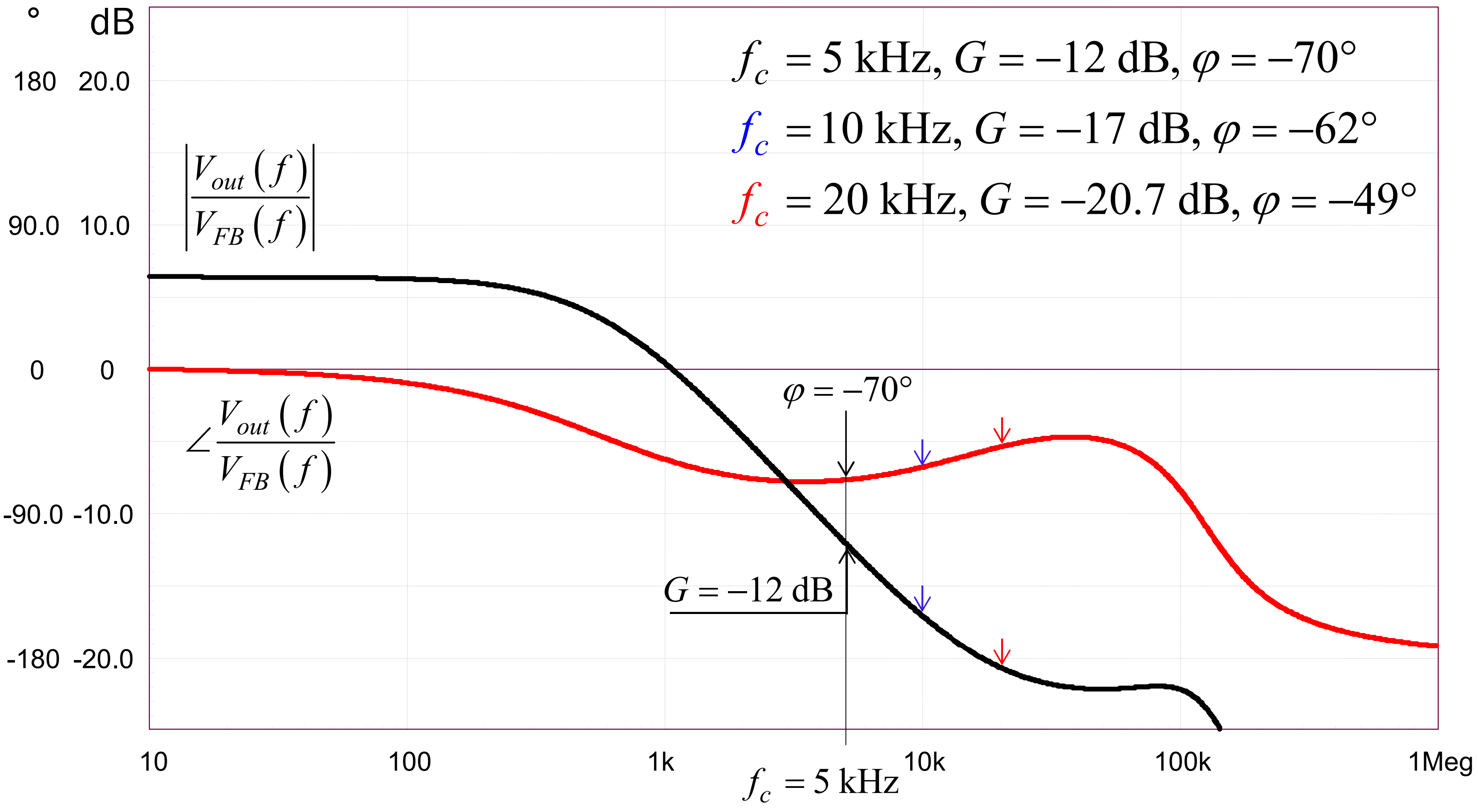
- ☐ A nonlinear average model is the ideal tool
  - ❖ Auto-bias the output to its operating point, I_{out} = 10 A



- ☐ Check all operating points are correct (V_{out} = 5 V)
- ☐ Extract magnitude and phase information from the Bode plot

You Need the Power Stage Response

- ☐ Select a crossover frequency f_c: read magnitude and phase



$f_c = 5\text{ kHz}, G = -12\text{ dB}, \varphi = -70°$

$f_c = 10\text{ kHz}, G = -17\text{ dB}, \varphi = -62°$

$f_c = 20\text{ kHz}, G = -20.7\text{ dB}, \varphi = -49°$

Capture these Values in the Sheet

- ☐ You can automate the compensation elements computation

parameters

Rupper=10k
Rlower=Rupper
fc=20k
pm=60

Power stage data

Gfc=-20.7
pfc=-49

k-factor automation

G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
C2=1/(2*pi*fc*G*k*Rupper)
C1=C2*(K^2-1)
R2=k/(2*pi*fc*C1)

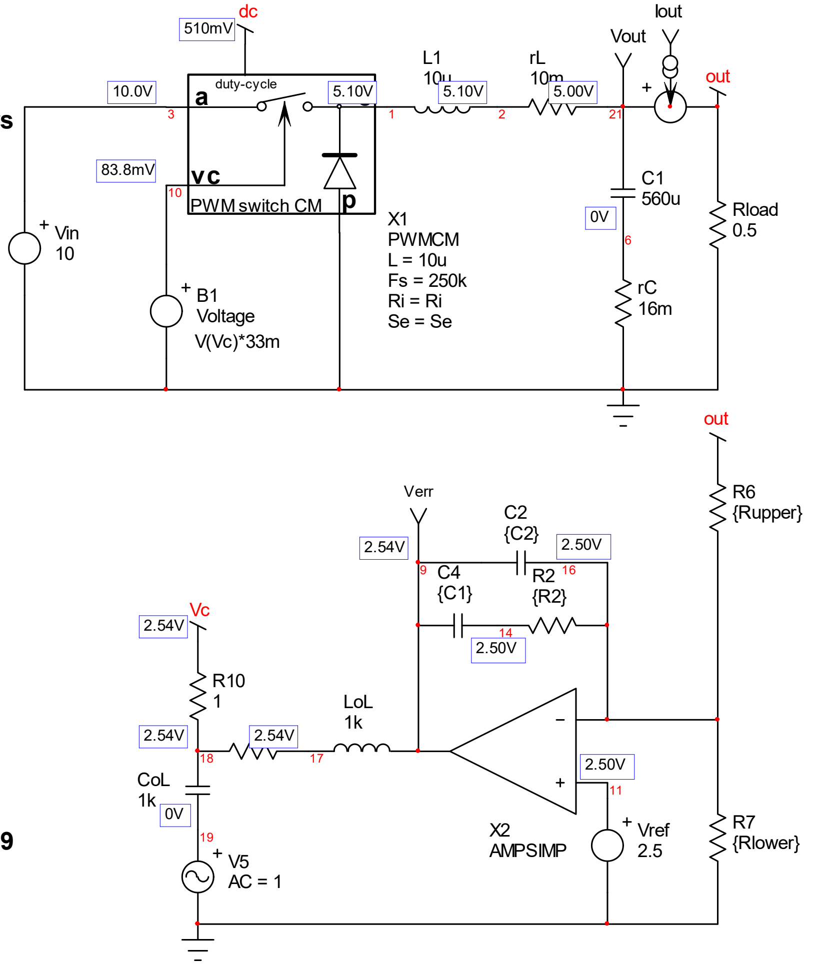
fp1=1/(2*pi*R2*C2)
fz1=1/(2*pi*R2*C1)

PWR stage response

fc=5 kHz, Gfc =-12 , pfc =-70
fc=10 kHz, Gfc =-17 , pfc =-62
fc=20 kHz, Gfc =-20.7 , pfc =-49

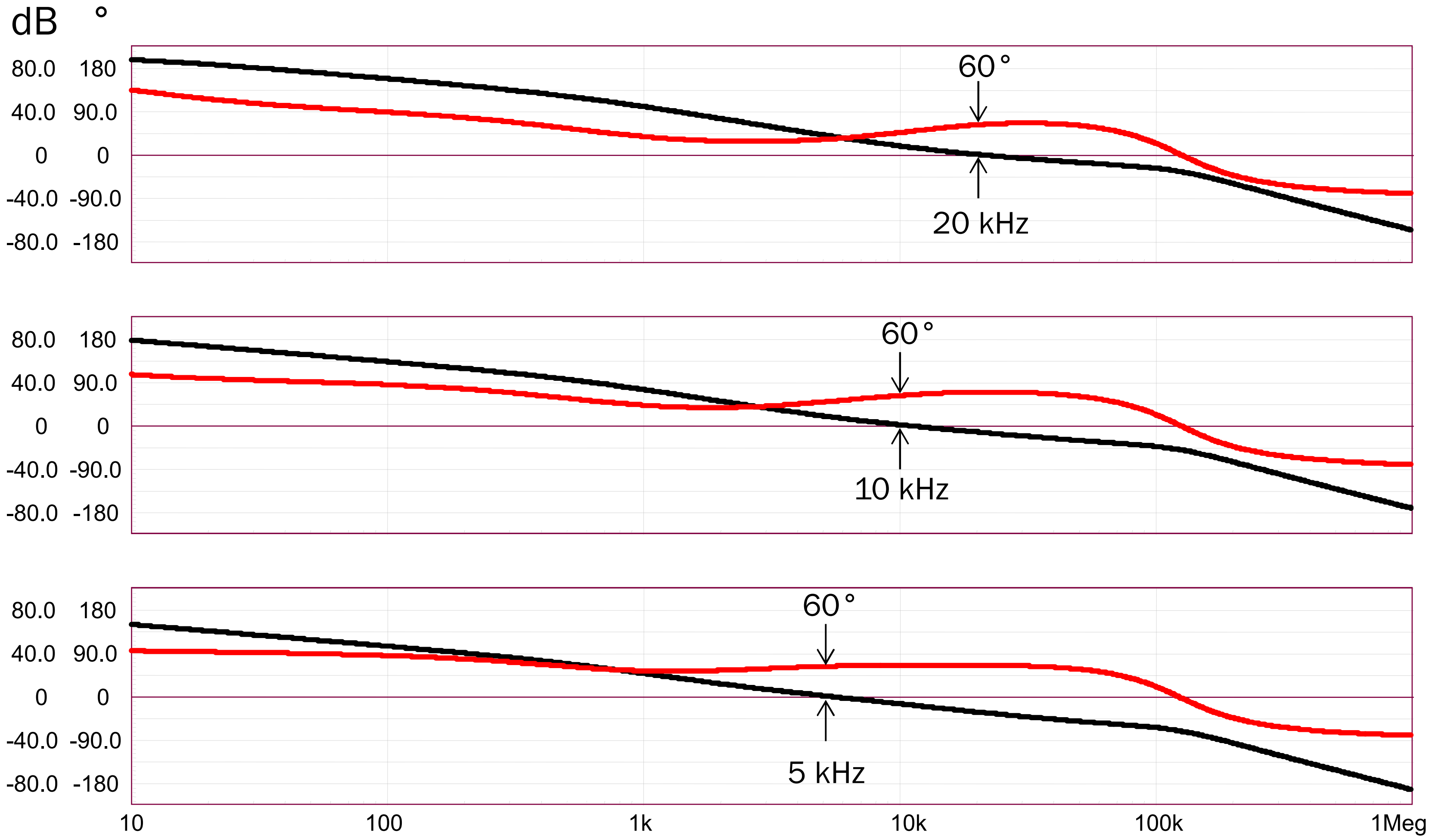
parameters

Ri=7.5m
Se=2.5k



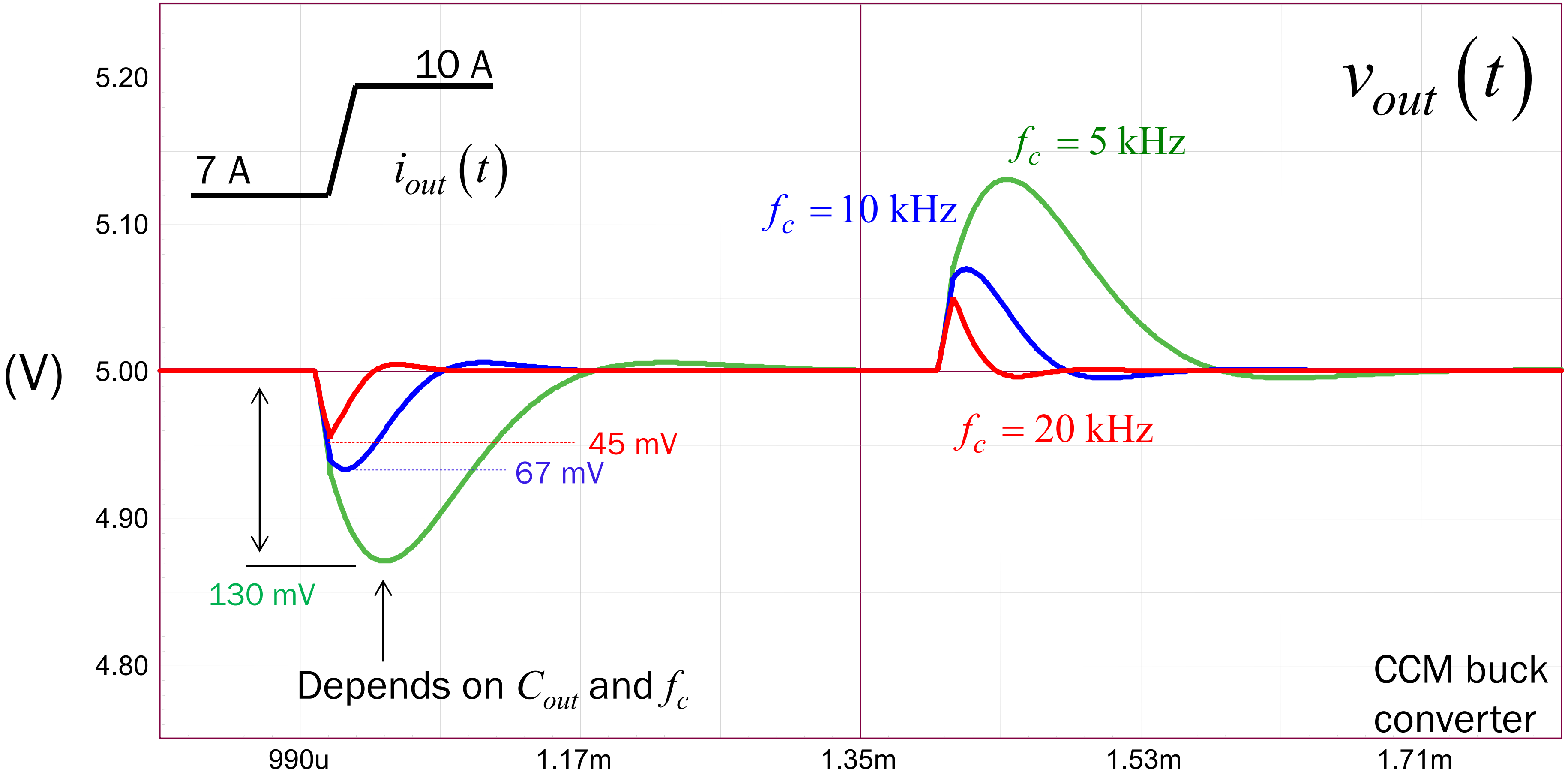
Check the Open-Loop Gain Data

- ☐ Verify compensation is correct at the three selected points



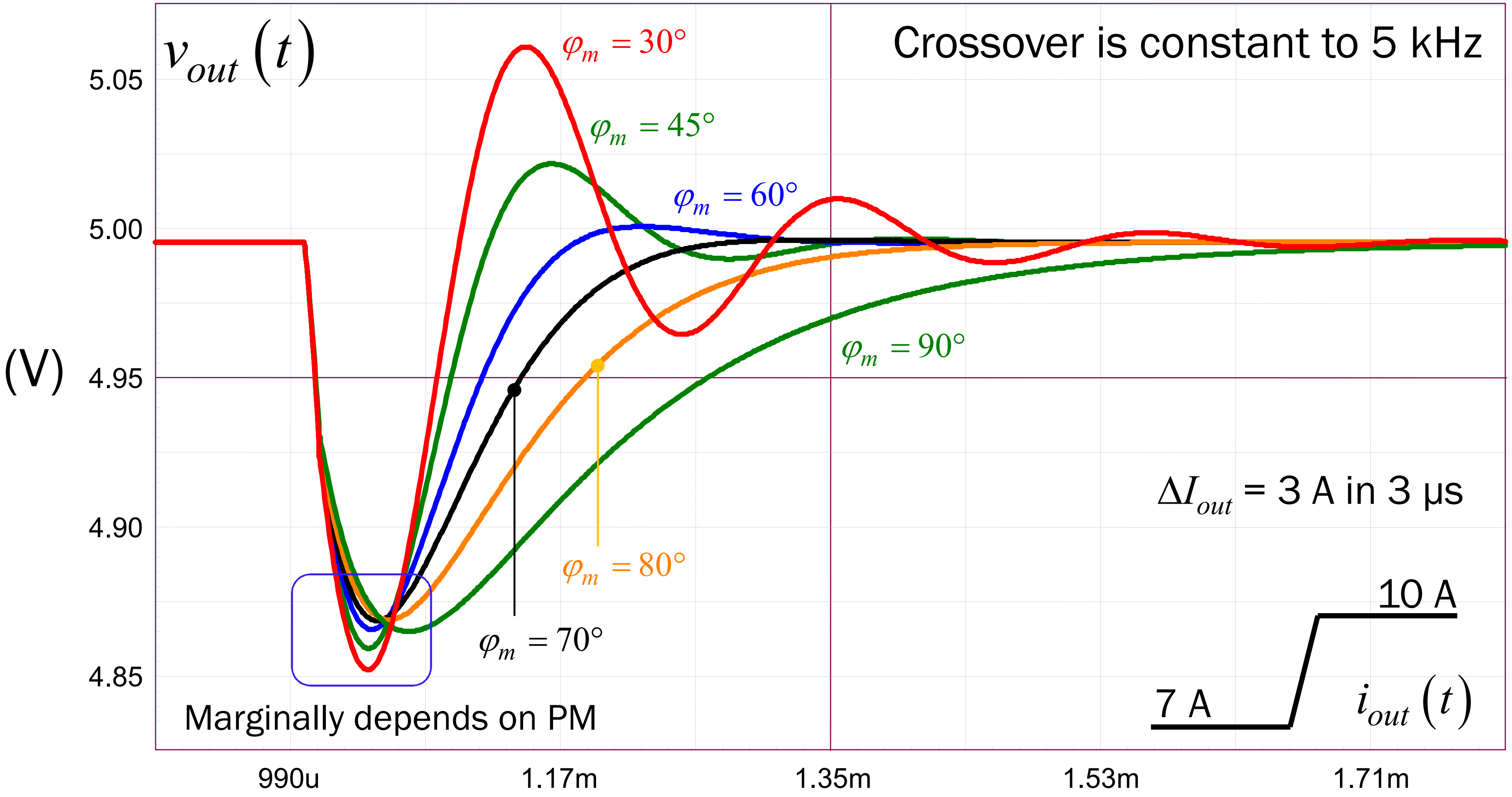
Step-Load the Output of the Converter

- ☐ With constant 60° phase margin the crossover affects the drop



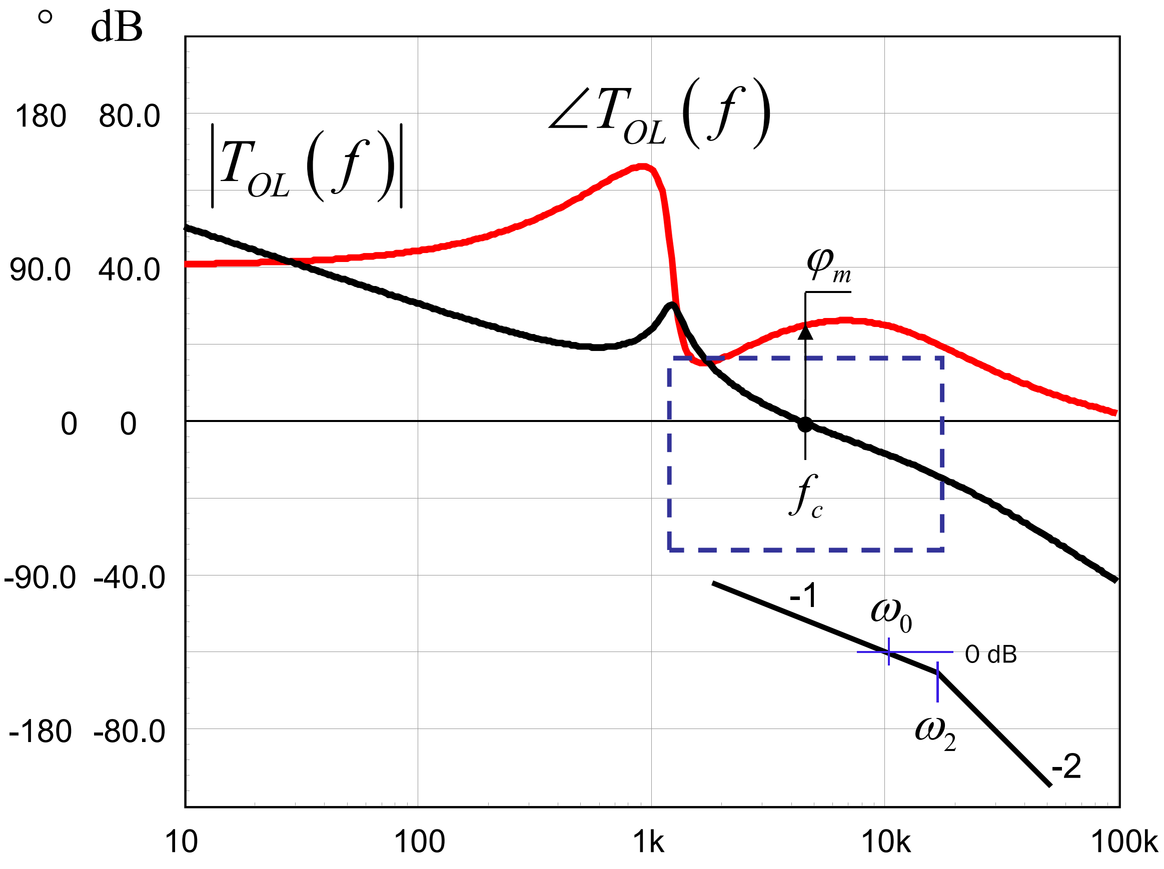
Sweep Phase Margin at Constant f_c

❑ The phase margin affects the overshoot and recovery



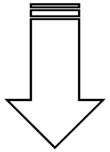
Link Phase Margin with Quality Factor

- ☐ We can observe the loop gain in the vicinity of crossover



Open-loop ac analysis

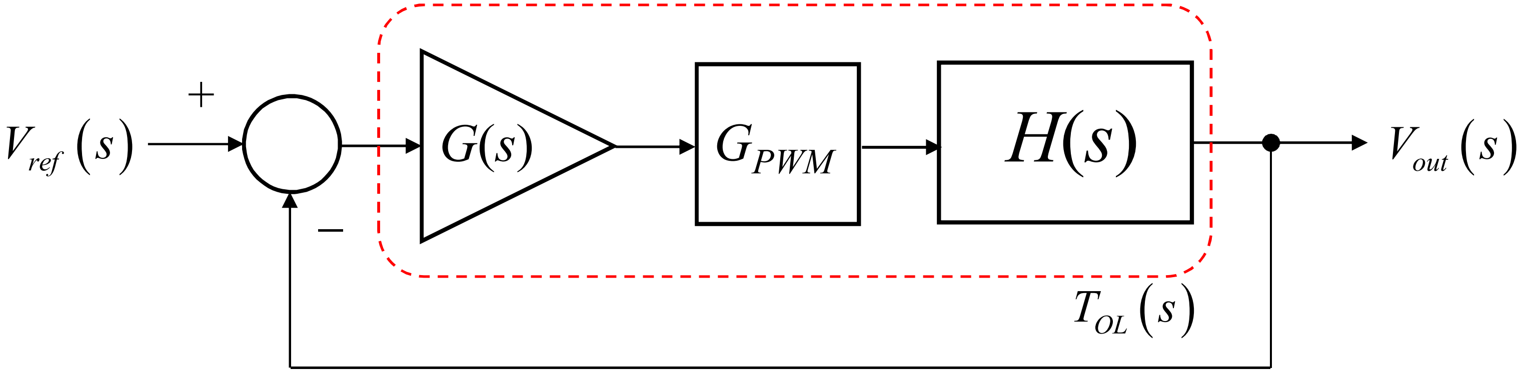
Approximation of $T(s)$ around f_c



$$T_{OL}(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Deriving the Closed-Loop Gain

- ☐ Consider a unity return control system



- ☐ We can derive the closed-loop transfer function

$$\frac{V_{out}(s)}{V_{ref}(s)} = \frac{T_{OL}(s)}{1+T_{OL}(s)} = \frac{1}{\frac{s^2}{\omega_0\omega_2}+\frac{s}{\omega_0}+1} = \frac{1}{1+\frac{s}{\omega_c Q}+\left(\frac{s}{\omega_c}\right)^2}$$

$$\omega_c = \sqrt{\omega_0\omega_2} \leftarrow$$ Open-loop data

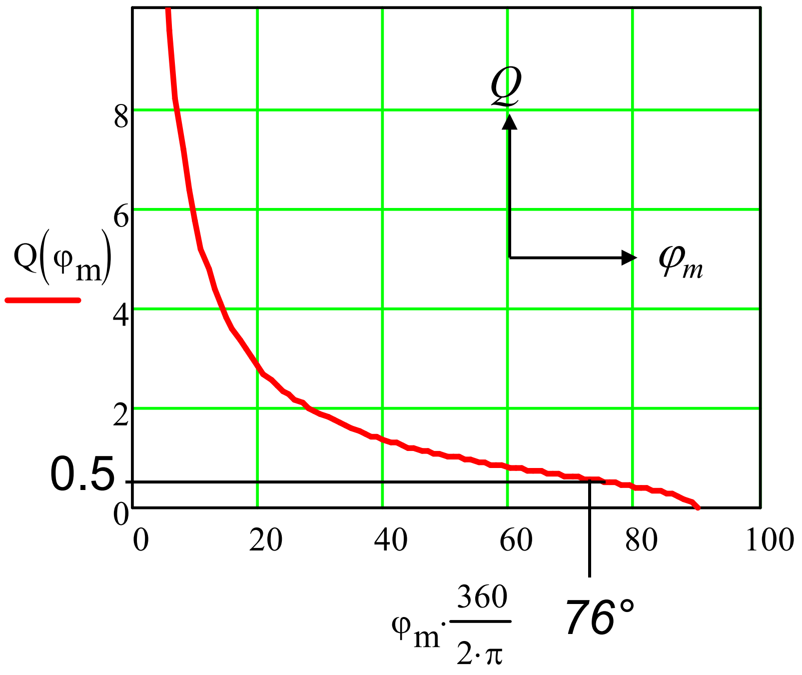
$$Q = \frac{\omega_0}{\omega_2}$$

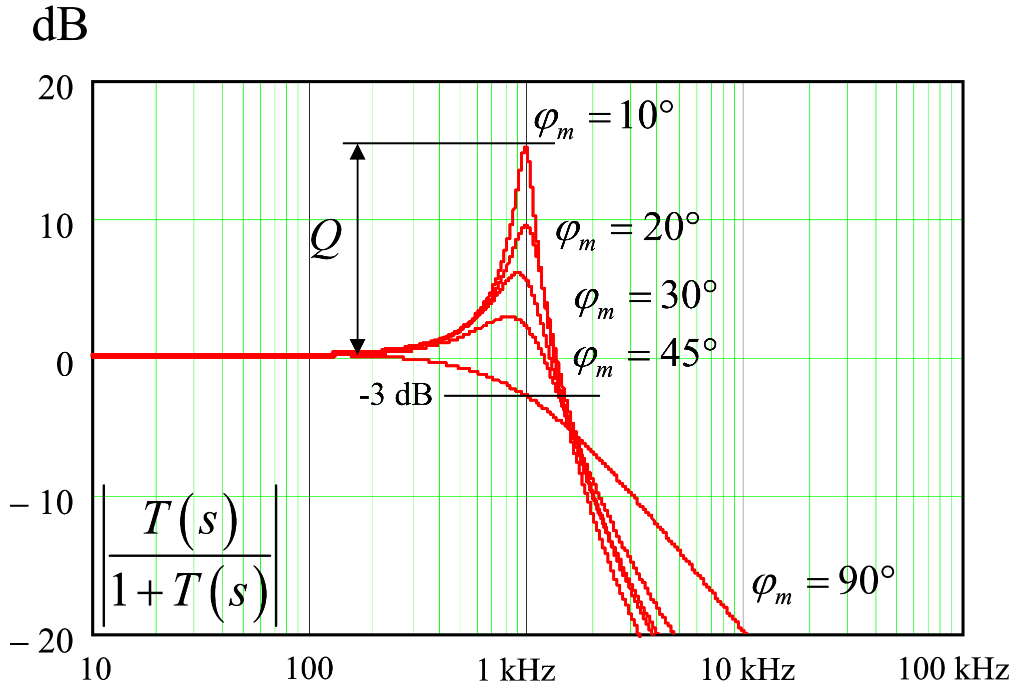
$\uparrow$ Closed-loop data

Phase Margin and Transient Response

- ☐ Choose phase margin based on the response you want

$$Q = \frac{\sqrt{\cos(\varphi_m)}}{\sin(\varphi_m)}$$



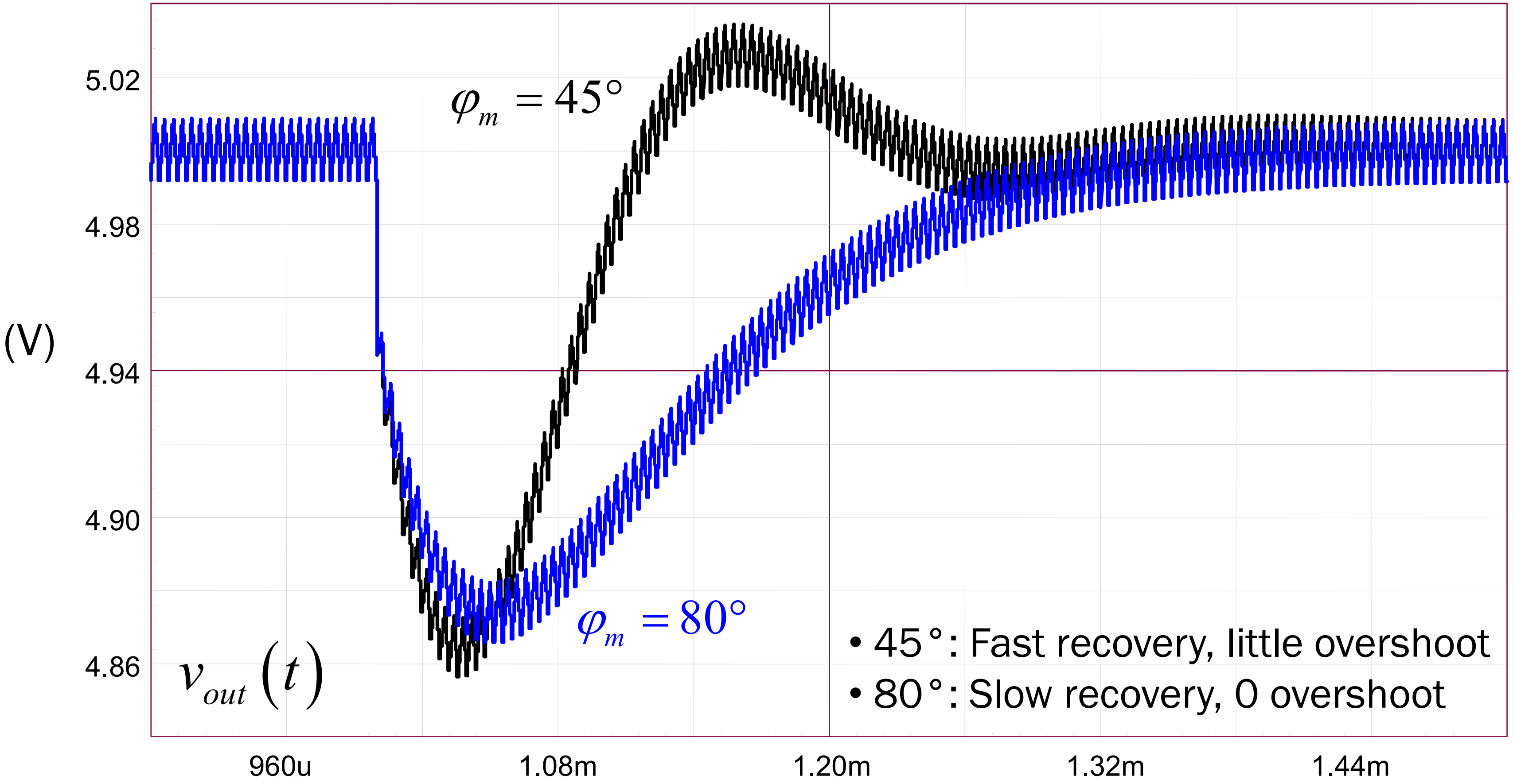


Closed-loop response

C. Basso, “The Dark Side of Loop Control Theory”, APEC 2012 Professional Seminar

What Transient Response is Wanted?

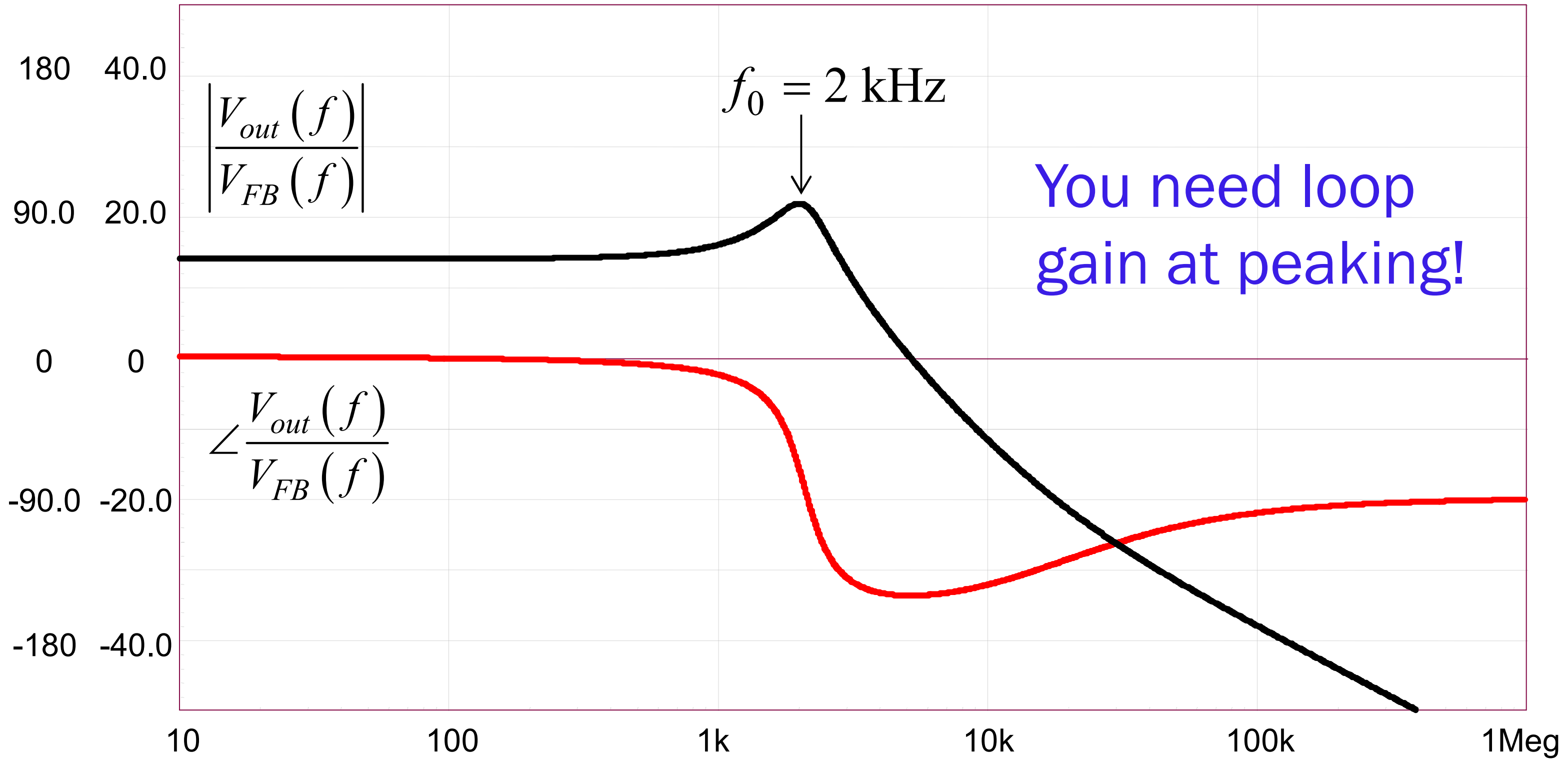
- ☐ Choose phase margin based on the response you want



- 45°: Fast recovery, little overshoot
- 80°: Slow recovery, 0 overshoot

How to Select Crossover Frequency?

- ☐ Before selecting a value, there are limits to respect

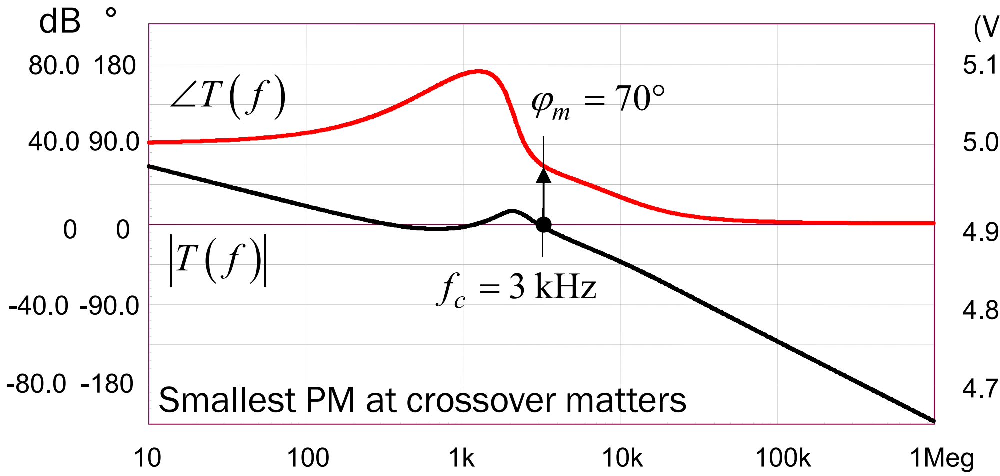


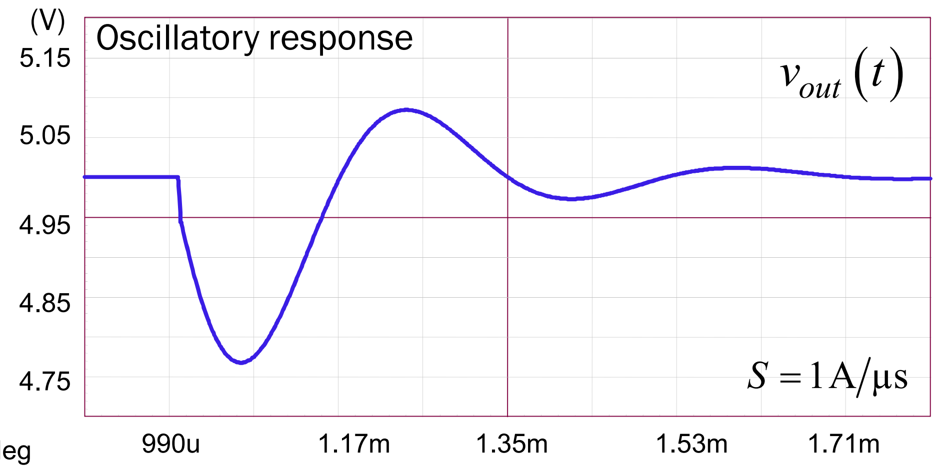
You need loop gain at peaking!

- ☐ Crossover for a CCM VM buck must be: $3 \cdot f_0 < f_c < \frac{F_{sw}}{2}$
- ❖ In this example: f_c > 6 kHz

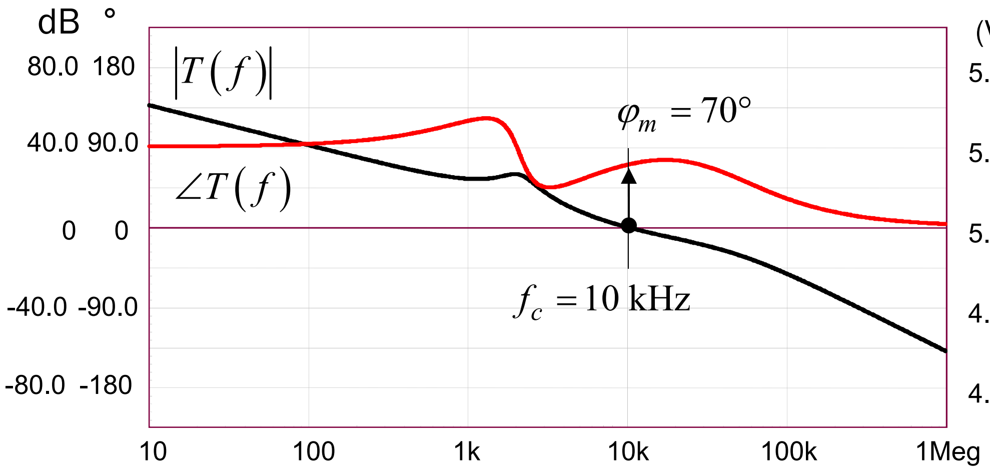
A Peaky Closed-Loop Output Impedance

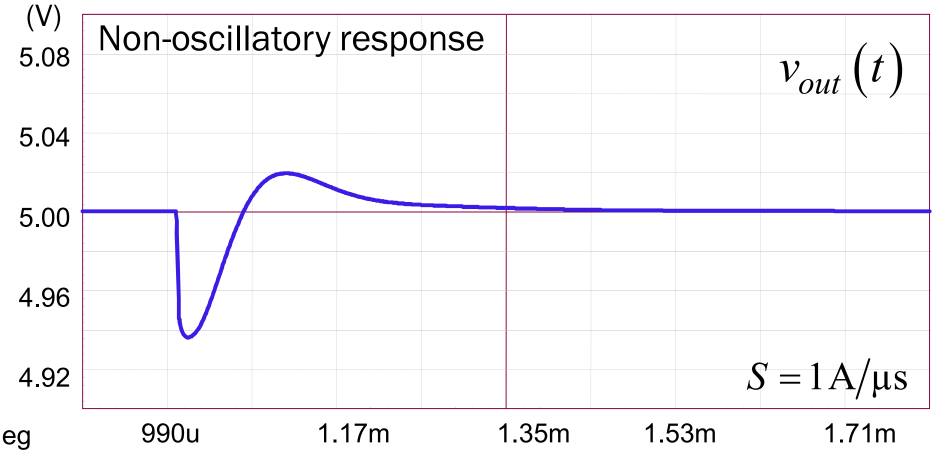
- ☐ Too low a crossover and perturbation can't be fought





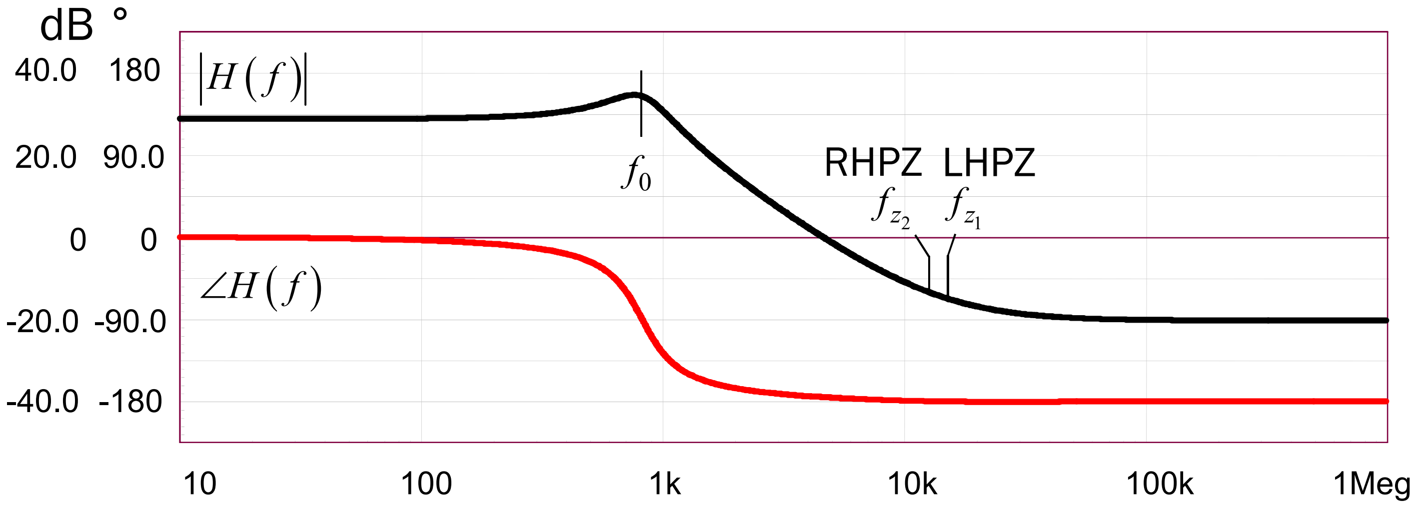
- ☐ With enough gain, response is non-oscillatory





Beware of the Deceitful RHP Zero

- ☐ Peaking also exists in the CCM VM boost and there is a RHPZ



$$f_0 = \frac{1-D}{2\pi\sqrt{LC}} = 817 \text{ Hz}$$

$$f_{z_1} = \frac{1}{2\pi r_C C} = 14.5 \text{ kHz}$$

$$f_{z_2} = \frac{R(1-D)^2}{2\pi L} = 13.8 \text{ kHz}$$

- ☐ Crossover must be selected beyond resonance and below RHPZ
- ❖ Stay below 30% of the lowest RHPZ position (low V_{in}, high I_{out})

$$3 \cdot f_0 < f_c < 0.3 \cdot f_{z_2}$$

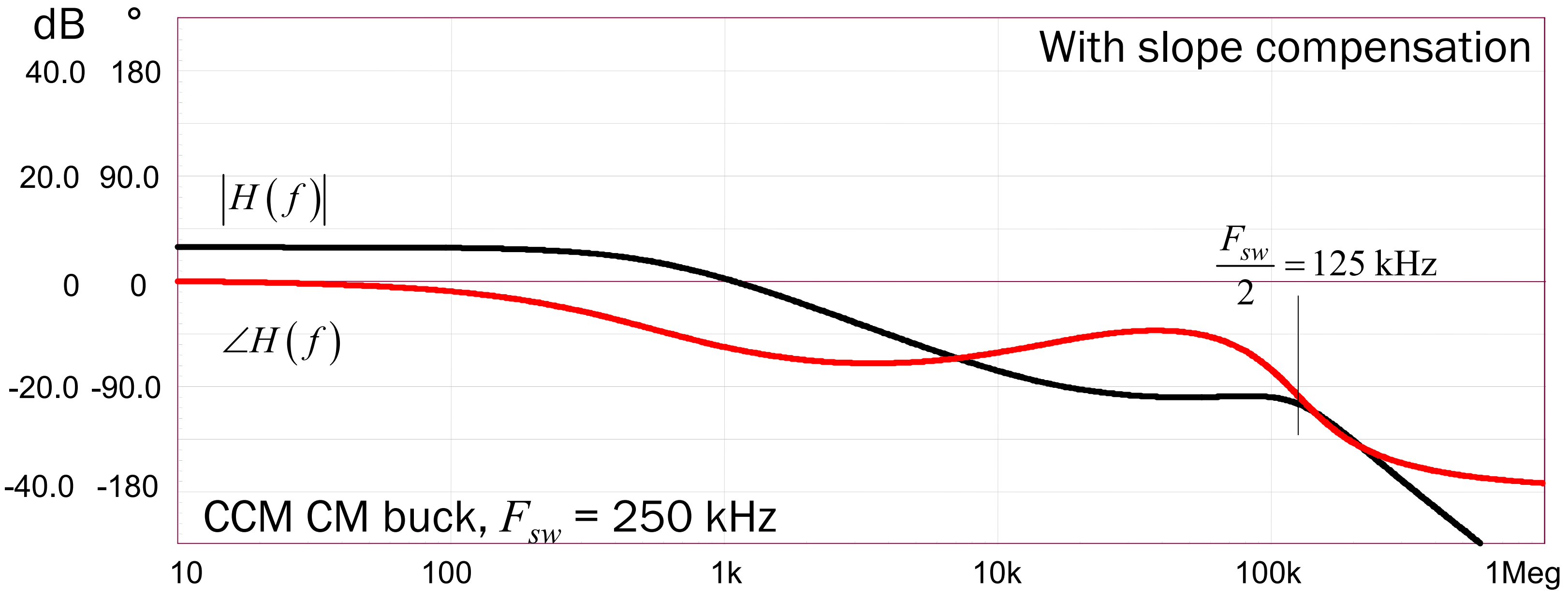
$V_{in} = 8 \text{ V}$
$L = 47\ \mu\text{H}$
$D = 47.8\%$
$R = 15\ \Omega$
$C = 220\ \mu\text{F}$

$$\Longrightarrow \quad 2.4 \text{ kHz} < f_c < 4.1 \text{ kHz}$$

Similar remarks for a buck-boost but different RHPZ

Less Constraints in Current Mode

- ☐ Once the poles are damped, the *theoretical* crossover limit is $F_{sw}/2$

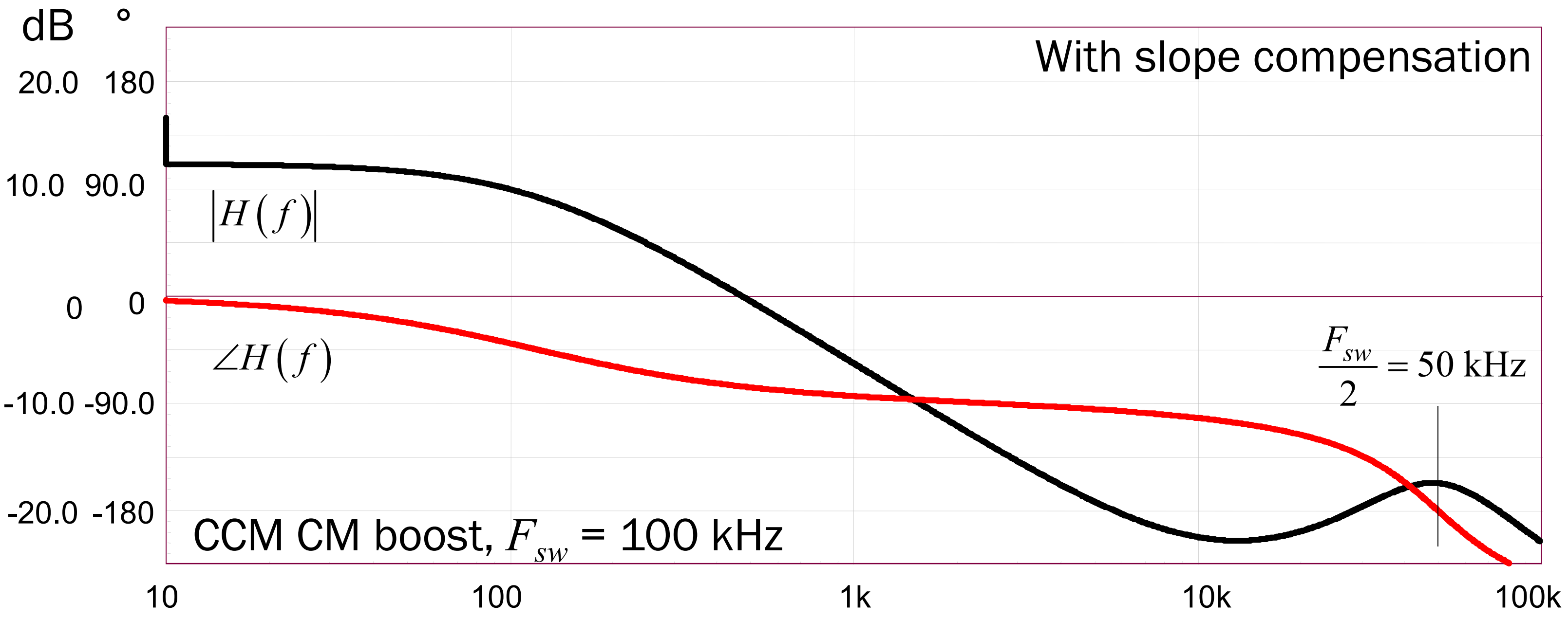


- ❖ Don't push f_c too far as it increases susceptibility to noise
- ❖ As f_c goes up, beware of various delays (conversion time, prop. del.)

⇒ Choose crossover to meet transient response, not more!

RHP Zero is Still There in Current Mode

❑ In boost and buck-boost CM, the RHPZ still fixes the upper limit



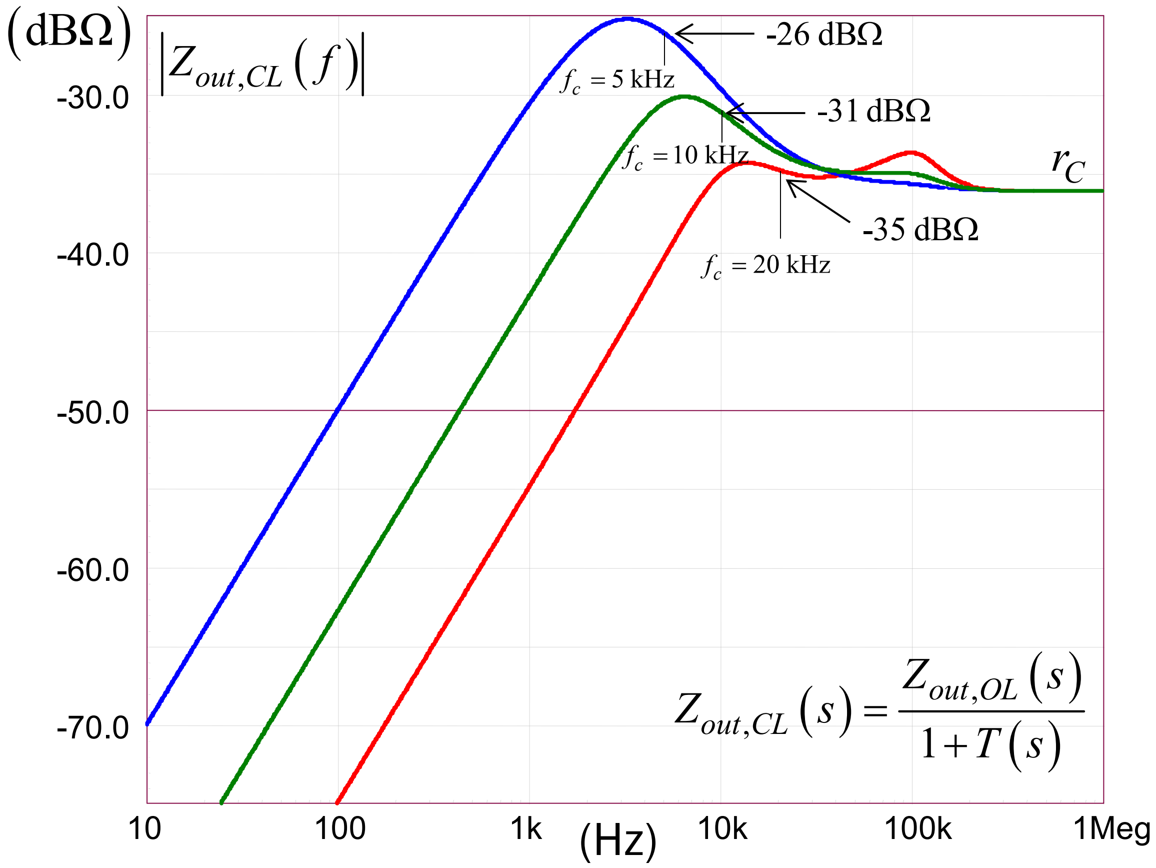
❖ Stay below 30% of the lowest RHPZ position (low V_{in}, high I_{out})

$$f_c < 0.3 \cdot f_{z_2} \Longrightarrow f_c < 4.1\text{ kHz}$$

Similar remarks for a buck-boost but different RHPZ

Output Impedance and Transient Response

- ☐ The closed-loop output impedance changes with crossover



$$Z_{out,CL}(s) = \frac{Z_{out,OL}(s)}{1+T(s)}$$

If C_{out}'s impedance dominates at f_c then

$$r_C << \frac{1}{2\pi f_c C_{out}}$$

$$\Downarrow$$

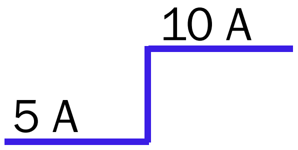
$$\Delta V_{out} \approx \frac{\Delta I_{out}}{2\pi f_c C_{out}}$$

- ☐ As crossover improves, Z_{out} approaches the minimum set by r_C

A Simple Guide to Crossover Selection

❑ Select the output capacitor based on ripple current, ESR etc.

✓ Choose the crossover frequency to meet undershoot specs

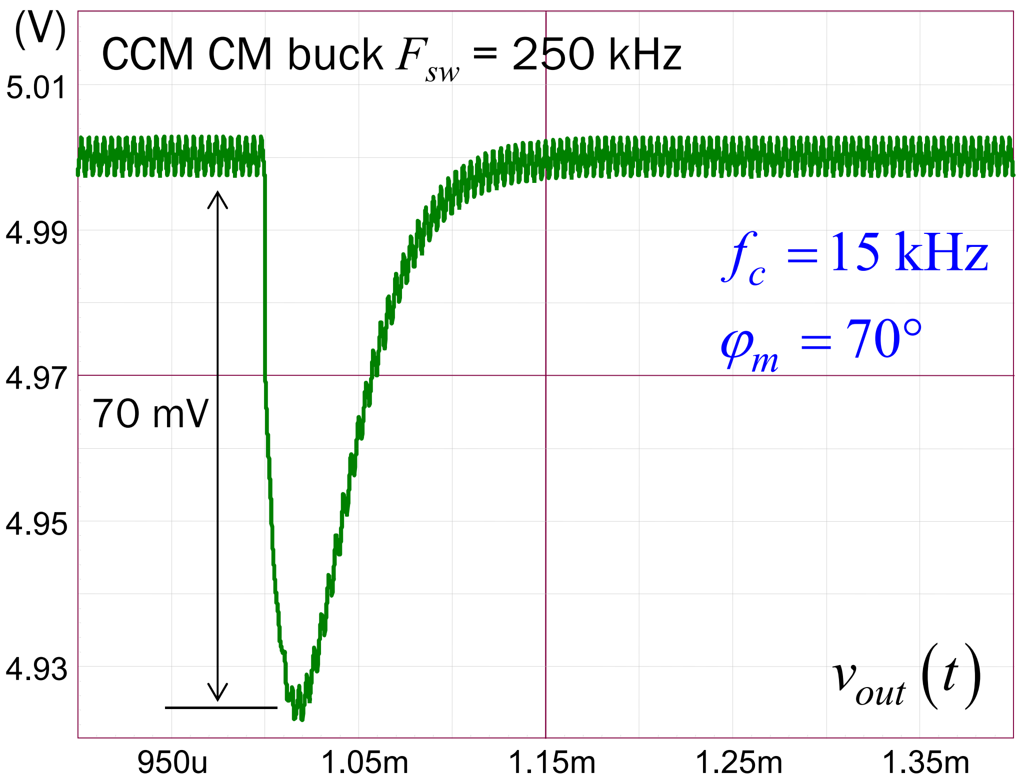


$$\Delta V_{out} < 100\text{ mV}$$

$$C_{out} = 560\ \mu\text{F}$$

$$r_C = 5\text{ m}\Omega$$

$$f_c \approx \frac{\Delta V_{out}}{2\pi C_{out} \Delta I_{out}} = 15\text{ kHz}$$



❑ It surely is an approximation as the system is nonlinear but it’s a guide

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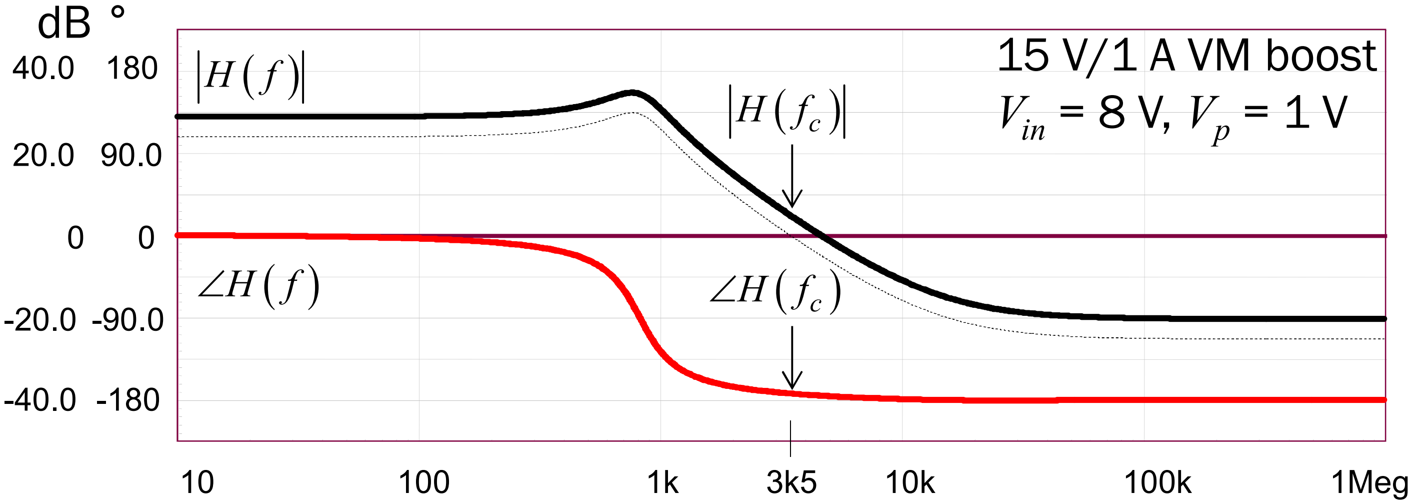
- ☐ Crossover Frequency and Phase Margin
- ☐ **Compensation Strategy**
- ☐ Experiments on Prototypes
- ☐ Conclusion

Stabilizing a Voltage-Mode Boost Converter

- ☐ Whether you use SPICE, Simplis®, Mathcad® or the bench…
  - ➢ You start the process with the power stage ac response



$$\left|H(f_c)\right| = 4.6\text{ dB}$$

$$\angle H(f_c) = -173°$$

- ☐ We select a crossover frequency f_c of 3.5 kHz
  - ❖ Read the power stage Bode plot at f_c:
  - ❖ Shift the curve down by 4.6 dB to crossover the 0-dB axis at f_c
- ☐ We choose a 50° phase margin

Place Poles and Zeros

☐ *k* factor is a way to go but conditional stability can happen in CCM

☐ We will adopt the following strategy:

✓ A double zero located slightly below the resonant frequency f_0

✓ A 1st pole adjusted to cancel the RHPZ or ESR contribution

✓ A 2nd pole is adjusted to meet the phase margin goal

$$V_{in} = 8\ \text{V}$$
$$L = 47\ \mu\text{H}$$
$$D = 47.8\%$$
$$R = 15\ \Omega$$
$$C = 220\ \mu\text{F}$$
$$r_C = 50\ \text{m}\Omega$$

$$f_0 = \frac{1-D}{2\pi\sqrt{LC}} = 817\ \text{Hz}$$

$$f_{z_1} = \frac{1}{2\pi r_C C} = 14.5\ \text{kHz}$$

$$f_{z_2} \approx \frac{R(1-D)^2}{2\pi \cdot L} = 13.5\ \text{kHz}$$

Compensation strategy:

$$f_{z_1} = f_{z_2} = 700\ \text{Hz}$$

$$f_{p_2} = 20\ \text{kHz}$$

$$f_{p_1} = ?$$

D. Venable, *The k factor: A New Mathematical Tool for Stability Analysis and Synthesis*, Proceedings of Powercon 10, 1983, pp. 1-12

Create Phase Boost at Crossover

- ☐ The cumulated phase lags must be less than 360°
- ✓ The compensator op amp lags by 180°
- ✓ The pole at the origin lags by 90°

⇨ $$boost = \varphi_m - \angle H\left(f_c\right) - 90° \approx 133°$$

- ☐ Place the second pole to boost the phase by 133° at f_c

$$G(s) = -G_0 \frac{\left(1+\frac{s_{z_1}}{s}\right)\left(1+\frac{s}{s_{z_2}}\right)}{\left(1+\frac{s}{s_{p_1}}\right)\left(1+\frac{s}{s_{p_2}}\right)}$$

$$boost = \tan^{-1}\left(\frac{f_c}{f_{z_1}}\right) + \tan^{-1}\left(\frac{f_c}{f_{z_2}}\right) - \tan^{-1}\left(\frac{f_c}{f_{p_1}}\right) - \tan^{-1}\left(\frac{f_c}{f_{p_2}}\right)$$

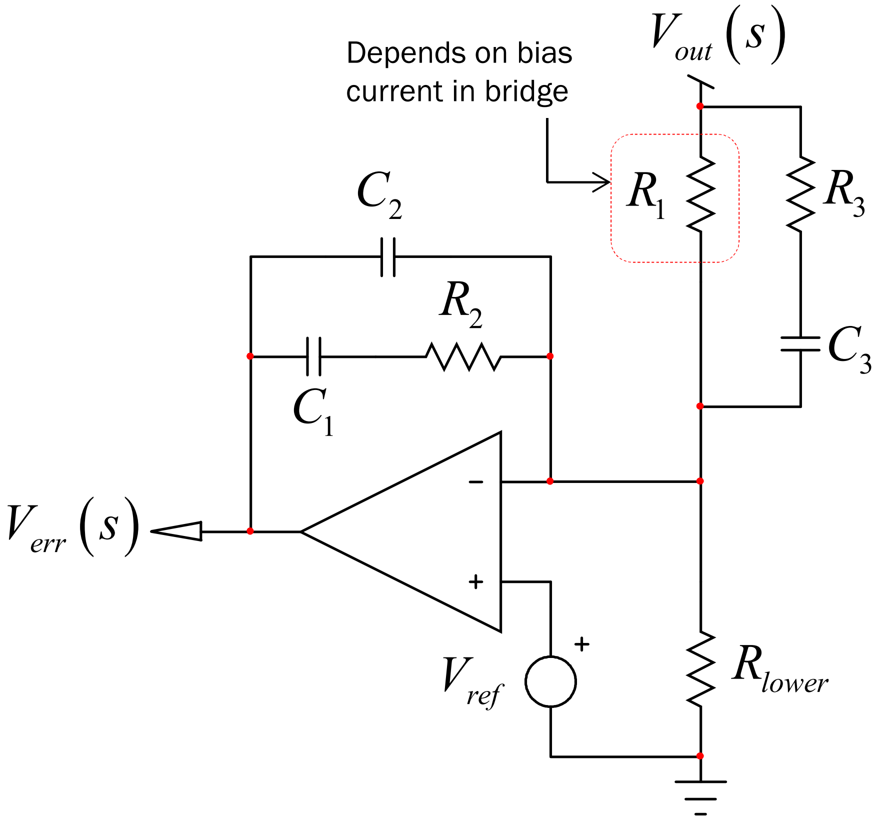
$$f_{p_1} = \frac{f_c}{\tan\left[\tan^{-1}\left(\frac{f_c}{f_{z_1}}\right) + \tan^{-1}\left(\frac{f_c}{f_{z_2}}\right) - boost - \tan^{-1}\left(\frac{f_c}{f_{p_2}}\right)\right]}$$

From compensation strategy:

⇨ $$f_{p_1} \approx 14\text{ kHz}$$

Adjust the Gain to Meet 0 dB at f_c

☐ The crossover gain is adjusted by resistance R_2



$$R_2 = \frac{\overset{\overset{\frac{1}{\left|H\left(f_c\right)\right|}}{\downarrow}}{G_{f_c}} R_1 f_{p_1}}{f_{p_1} - f_{z_1}} \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}}$$

$$C_1 = \frac{1}{2\pi f_{z_1} R_2} \qquad C_2 = \frac{C_1}{2\pi f_{p_1} C_1 R_2 - 1}$$

$$C_3 = \frac{f_{p_2} - f_{z_2}}{2\pi R_{upper} f_{p_2} f_{z_2}} \qquad R_3 = \frac{R_1 f_{z_2}}{f_{p_2} - f_{z_2}}$$

☐ Check all values are positive or revise phase margin goals!

C. Basso, *Designing Control Loops for Linear and Switching Power Supplies*, Artech House, 2012

Automate the Calculation in SPICE

☐ A dedicated macro computes the component values

parameters

Rupper=5k
fc=3.5k
Gfc=4.58
pfc=-173.476
pm=50

boost=pm-(pfc)-90

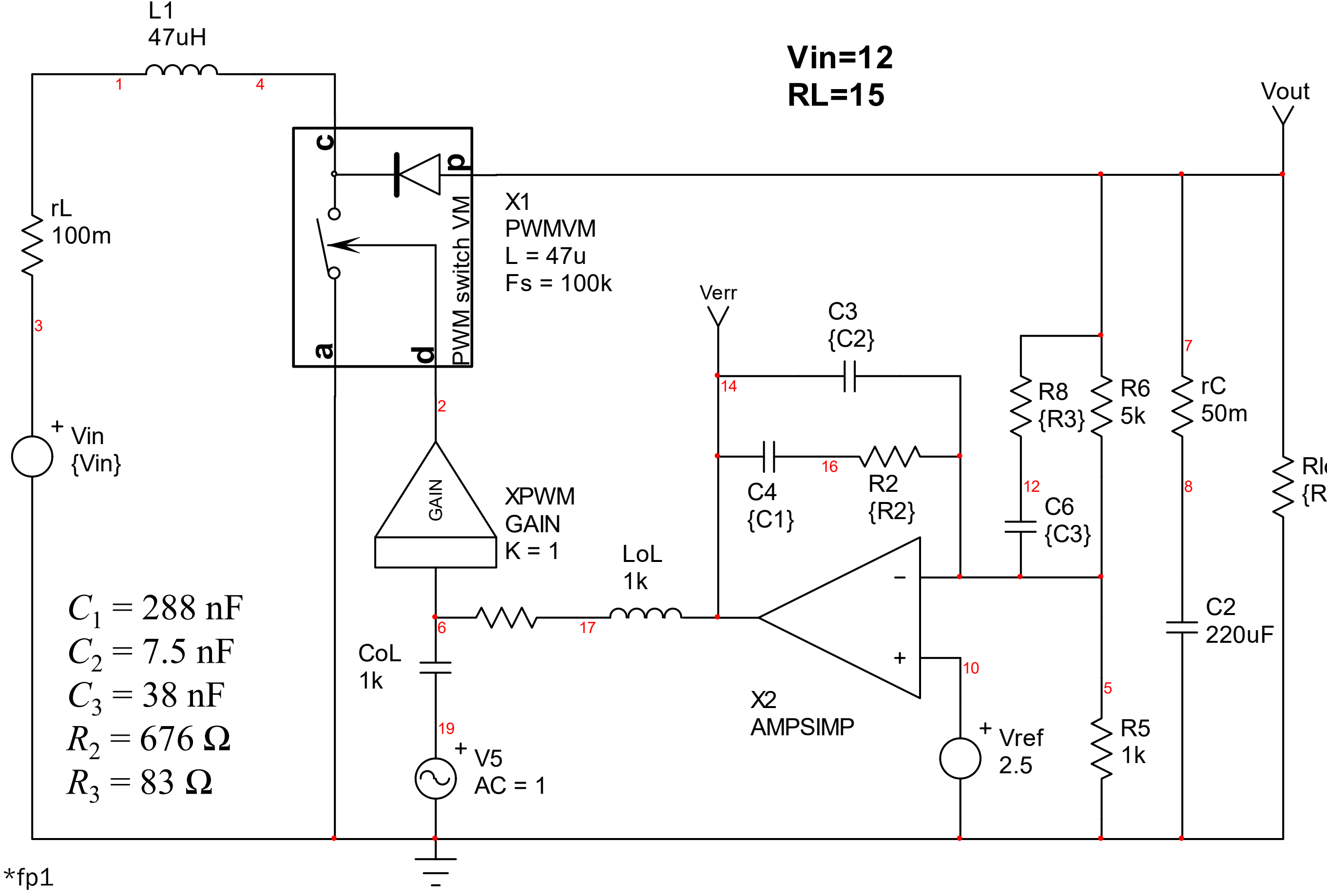
G=10^(-Gfc/20)
pi=3.14159

fz1=700
fz2=700
fp2=20k

C1=1/(2*pi*fz1*R2)
C2=C1/(C1*R2*2*pi*fp1-1)
C3=(fp2-fz2)/(2*pi*Rupper*fp2*fz2)
R3=Rupper*fz2/(fp2-fz2)

a=sqrt((fc^2/fp1^2)+1)
b=sqrt((fc^2/fp2^2)+1)
c=sqrt((fz1^2/fc^2)+1)
d=sqrt((fc^2/fz2^2)+1)

R2=((a*b/(c*d))/(fp1-fz1))*Rupper*G*fp1
fp1=fc/tan((2*atan(fc/fz1)-tan(fc/fp2))-boost*pi/180)



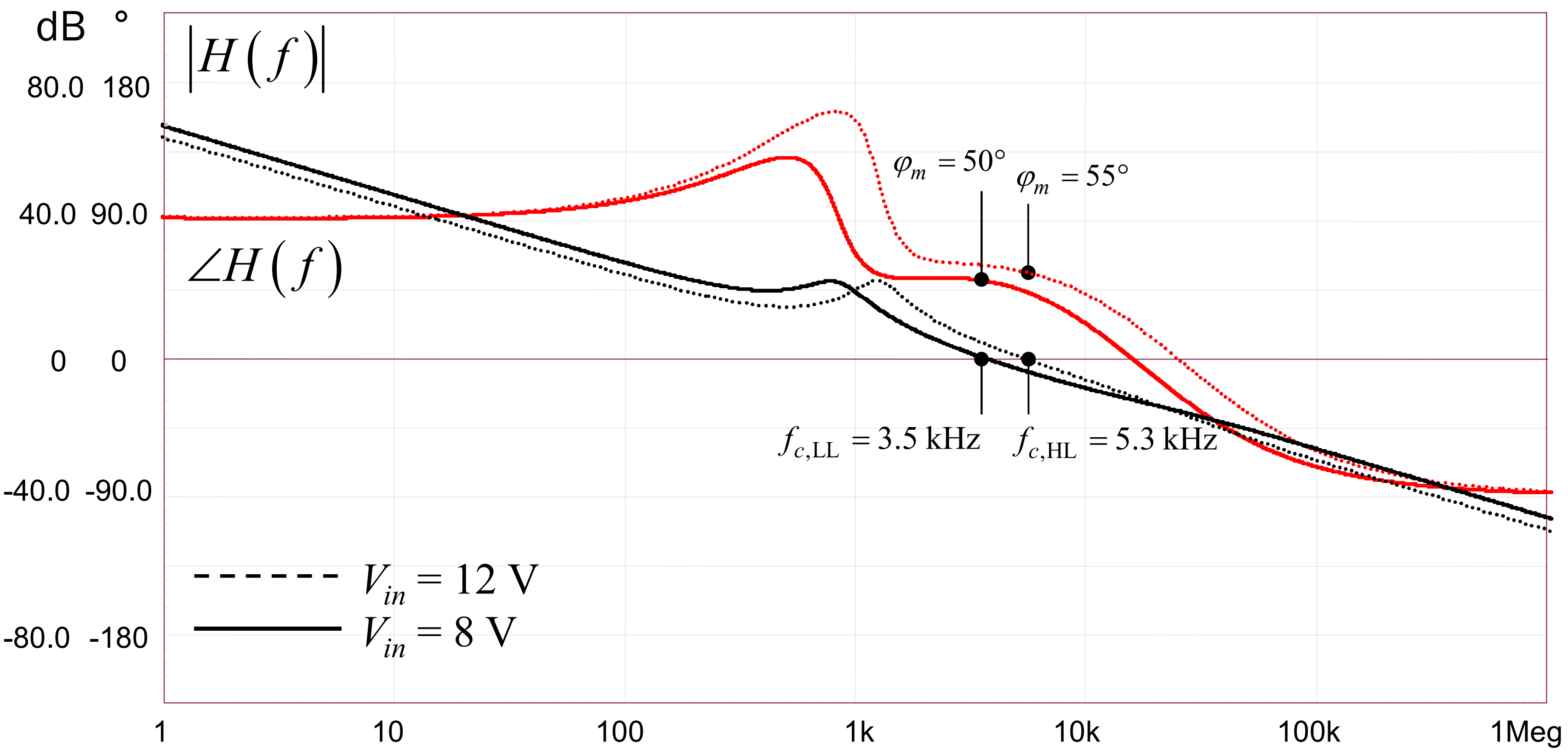
parameters

Vin=12
RL=15

$$C_1 = 288\ \text{nF}$$
$$C_2 = 7.5\ \text{nF}$$
$$C_3 = 38\ \text{nF}$$
$$R_2 = 676\ \Omega$$
$$R_3 = 83\ \Omega$$

Check the Compensated Converter

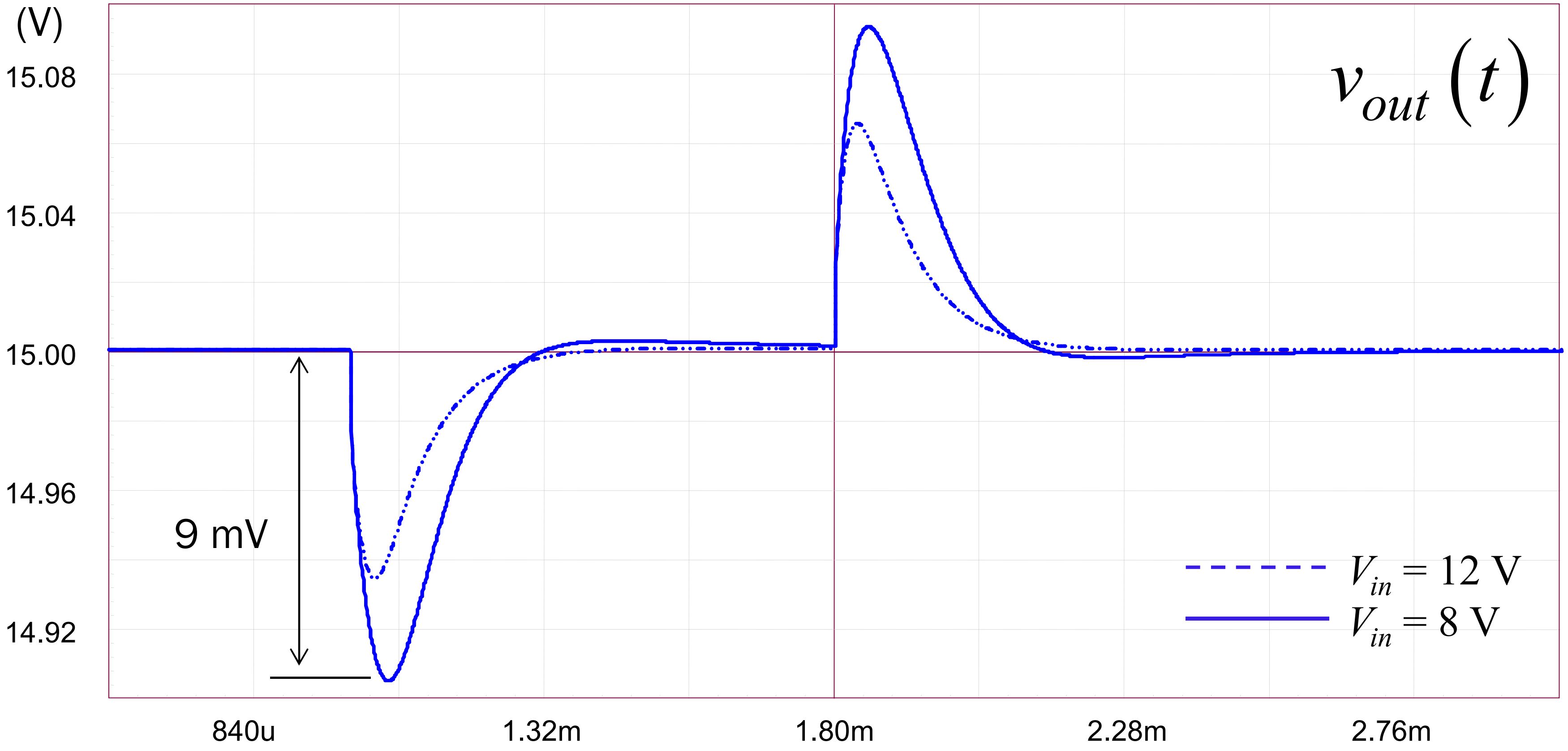
❑ Verify phase and gain margins at low- and high-line conditions



❖ If resonance is too high at high line, increase output capacitor

Check Transient Response

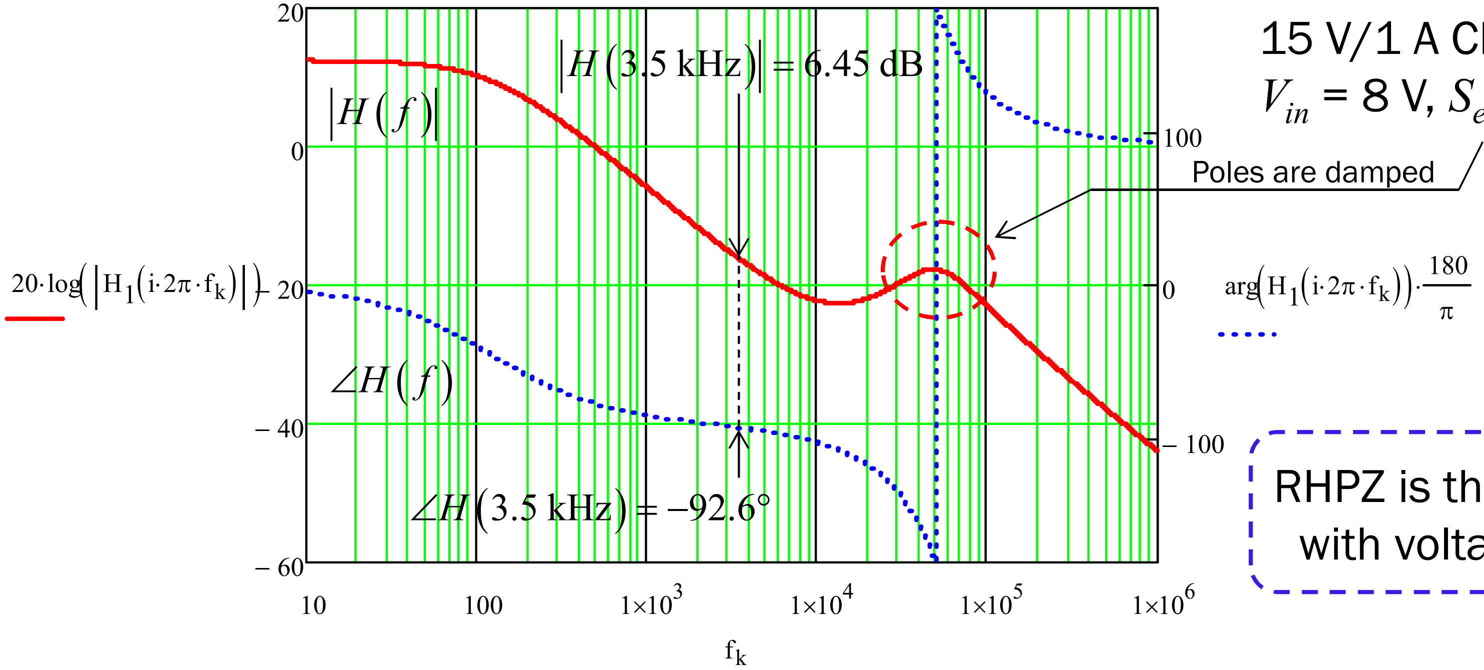
- ☐ The output current is varied from 0.5 to 1 A with a 1-A/µs slope



- ☐ Better response at high line with a slightly higher crossover

Stabilizing a Current-Mode Boost Converter

- ☐ You start with the control-to-output transfer function



15 V/1 A CM boost
V_{in} = 8 V, S_e = 28 kV/s

Poles are damped

RHPZ is the same as with voltage mode

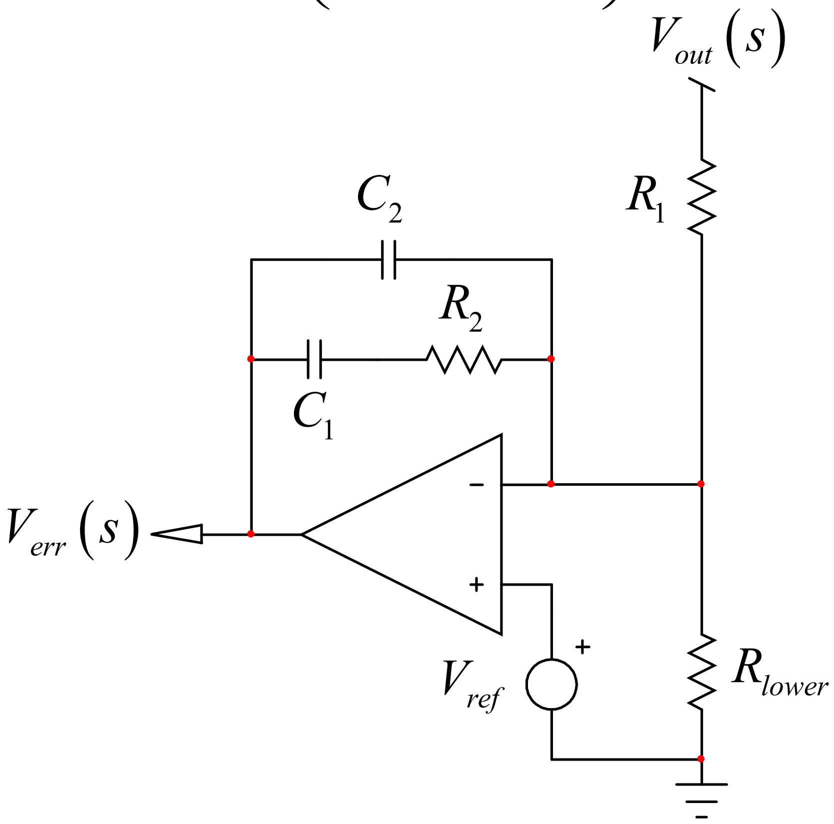
- ☐ For a phase boost below 90° we will use a type-2 compensator

$$boost = \varphi_m - \angle H\left(f_c\right) - 90° \approx 62.6°$$

Place Poles and Zeros

- ☐ *k* factor lends itself well to stabilizing 1st-order circuits

$$k = \tan\left(\frac{boost}{2} + \frac{\pi}{4}\right) = 4.1 \qquad f_z = \frac{f_c}{k} \qquad f_p = k \cdot f_c$$



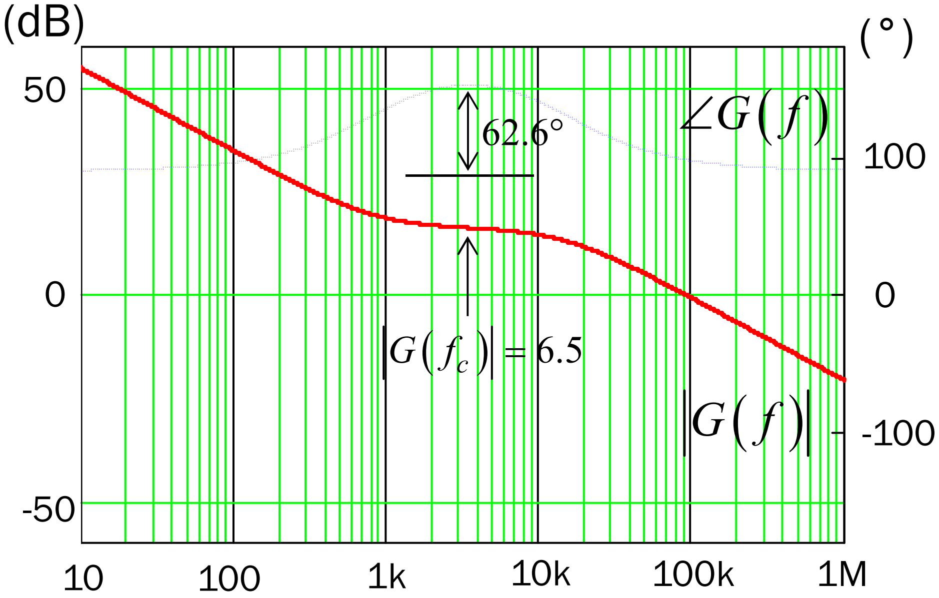
$$R_2 = \frac{\overset{\frac{1}{|H(f_c)|}\downarrow}{G_{f_c}} R_1 f_p}{f_p - f_z} \frac{\sqrt{\left(\frac{f_c}{f_p}\right)^2 + 1}}{\sqrt{\left(\frac{f_z}{f_c}\right)^2 + 1}} = 34.3\ \text{k}\Omega$$

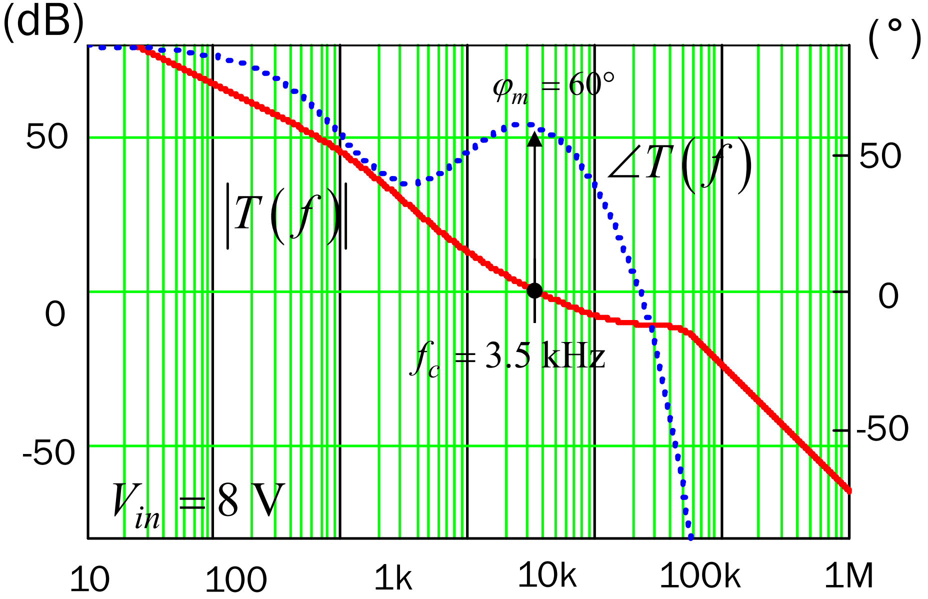
$$C_1 = \frac{1}{2\pi f_z R_2} = 5.4\ \text{nF}$$

$$C_2 = \frac{C_1}{2\pi f_p C_1 R_2 - 1} = 343.7\ \text{pF}$$

Verify Crossover and Phase Margin

- ❑ Plot the loop gain once compensator is designed





- ❑ Mathcad® can read the loop gain plot and extract data

$f_{cc} := \mathrm{root}\left(\left|T_{OL}(i\cdot 2\pi\cdot f_c)\right| - 1, f_c\right)$

$f_{cc} = \boxed{3.5\cdot\mathrm{kHz}}$

$PM_{fc} := \arg\left(T_{OL}(i\cdot 2\pi\cdot f_c)\right) = \boxed{60\cdot°}$

$-20\cdot\log\left(\left|T_{OL}(i\cdot 2\pi\cdot f_g)\right|, 10\right) = \boxed{10.53891}$

$f_g := \mathrm{root}\left(\arg\left(T_{OL}(i\cdot 2\cdot\pi\cdot f_g)\right), f_g\right)$

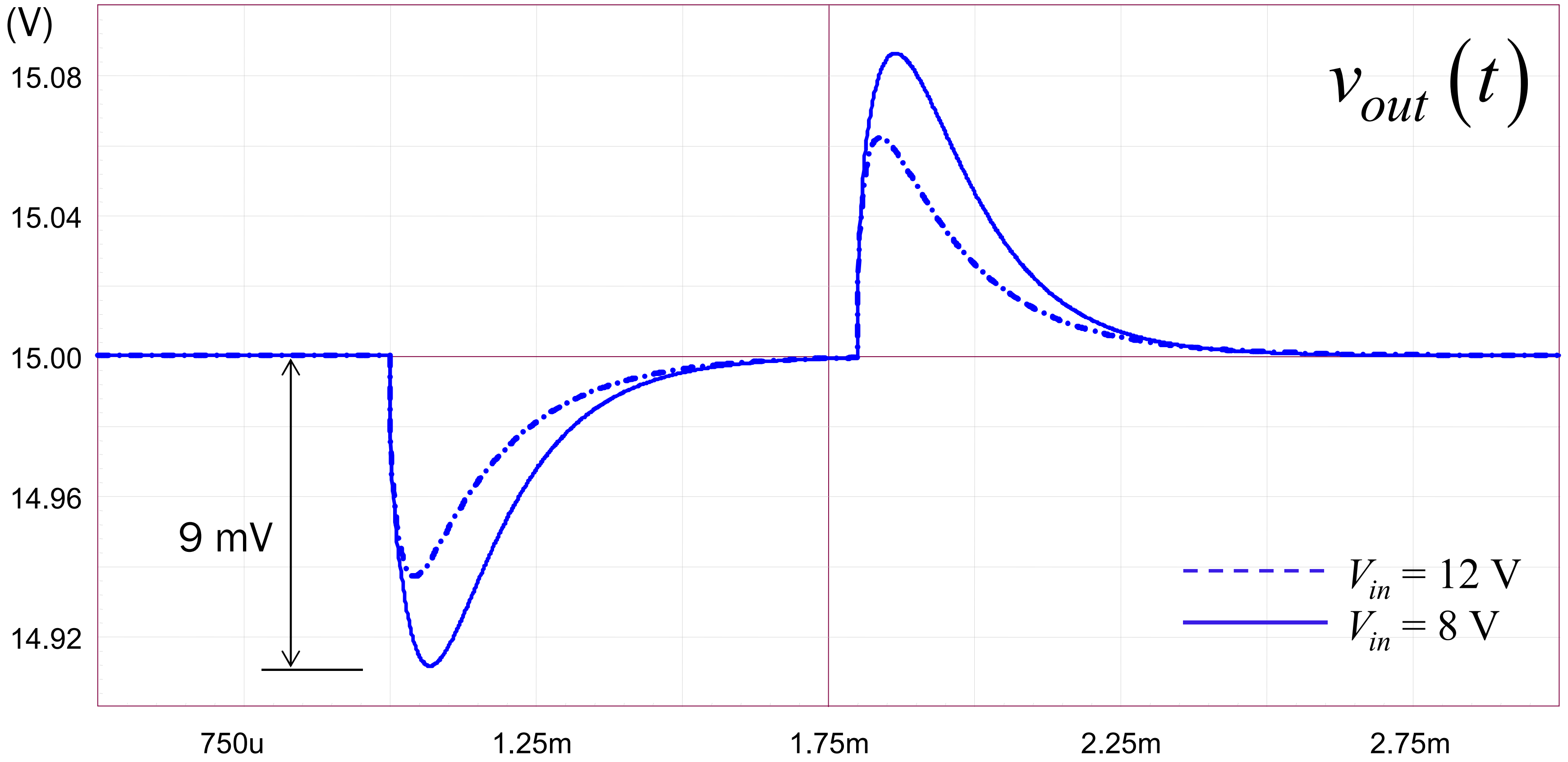
$f_g = 22.30226\mathrm{kHz}$ f(pi)

Crossover

Gain margin

Check Transient Response

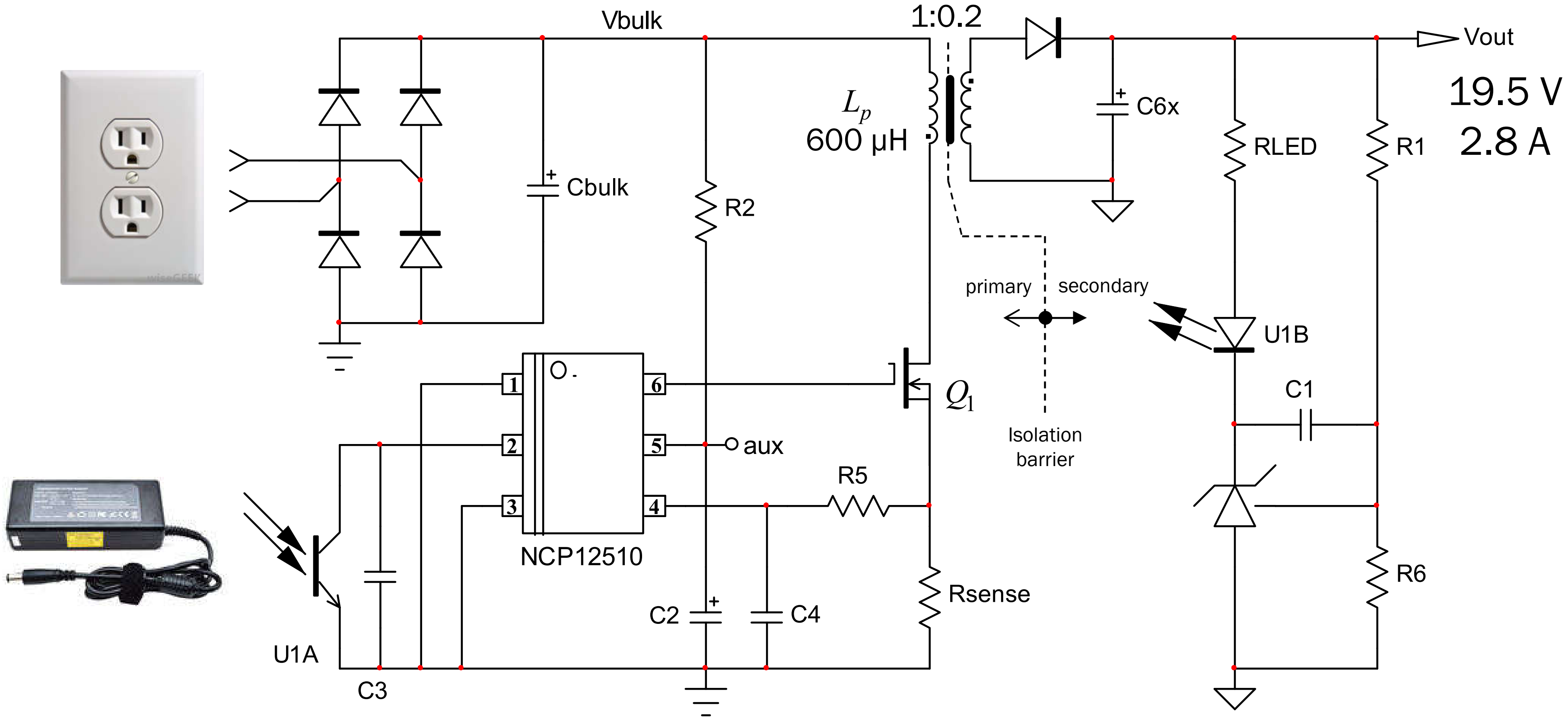
- ☐ The output current is varied from 0.5 to 1 A with a 1-A/µs slope



- ☐ Better response at high line with a slightly higher crossover

Stabilizing a Current-Mode Flyback Converter

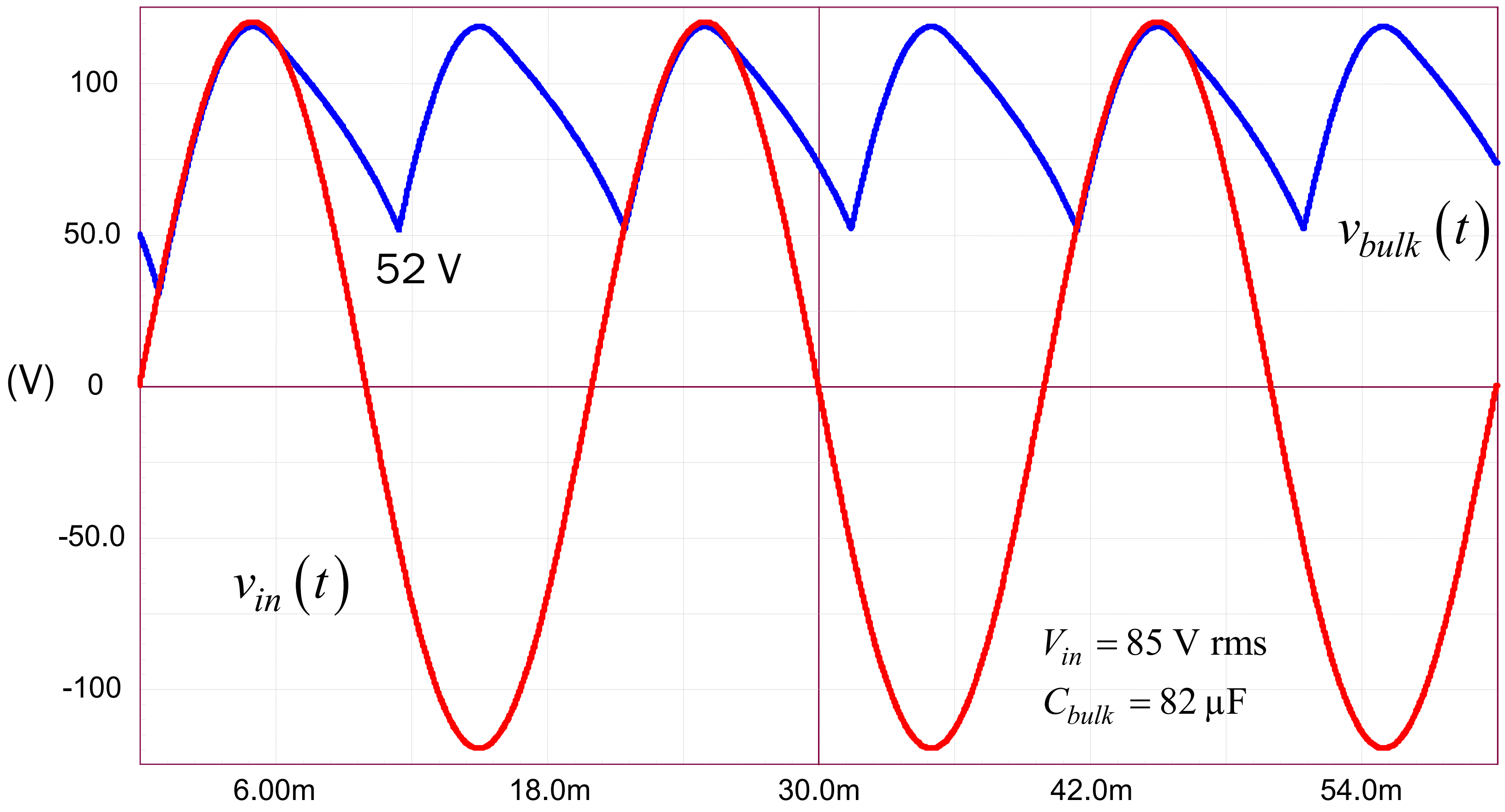
- ☐ This is an isolated power converter requiring an optocoupler



- ☐ The input voltage is the rectified mains and exhibits ripple

Check the True Minimum Input Voltage

- ☐ The rectified mains dips to a valley voltage at maximum P_{out}



- ☐ The power delivery and stability must be ensured down to 52 V

Evaluate the Compensation Ramp

☐ This is a CCM-operated current-mode circuit: check the poles

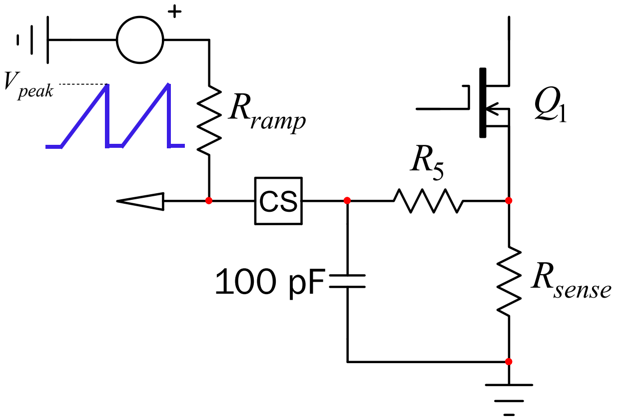
$$Q=\frac{1}{\pi\left(m_cD'-0.5\right)}\qquad \frac{V_{out}}{V_{in}}=\frac{ND}{1-D}\longrightarrow D=\frac{V_{out}}{V_{out}+NV_{in}}=\frac{19}{19+0.2\times 52}=83\%$$

Check IC D_{max} ⚠

☐ Damping is necessary!

$$Q=\frac{1}{\pi\left(m_cD'-0.5\right)}=-0.96\longrightarrow m_c=\frac{\frac{1}{\pi}+0.5}{1-D}=\frac{0.818}{1-0.83}\approx 4.9$$

☐ Adjust series resistance to tune slope compensation level



$$S_{ramp}=\frac{V_{peak}}{D_{\max}T_{sw}}=\frac{2.5}{0.8\times 15u}=208\text{ mV/µs}$$

$$m_c=4.9\longrightarrow S_a=S'_n\left(m_c-1\right)=\frac{V_{in}}{L_p}R_{sense}\left(m_c-1\right)$$

$$S_a=\frac{52}{600u}\times 3.9\times 0.25=84.5\text{ mV/µs}$$

$$R_5=\frac{S_a}{S_{ramp}}R_{ramp}=\frac{84m}{208m}\times 20k\approx 8\text{ k}\Omega$$

Will affect maximum peak current!

Power Stage Dynamic Response

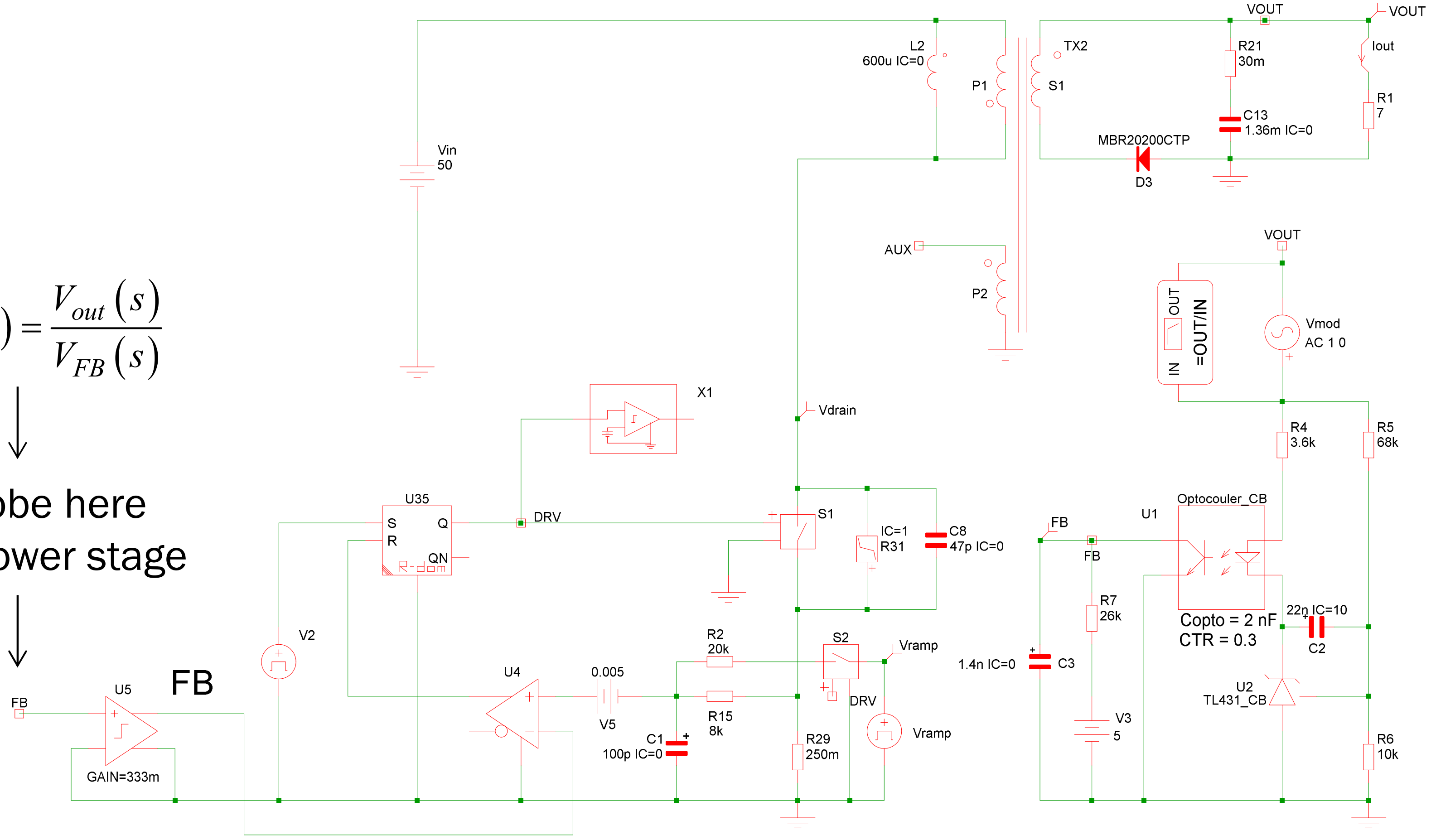
- ❑ Check power stage dynamic response at minimum input

$$H(s) = \frac{V_{out}(s)}{V_{FB}(s)}$$

↓

Probe here
for power stage

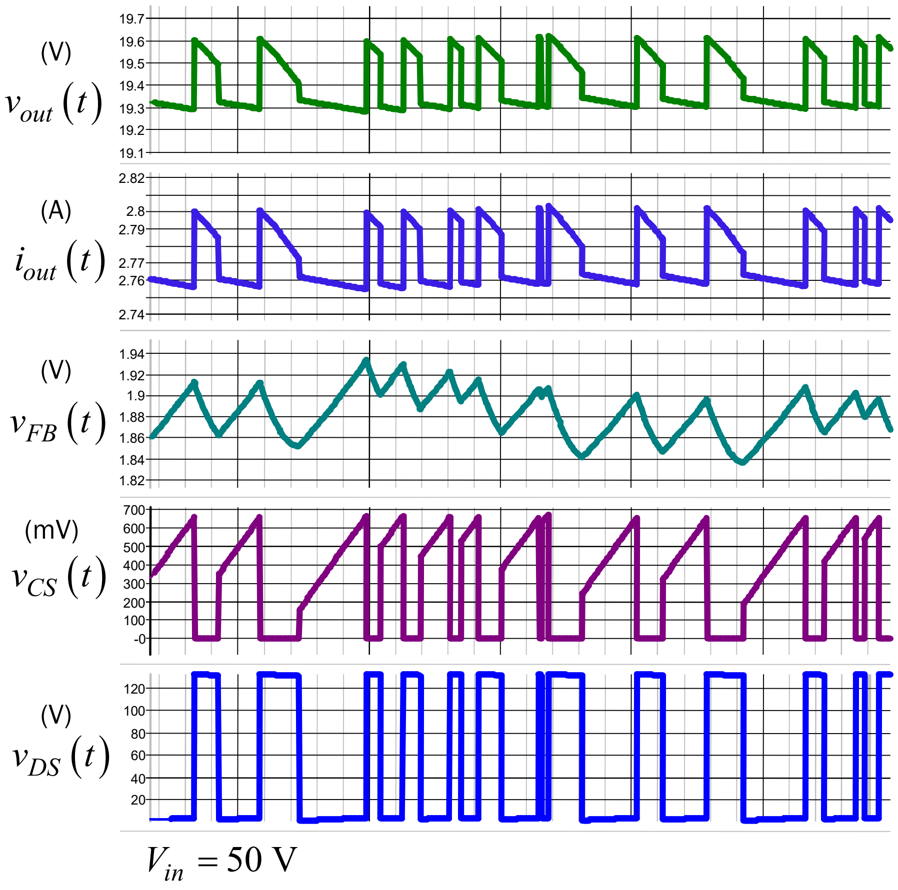
↓

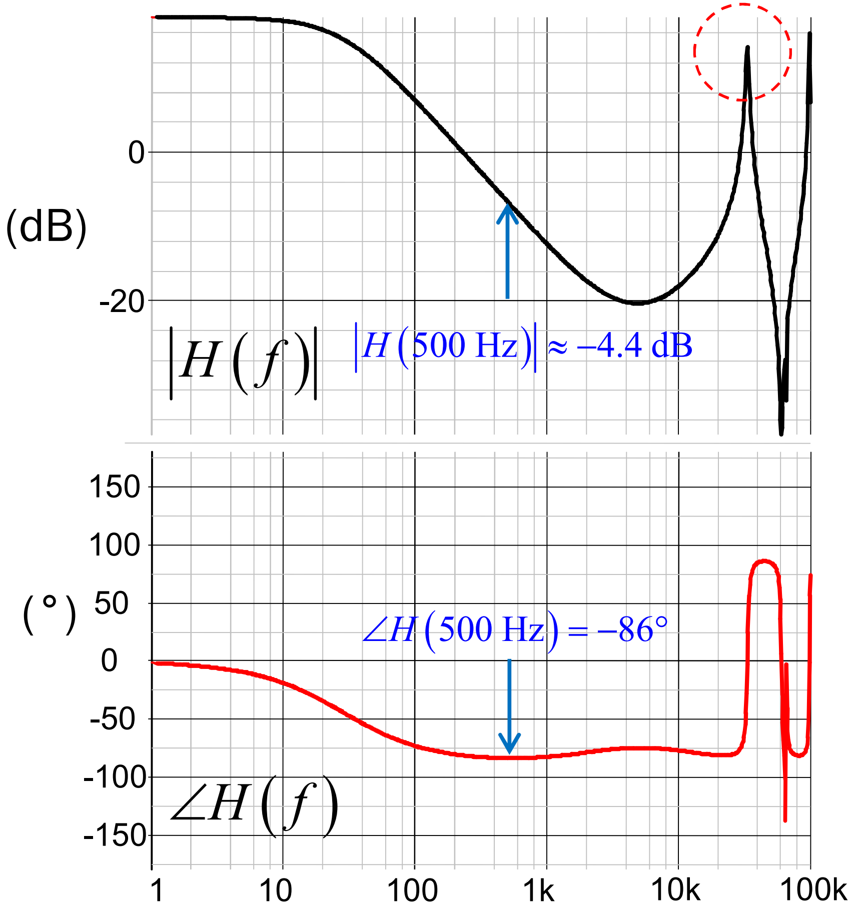


- ❖ Make sure parasitics are well modeled and match reality

Simulation Confirms Complex Poles

- Damping is an absolute necessity at the lowest input



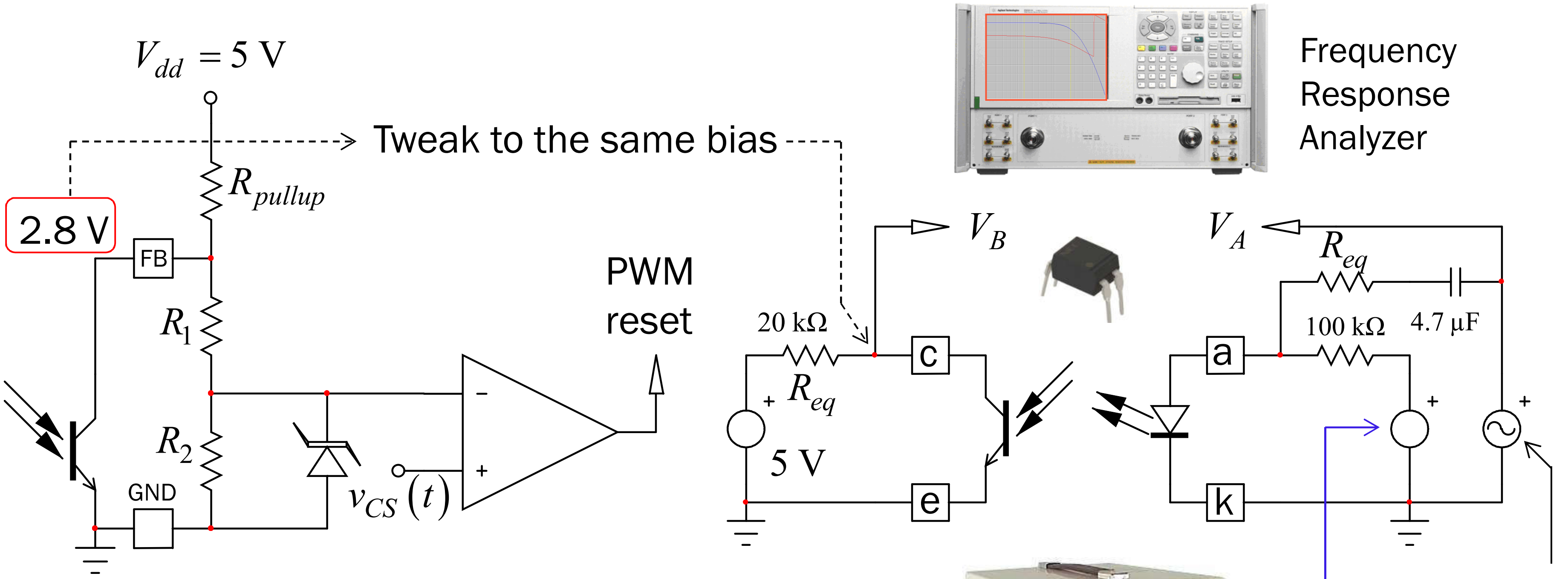


- Simplis® cannot find a stable point, a little bit of ramp is needed

Know what Affects the Return Path

☐ The optocoupler hosts a low-frequency pole: characterize it

❖ Reproduce the controller internals where the opto connects

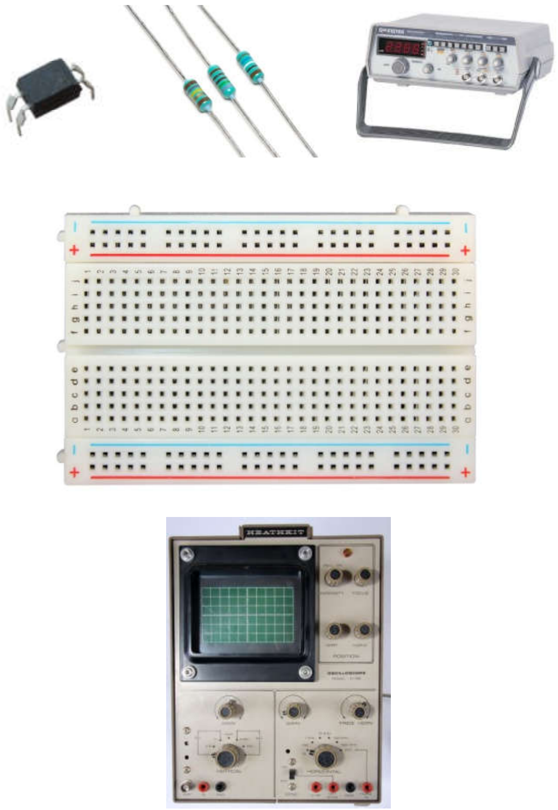


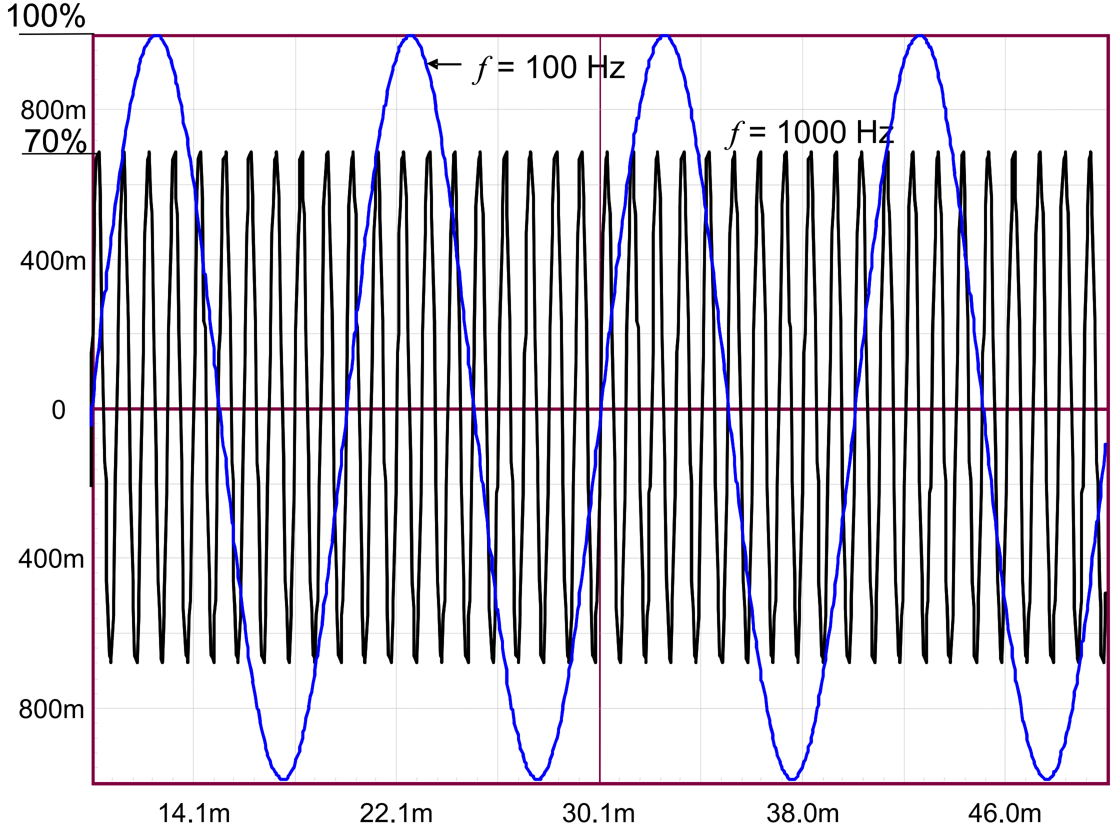
✓ Characterize a significant amount of optocouplers – 30

✓ Measure pole and CTR

A Simple Oscilloscope is Enough

- ☐ Pole extraction can also be done with an oscilloscope

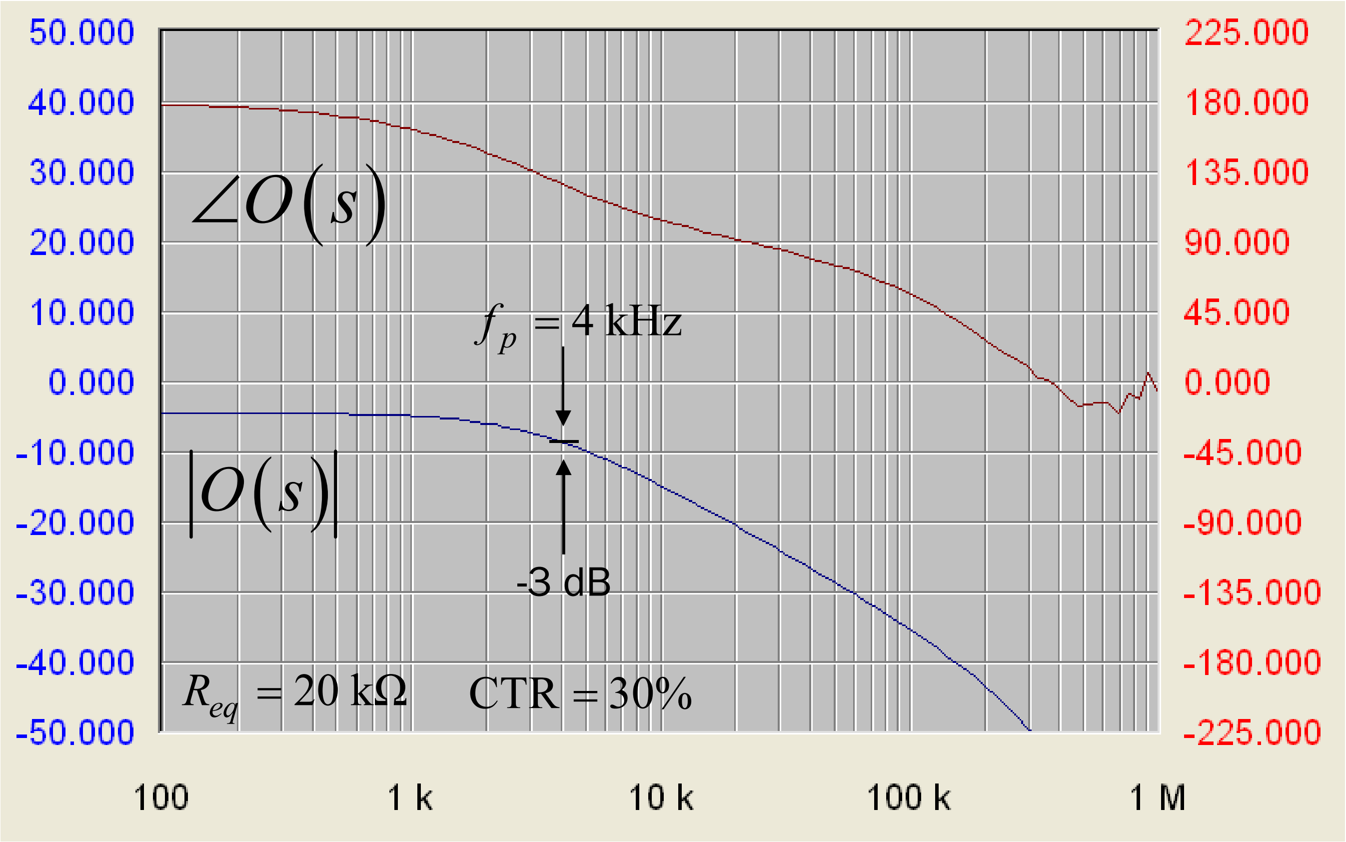




- ☐ 10 divisions is the low-frequency reference, 7 divisions is f_p

The Collector Resistance Affect the Pole

- ☐ A low value pull-up resistor would offer a better bandwidth…



$f_p = 4$ kHz

R_{eq}

20 kΩ

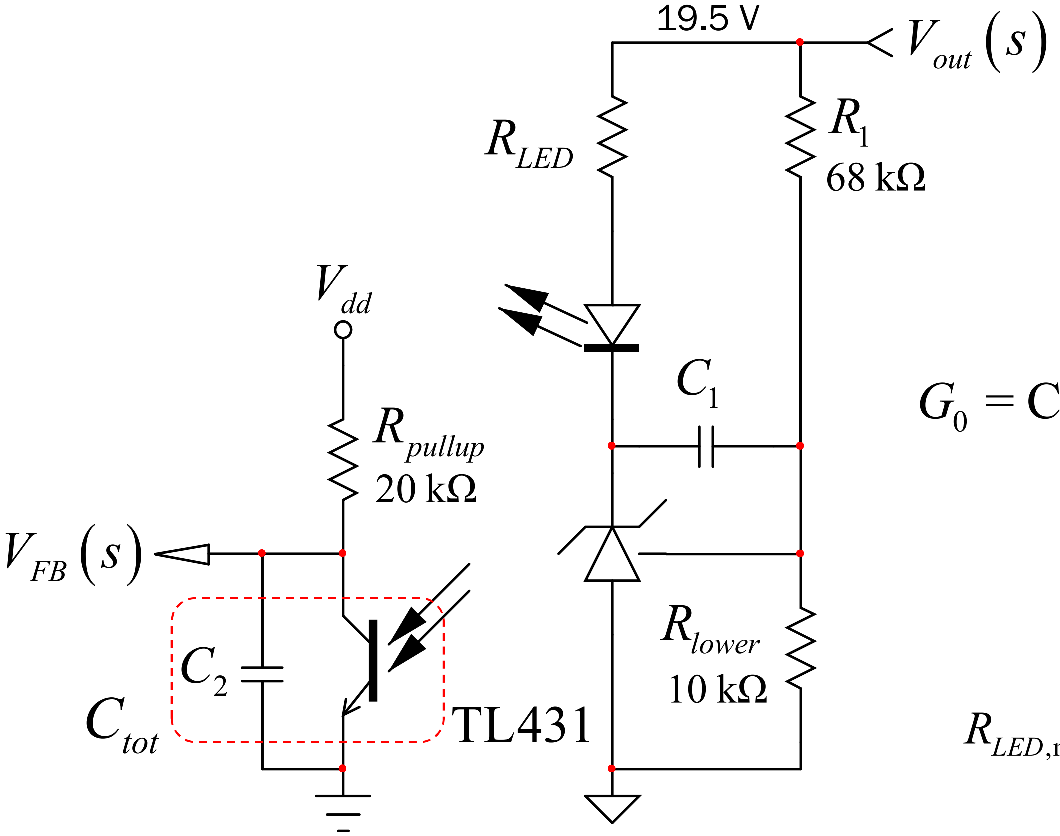
$C_{opto} \approx 2$ nF

…but would affect the standby power at $P_{out} = 0$

SFH615A-2

Build a Compensator with TL431

- ❑ The TL431 requires a single capacitor for a type-2 response



$$G(s) = -G_0 \frac{1+\frac{\omega_z}{s}}{1+\frac{s}{\omega_p}}$$

$$G_0 = \mathrm{CTR}\frac{R_{pullup}}{R_{LED}} \qquad \omega_z = \frac{1}{R_1 C_1} \qquad \omega_p = \frac{1}{R_{pullup} C_{tot}}$$

$$\uparrow$$

$$C_{tot} = C_2 \,||\, C_{opto}$$

⚠ $\downarrow$ check bias!

$$R_{LED,\max} \leq \frac{V_{out} - V_f - V_{TL431,\min}}{V_{dd} - V_{CE,sat}} R_{pullup} \mathrm{CTR}_{\min}$$

- ❑ The fast lane clamps down on the minimum gain

http://cbasso.pagesperso-orange.fr/Downloads/PPTs/Chris%20Basso%20APEC%20seminar%202011.pdf

Place Poles and Zeros to Boost the Phase

☐ Determine the necessary boost for a phase margin of 70°

$$boost = \varphi_m - \angle H\left(f_c\right) - 90° = 66°$$

☐ *k* factor is a good tool for stabilizing current mode converters

$$k = \tan\left(\frac{boost}{2} + \frac{\pi}{4}\right) = 4.7 \qquad f_z = \frac{f_c}{k} \quad f_p = k \cdot f_c$$

❖ Maximum crossover depends on worst-case RHPZ at 50 V:

$$f_{z_2} = \frac{(1-D)^2 R_{load}}{2\pi D L_p N^2} \approx 1.6\ \text{kHz} \longrightarrow f_c < 0.3 \cdot f_{z_2} \longrightarrow f_c = 500\ \text{Hz}$$

☐ Place the zero and the pole at:

$$f_z = f_c^{\,2} / k = 500^2 / 4.7 \approx 106\ \text{Hz} \qquad f_p = k \cdot f_c = 4.7 \times 500 \approx 2.3\ \text{kHz}$$

Adjust Mid-Band Gain to Crossover at f_c

- The mid-band gain depends on the LED series resistance

from the mag. graph

$$G_0 = \text{CTR}\frac{R_{pullup}}{R_{LED}} = 10^{\frac{\boxed{-G_{f_c}}}{20}} = 10^{\frac{4.4}{20}} = 1.66$$

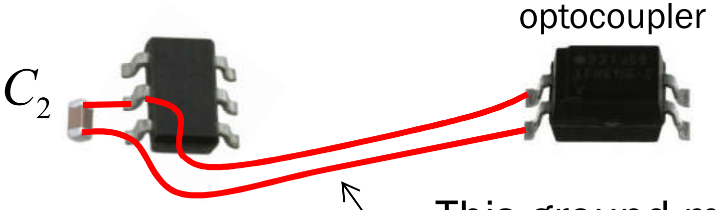
$$\Rightarrow \quad R_{LED} = 3.6\,\text{k}\Omega \xrightarrow{\text{ok}} R_{LED,\max} \le 20.4\,\text{k}\Omega$$

⚠

- Determine the capacitor values

$$C_1 = \frac{1}{2\pi R_1 f_z} = 22\text{ nF} \quad C_{tot} = \frac{1}{2\pi R_{pullup} f_p} = 3.4\text{ nF} \longrightarrow C_2 = C_{tot} - C_{opto} = 1.4\text{ nF}$$

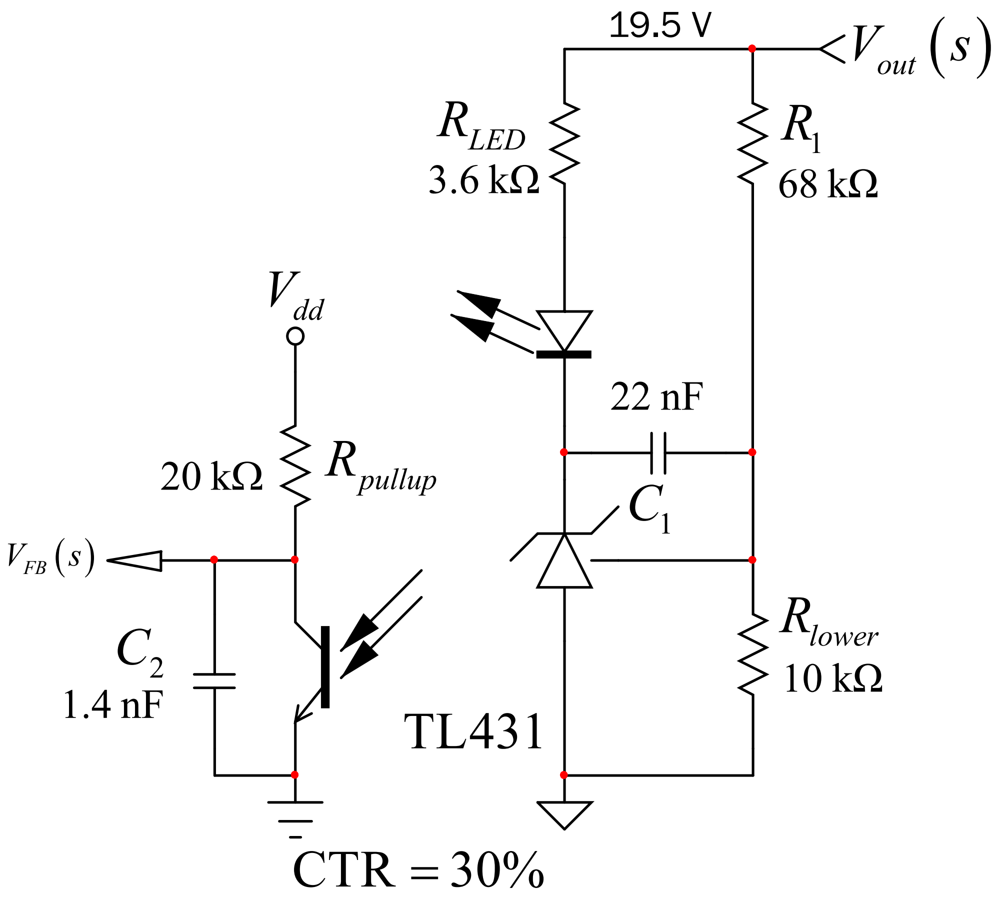
- C_2 must be located close to the control circuit

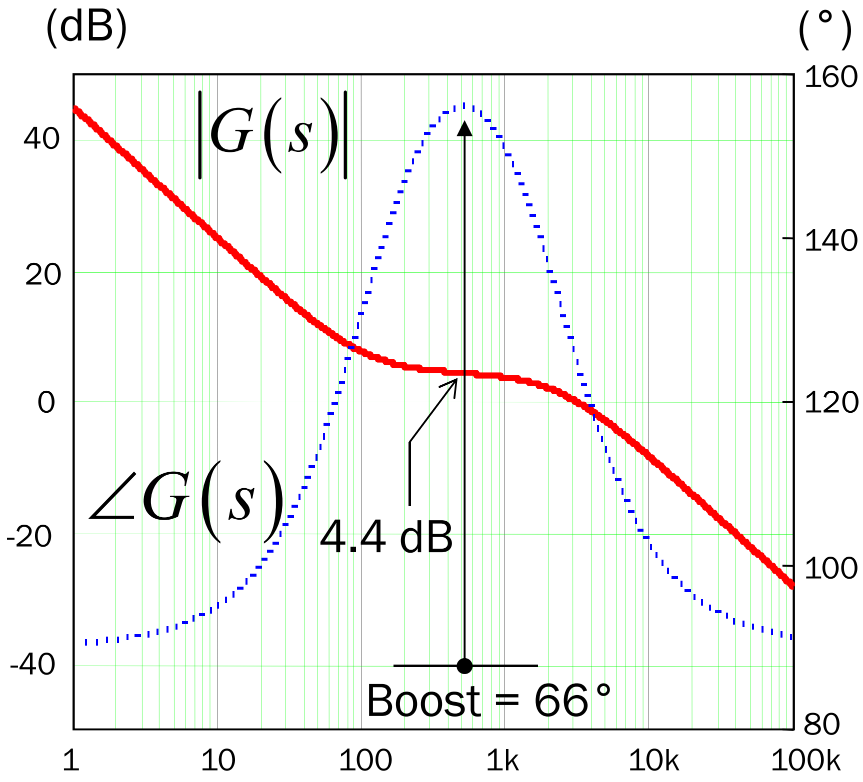


This ground must be quiet - go to the IC GND

Check Individual Response

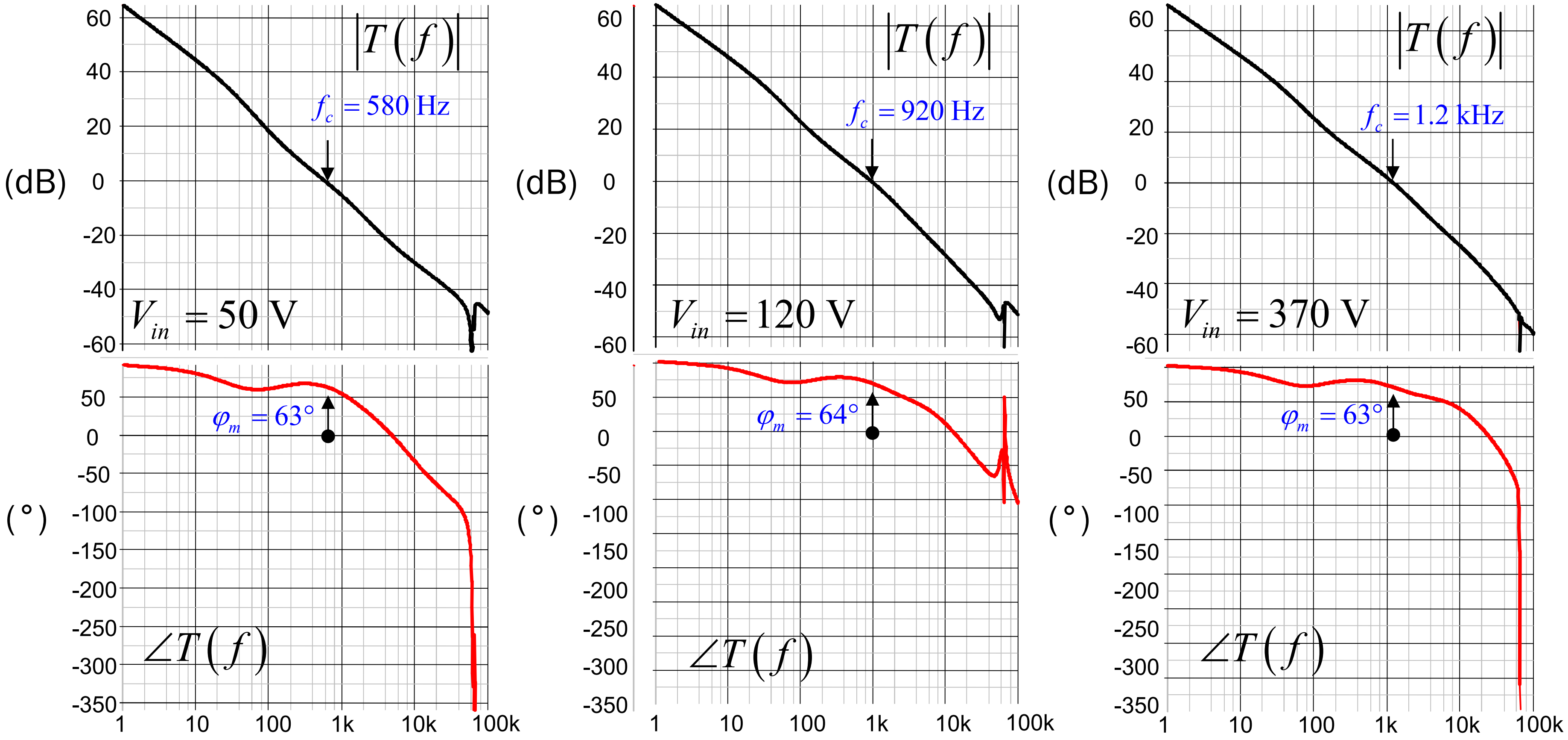
- ☐ Use Mathcad® or SPICE to confirm the compensator response





Check Loop Gain T across Line/Load

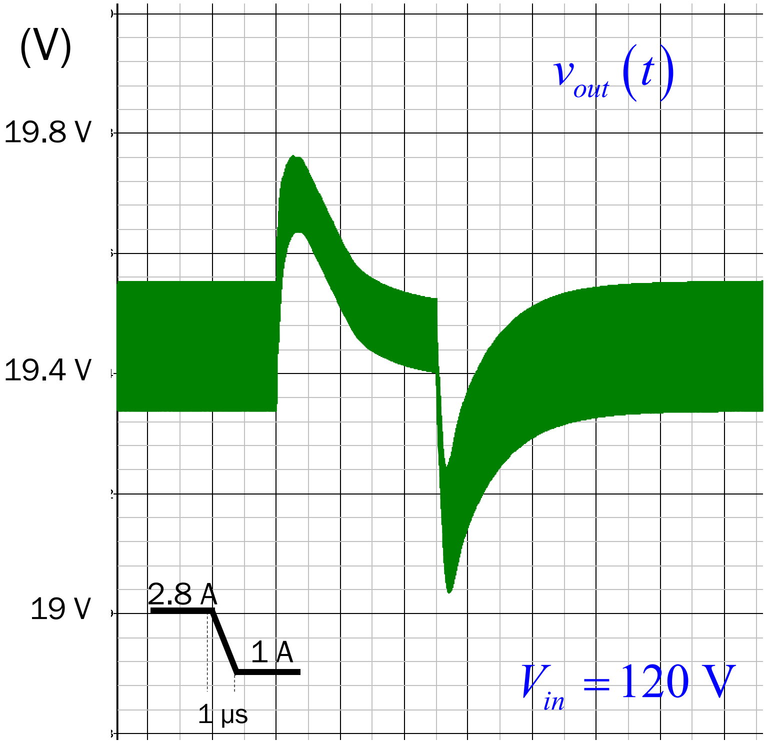
- ☐ The loop is compensated as per specifications

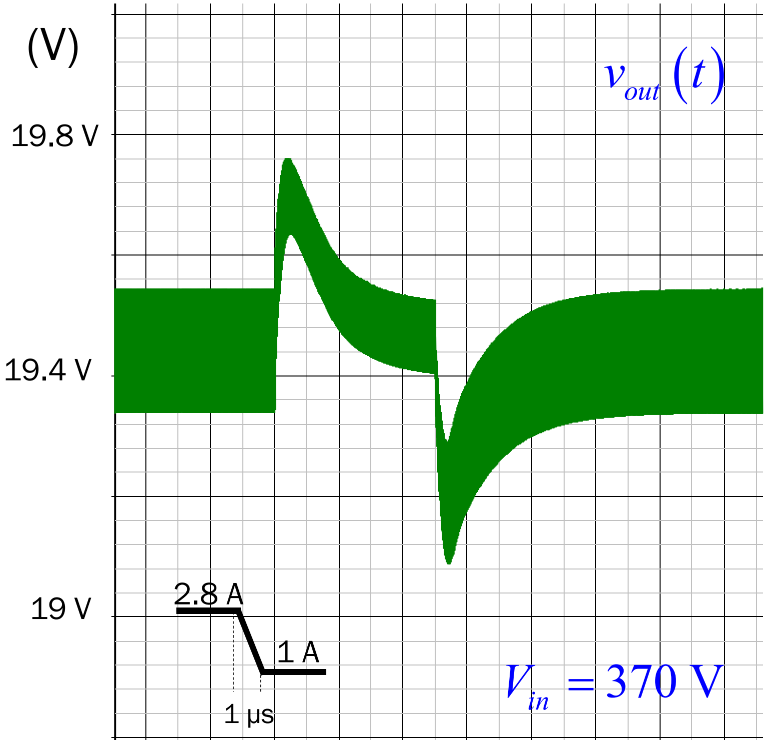


- ❖ CTR variations will affect crossover, check margins

Load Step on the Output

- ☐ Check transient response at both line extremes





- ☐ As converter heats up, L_p goes down. Check at high temp also

Course Agenda

☐ Blocks in a Switching Converter

☐ Introduction to Small-Signal Modeling

☐ Analytical Analysis of an Output Stage

☐ Simulation Models - Averaged or Switched?

☐ Crossover Frequency and Phase Margin

☐ Compensation Strategy

☐ Experiments on Prototypes

☐ Conclusion

Building Prototypes

☐ Bench experiment is a mandatory step

✓ Are the assumptions adopted for the analytical model confirmed?
➢ Feed the model back with real measurements, refine simulations
✓ Check the behavior in temperature, particularly stability

☐ Characterize the selected components

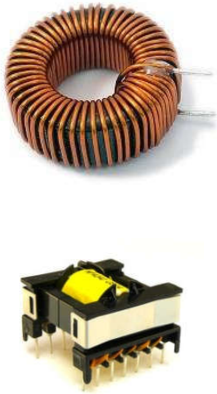
✓ Are parasitics within the data-sheet specs (r_C and r_L)?
✓ What is the inductor saturation current?

ESR?
f_0?
C at bias?

Know the components you will later use in a high-volume project.

⇩

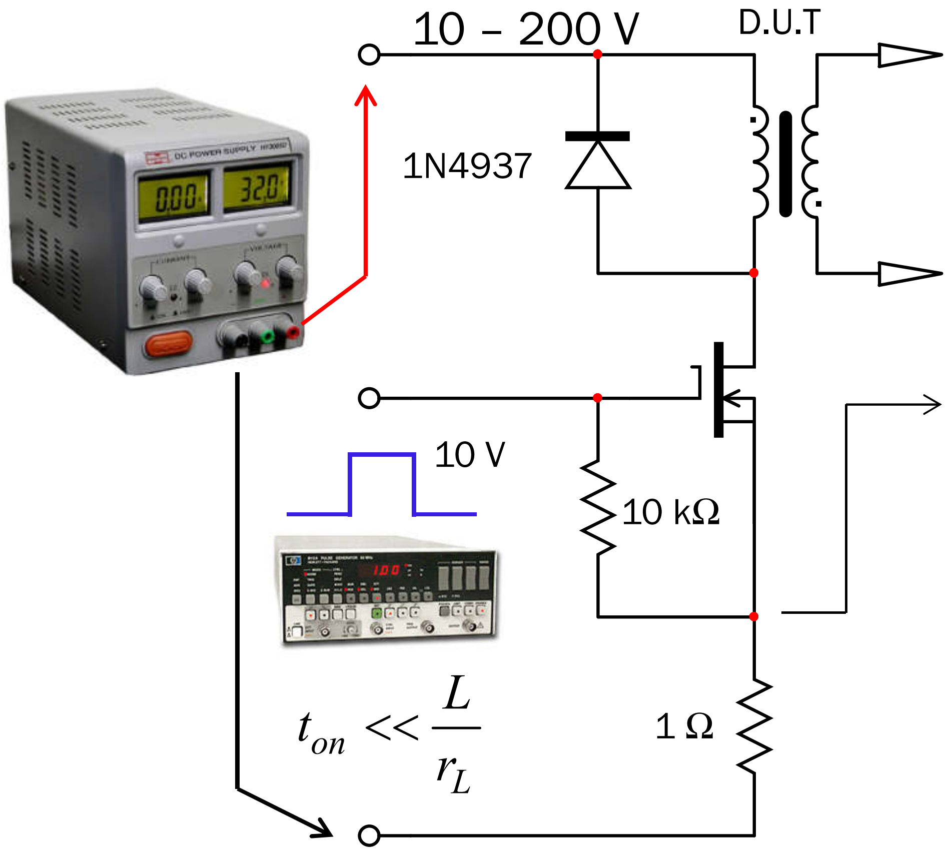
Characterization



ESR?
f_0?
Capacitance
I_{sat}?

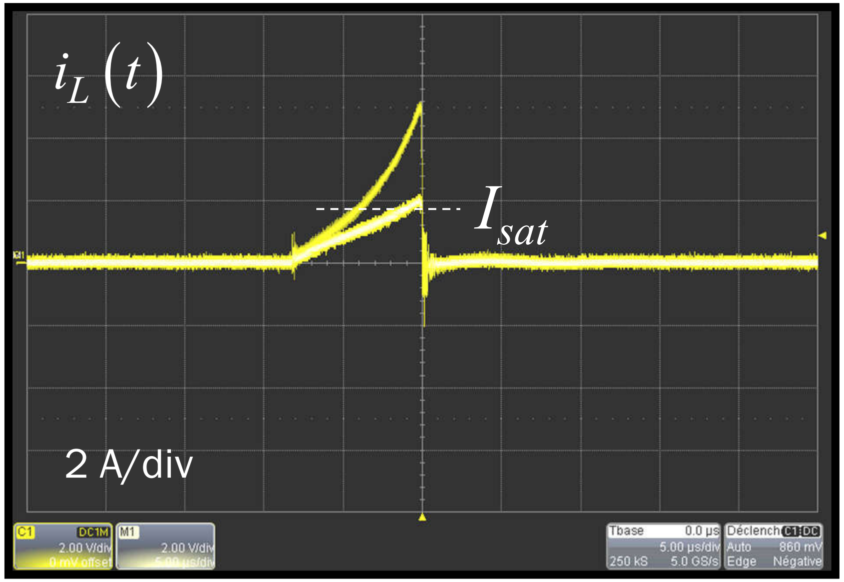
Checking the Saturation Current

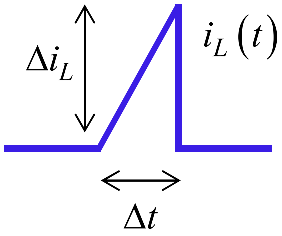
- ☐ Measure the inductance in the linear zone
- ☐ Check the saturation current at the maximum operating temp.



$$t_{on} << \frac{L}{r_L}$$

1.8 A maximum



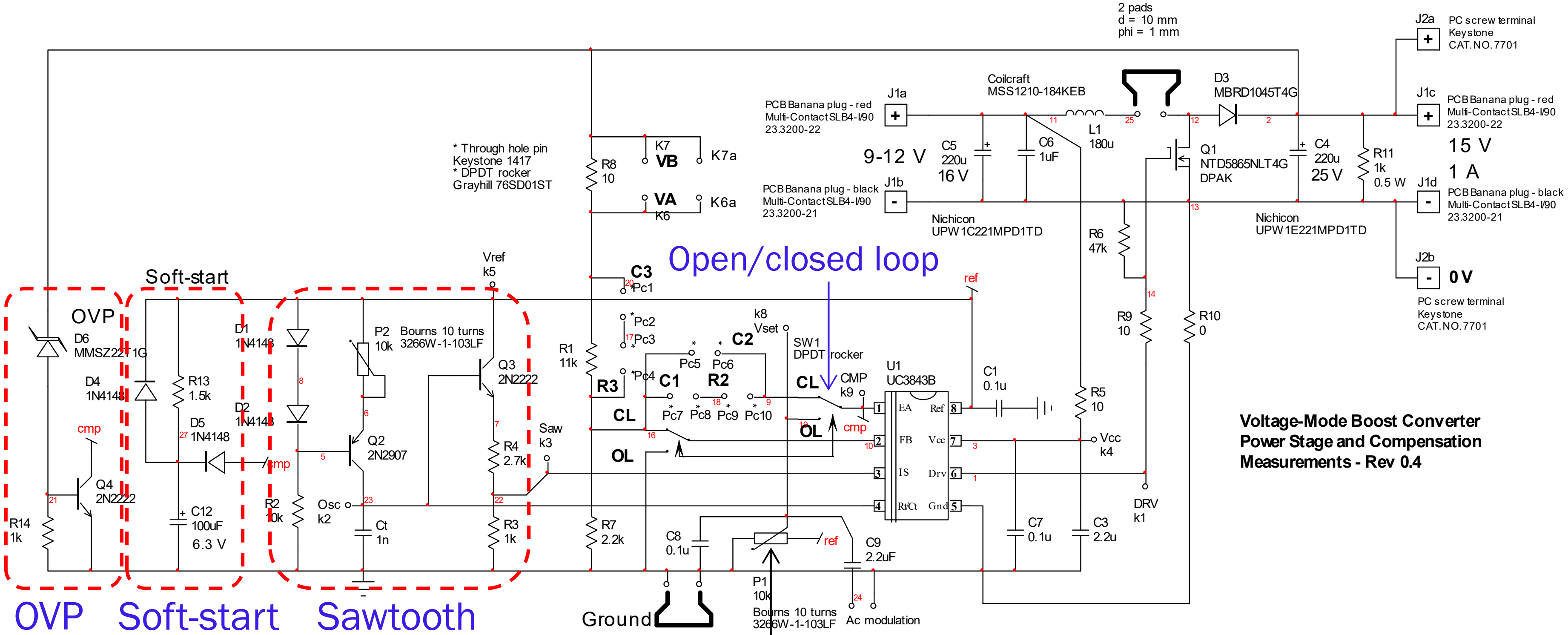


$$\Rightarrow \quad L = V_{in}\frac{\Delta t}{\Delta i_L}$$

D.U.T: device under test

A Boost Converter in Voltage Mode

- ☐ A converter using a UC3843 has been assembled



Open/closed loop

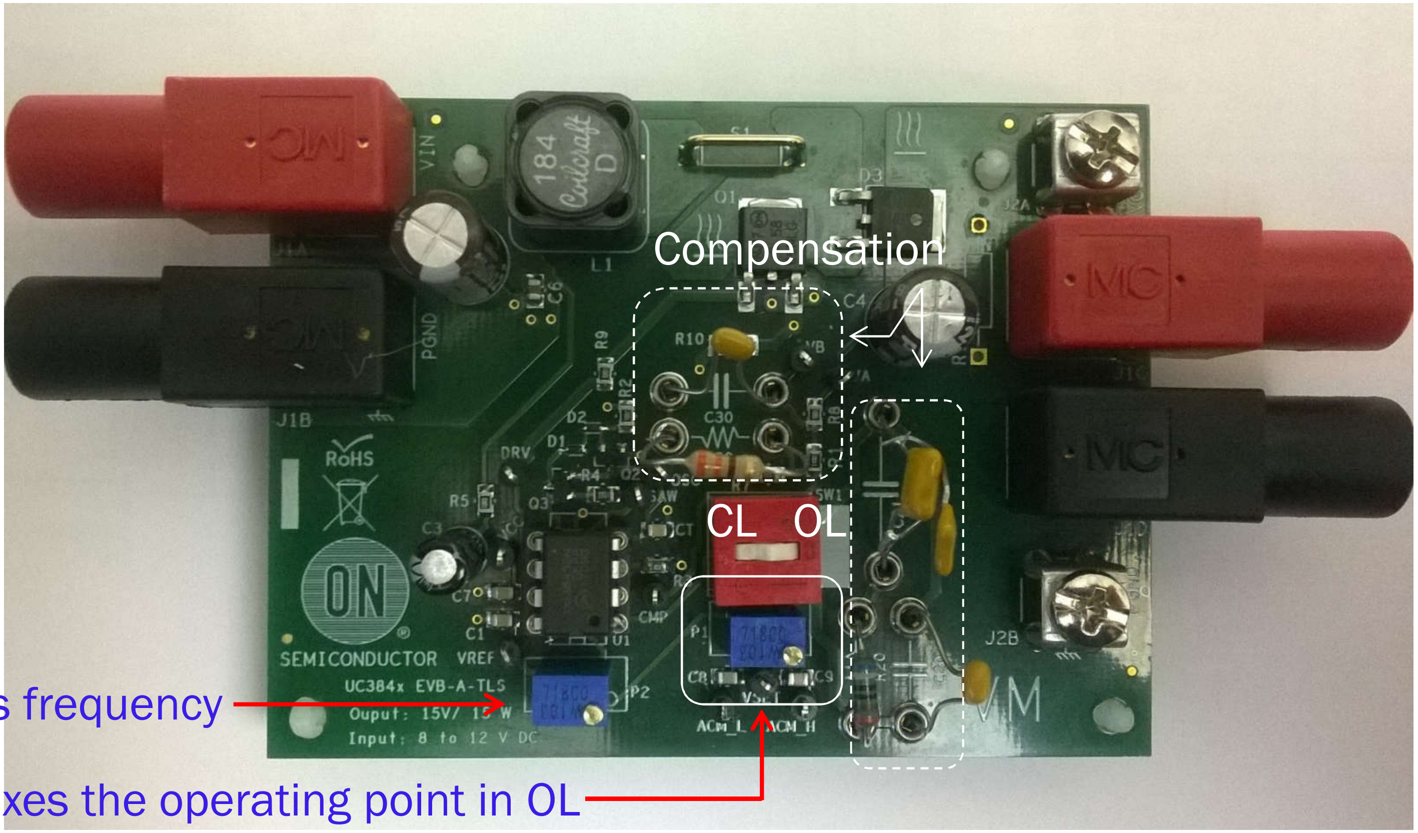
OVP Soft-start Sawtooth generator

Fixes bias point

- ❖ The soft-start avoids inductor saturation at power on
- ❖ The OVP circuit protects in no-load operations

A Convenient VM Board to Test

- ☐ The board includes switches to measure open and closed loops



9 – 12 V

15 V
1 A

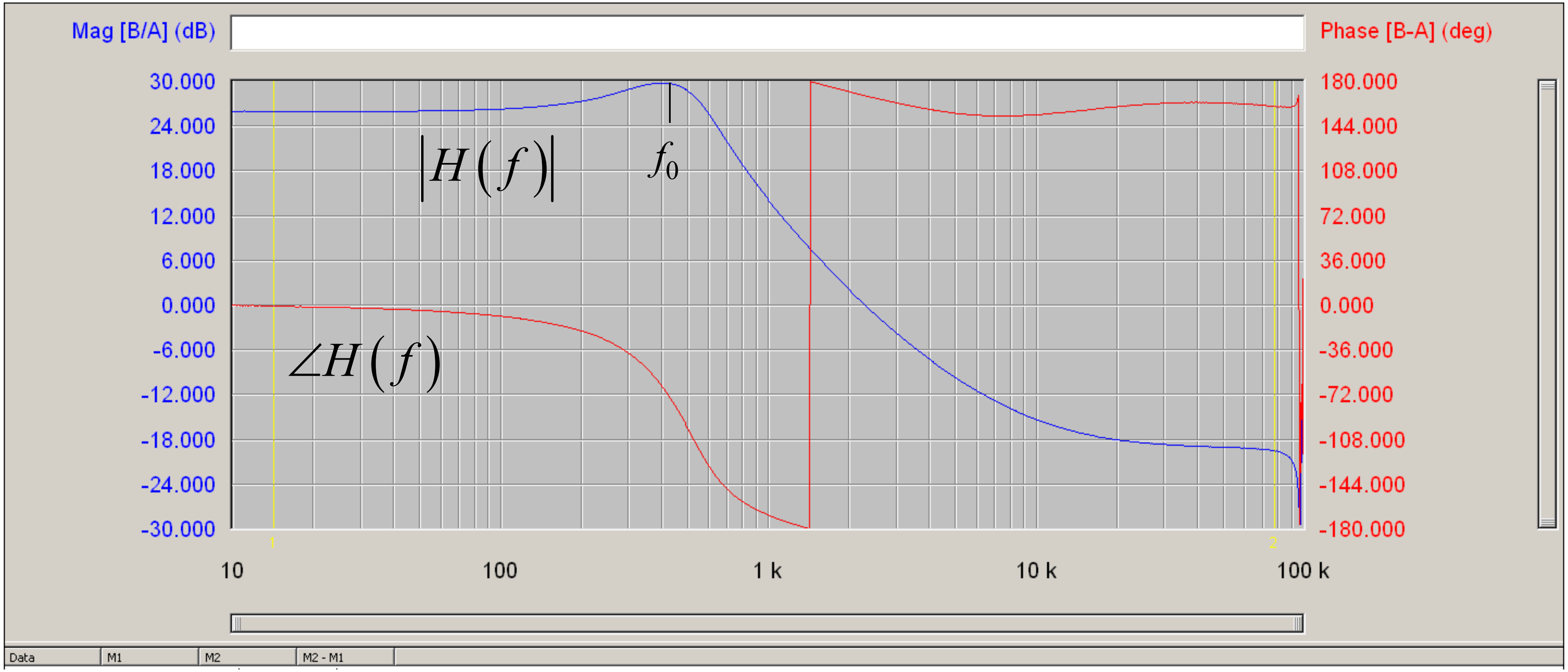
Fixes frequency

Fixes the operating point in OL

- ☐ It works from 9 to 12 V and delivers 15 V at a 1-A output current

Get the Power Stage Response

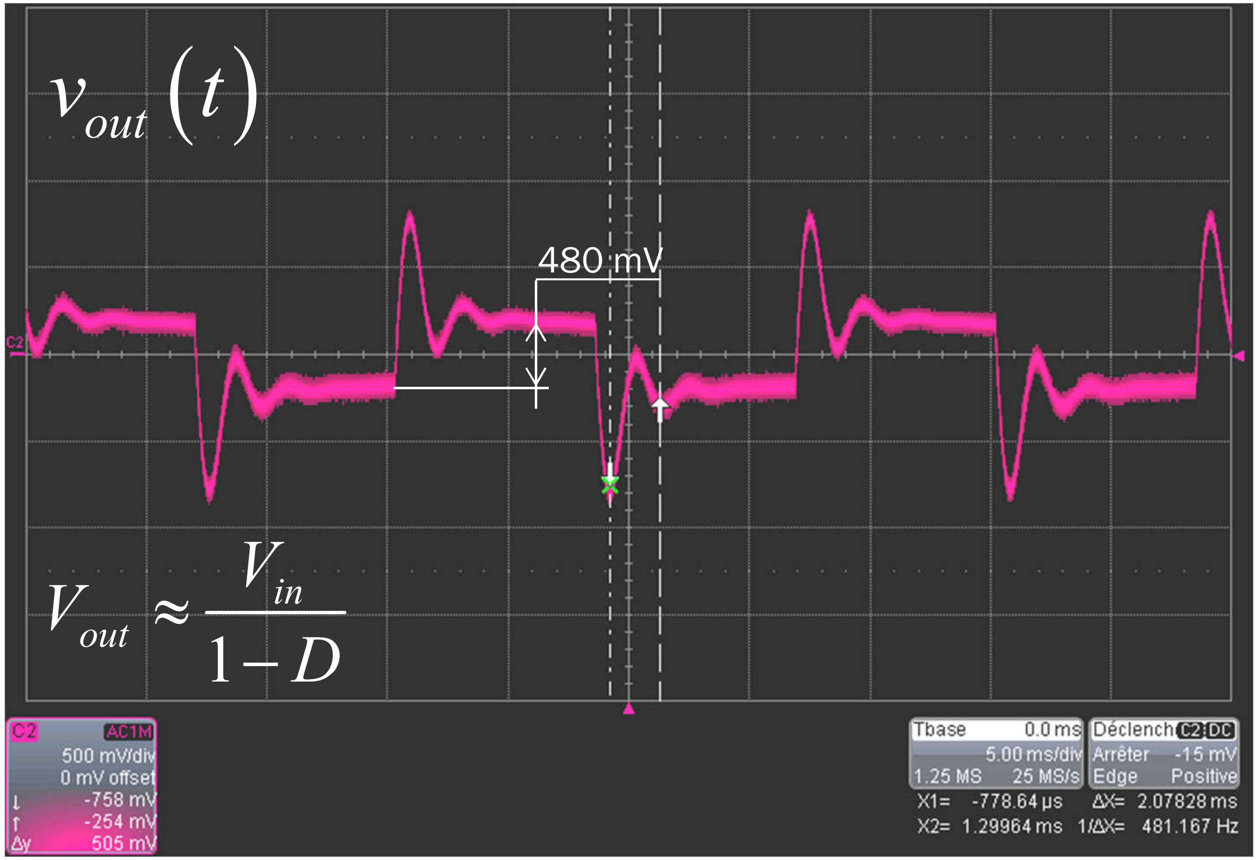
- ❑ The first thing is to measure the power stage dynamic response



- ❑ Measured for V_{in} = 9 V and I_{out} = 1 A

The Open-Loop Response is Ringing

- ☐ The LC filter rings at the resonant frequency



$$v_{out}(t)$$

$$V_{out} \approx \frac{V_{in}}{1-D}$$

```
C2                AC1M
            500 mV/div
            0 mV offset
↓              -758 mV
↑              -254 mV
Δy              505 mV
```

```
Tbase          0.0 ms   Déclench  C2 DC
          5.00 ms/div   Arrêter    -15 mV
1.25 MS      25 MS/s    Edge     Positive
X1=    -778.64 µs       ΔX=   2.07828 ms
X2=   1.29964 ms      1/ΔX=   481.167 Hz
```

0.5 A to 1 A
in 1 µs/A

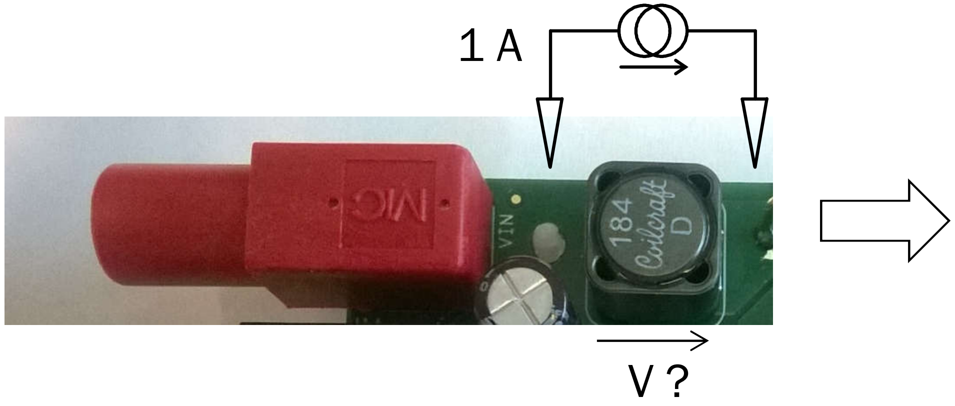
$$R_{out} \approx \frac{0.48}{0.5}$$

$$= 0.96\ \Omega$$

- ❖ This is why you need enough gain at f_0 to damp oscillations

Measure Parasitics on the Board

- Inject a 1-A current in the inductor, measure the drop

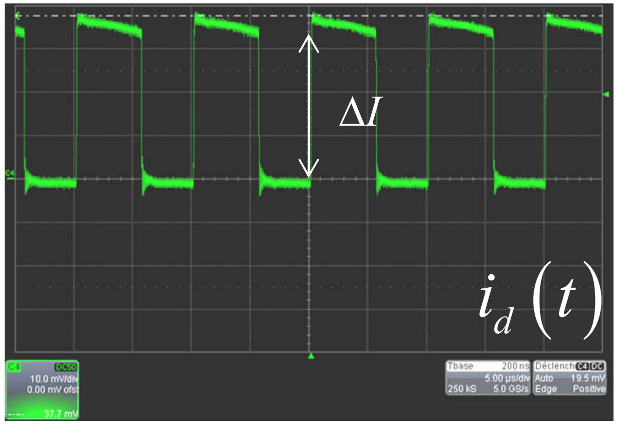


Measured at terminals

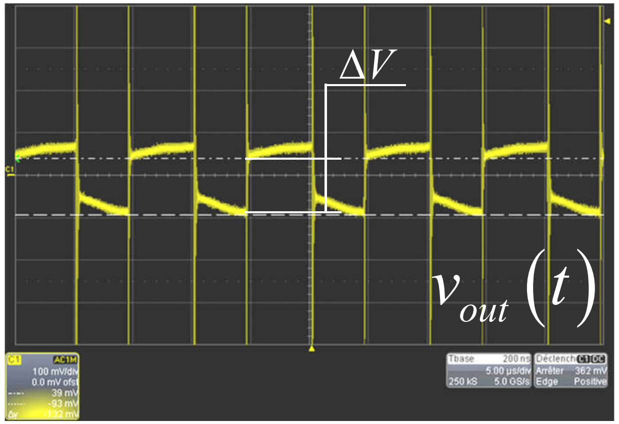
195 mV

$$r_L = \frac{195m}{1} = 195\ \text{m}\Omega$$

- Display the diode current and output voltage in open loop



Diode current, ΔI = 1.85 A



Output voltage, ΔV = 132 mV

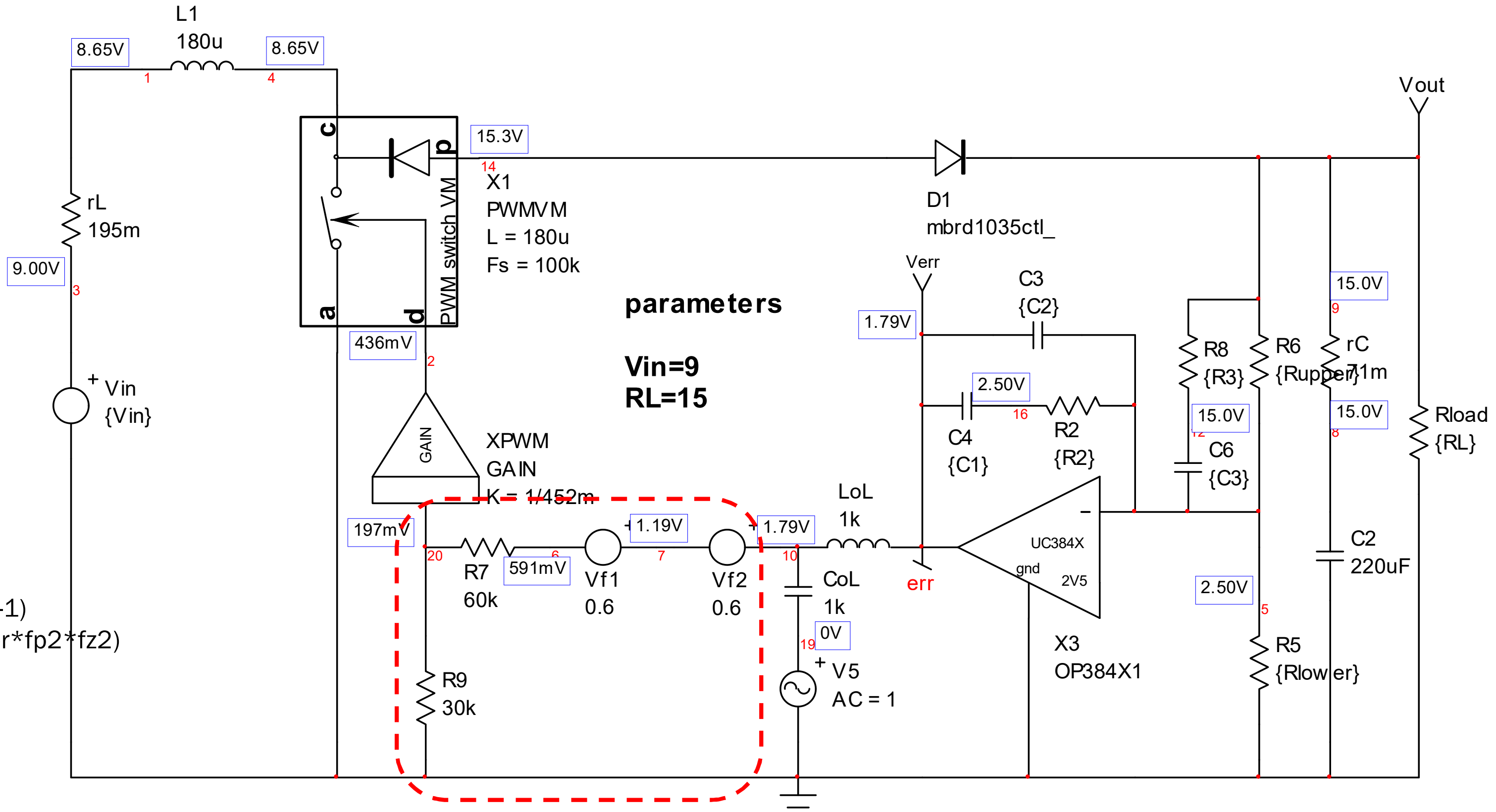
$$r_C \approx \frac{132m}{1.85} = 71\ \text{m}\Omega$$

Build a Model for the Boost Converter

- ☐ Analyze the dynamic response with SPICE and compensate

parameters

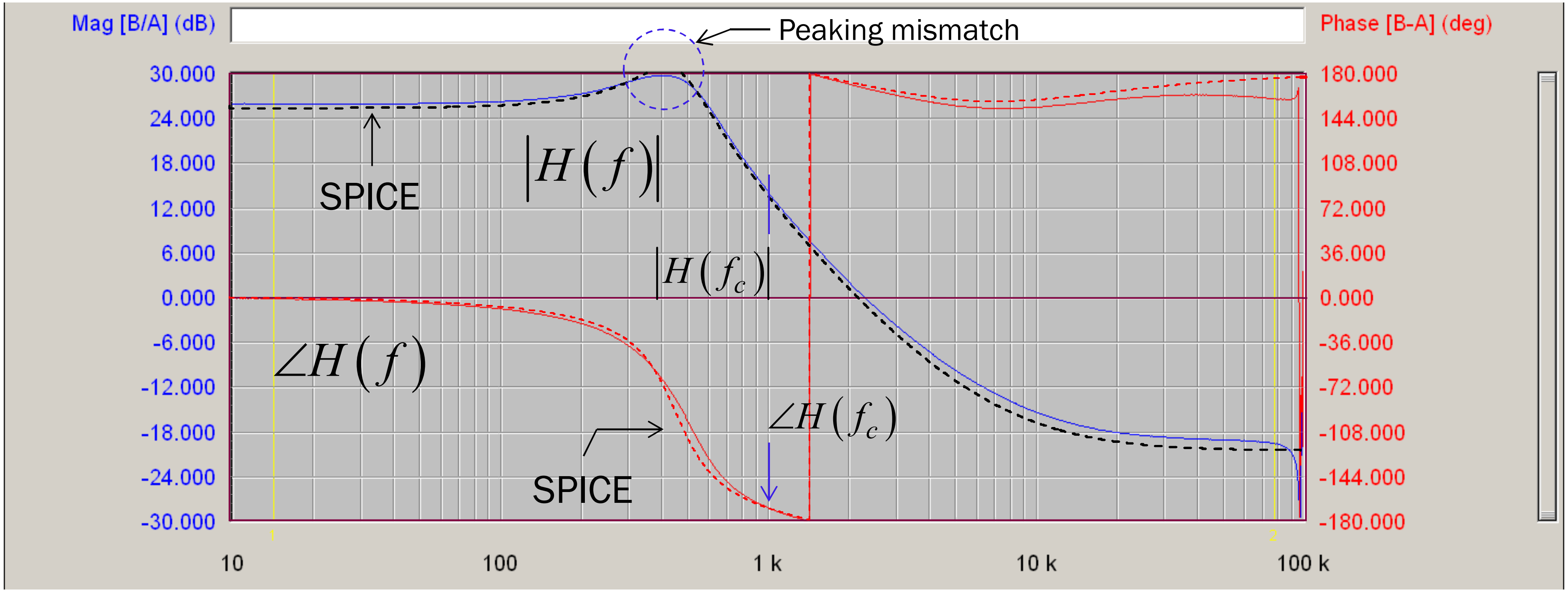
Ibias=15u
Vout=15
Vref=2.5
Rupper=(Vout-Vref)/Ibias
Rlower=Vref/Ibias
fc=1k
Gfc=13.8
pfc=-170
pm=50
boost=pm-(pfc)-90
G=10^(-Gfc/20)
pi=3.14159
fz1=300
fz2=300
fp2=10k
C1=1/(2*pi*fz1*R2)
C2=C1/(C1*R2*2*pi*fp1-1)
C3=(fp2-fz2)/(2*pi*Rupper*fp2*fz2)
R3=Rupper*fz2/(fp2-fz2)
a=sqrt((fc^2/fp1^2)+1)
b=sqrt((fc^2/fp2^2)+1)
c=sqrt((fz1^2/fc^2)+1)
d=sqrt((fc^2/fz2^2)+1)
fp1=fc/tan((2*atan(fc/fz1)-tan(fc/fp2))-boost*pi/180)
R2=((a*b/(c*d))/(fp1-fz1))*Rupper*G*fp1
G0=((R2*C1)/(Rupper*(C1+C2)))*c*d/(a*b)



UC3843 internals

Check the Model Reflects Reality

❑ Scale then superimpose SPICE and analyzer responses

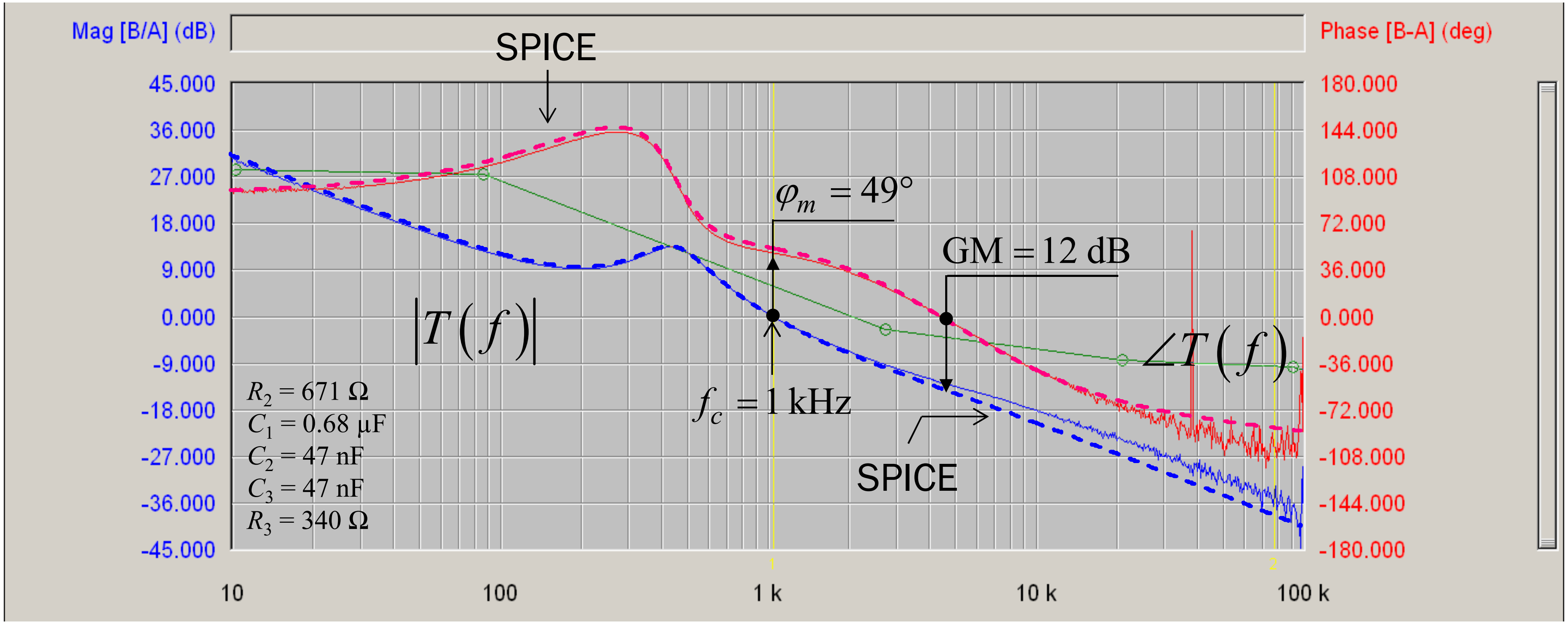


❑ Extract gain and phase values for a 1-kHz crossover goal

$$|H(f_c)| = 13.8\text{ dB} \quad \angle H(f_c) = -170°$$

Close the Loop and Check Margins

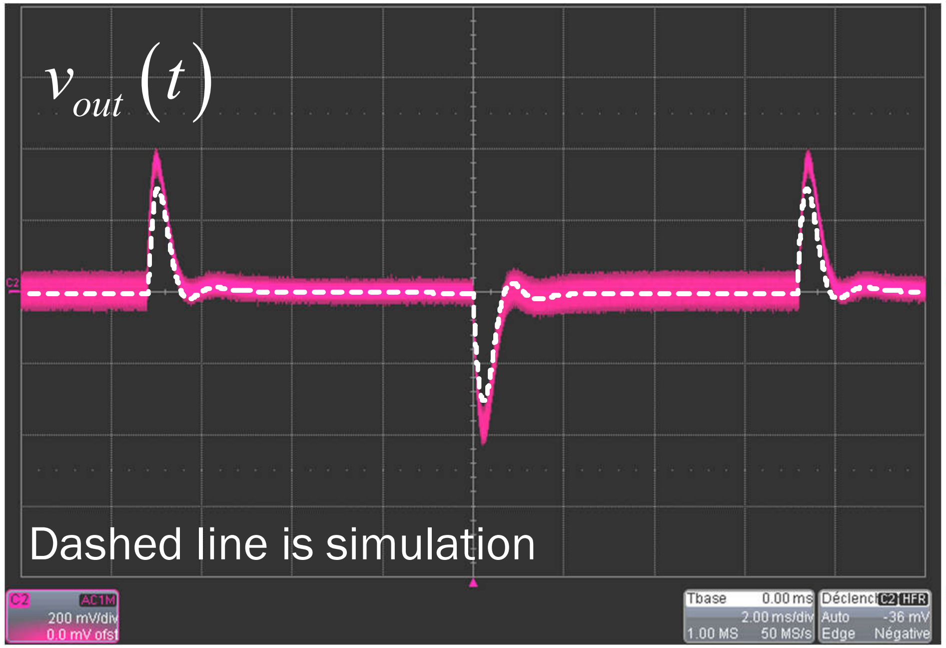
- Apply compensation strategy and plot/measure open-loop gain



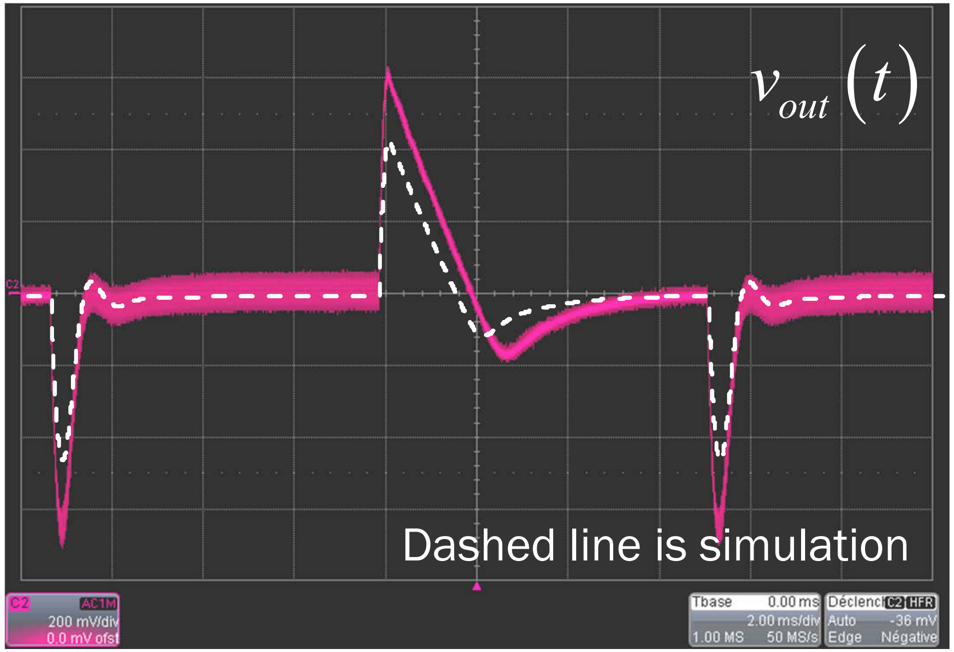
$$f_{z_1} = f_{z_2} = 300\text{ Hz} \qquad f_{p_1} = 5.2\text{ kHz}\ f_{p_2} = 10\text{ kHz}$$

Check Response to a Load Step

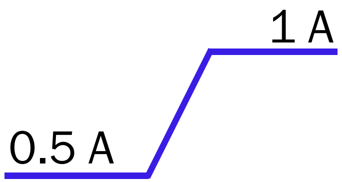
- ☐ Step the output at different load current spans



Dashed line is simulation

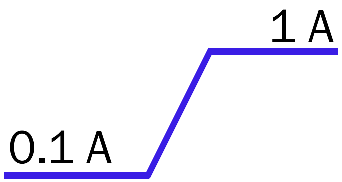


Dashed line is simulation



S = 1 A/µs

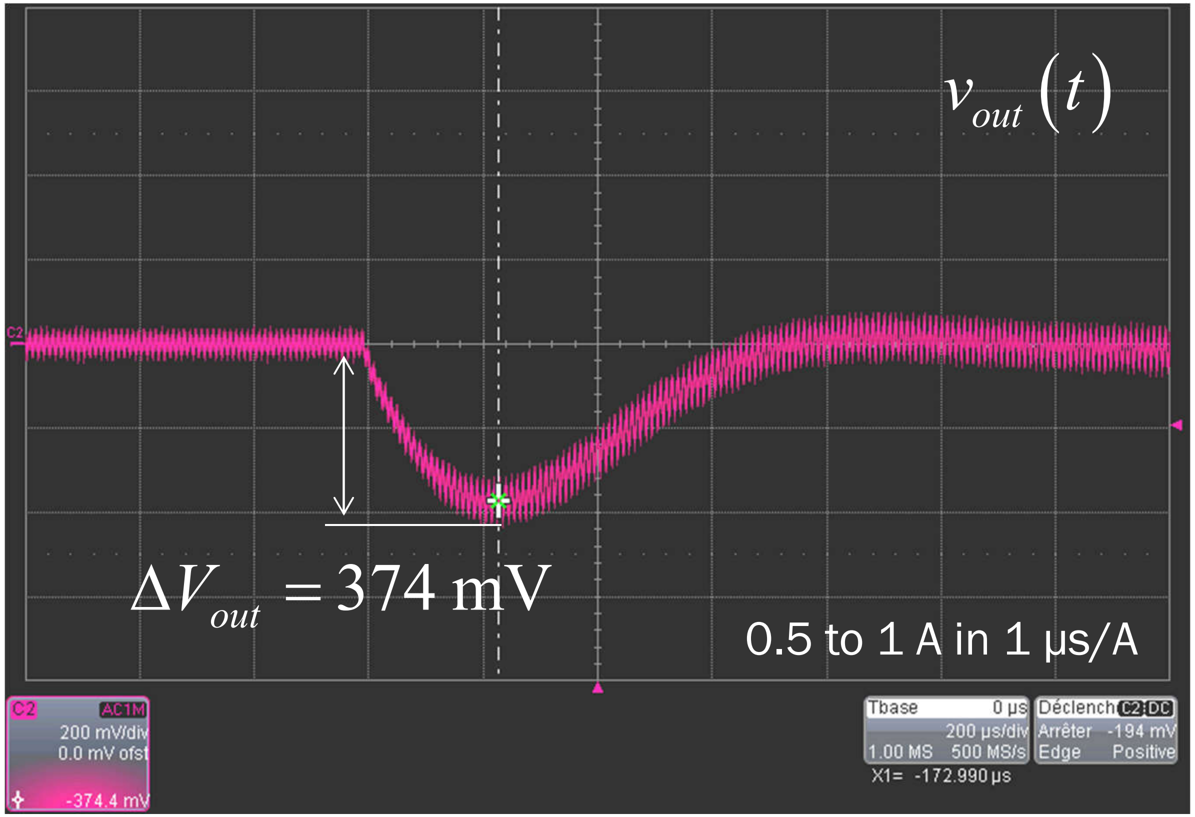
SPICE predicts a deviation within acceptable limits even in a highly non-linear mode



S = 1 A/µs

Check the Dynamic Response Amplitude

❑ Verify simplified output voltage deviation formula



$$\Delta V_{out} = 374 \text{ mV}$$

0.5 to 1 A in 1 µs/A

$$C_{out} = 220\ \mu\text{F}$$

$$r_C = 77\ \text{m}\Omega$$

$$\Delta I_{out} = 0.5\ \text{A}$$

$$f_c = 1\ \text{kHz}$$

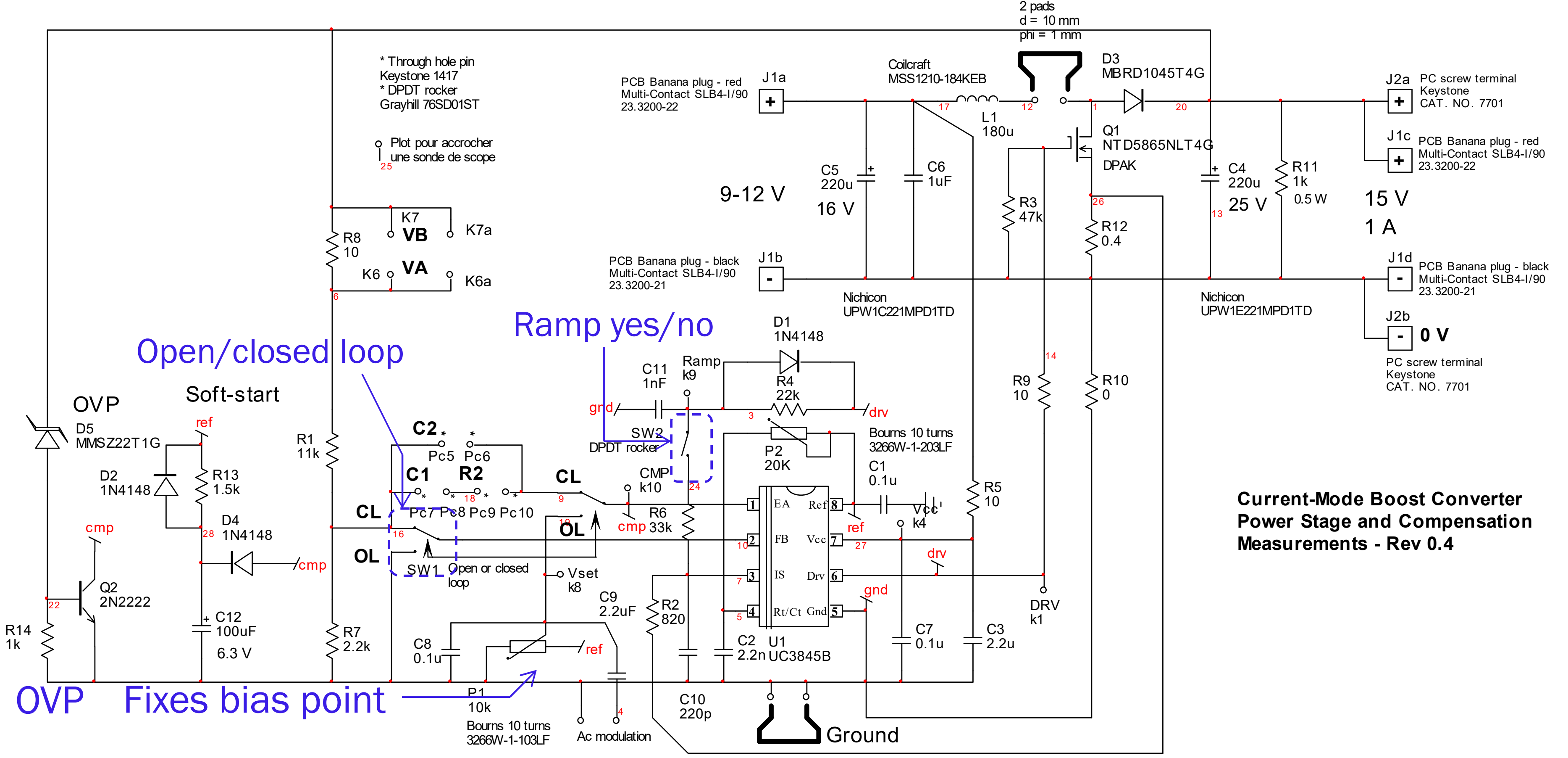
$$C_{out} = 220\ \mu\text{F}$$

$$\Delta V_{out} \approx \frac{\Delta I_{out}}{2\pi f_c C_{out}} = 362\ \text{mV}$$

❖ The approximated formula works well in this case

A Boost Converter in Current Mode

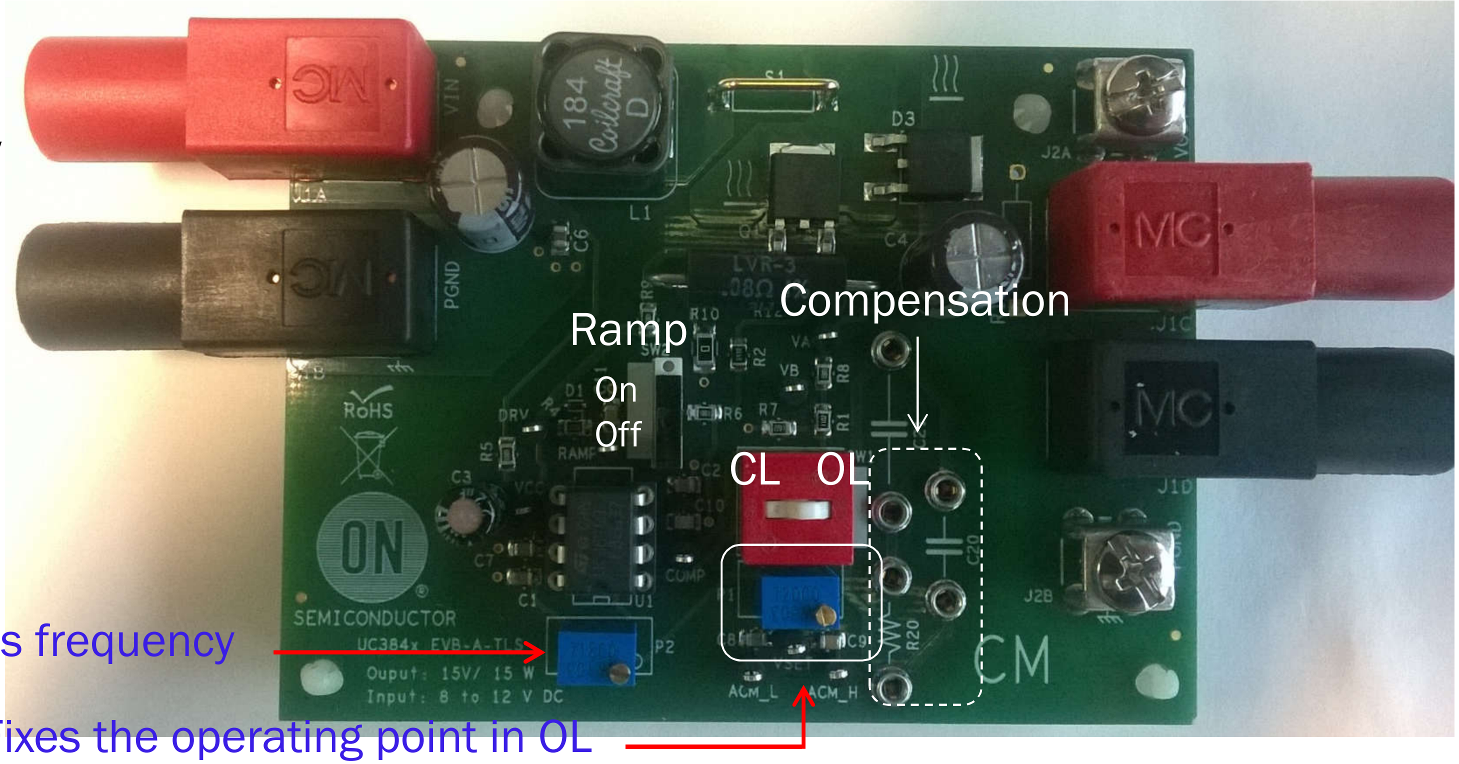
- ❑ The converter still uses a UC3843 configured in current mode



- ❖ Similar protections as in the voltage-mode case

A Convenient CM Board to Test

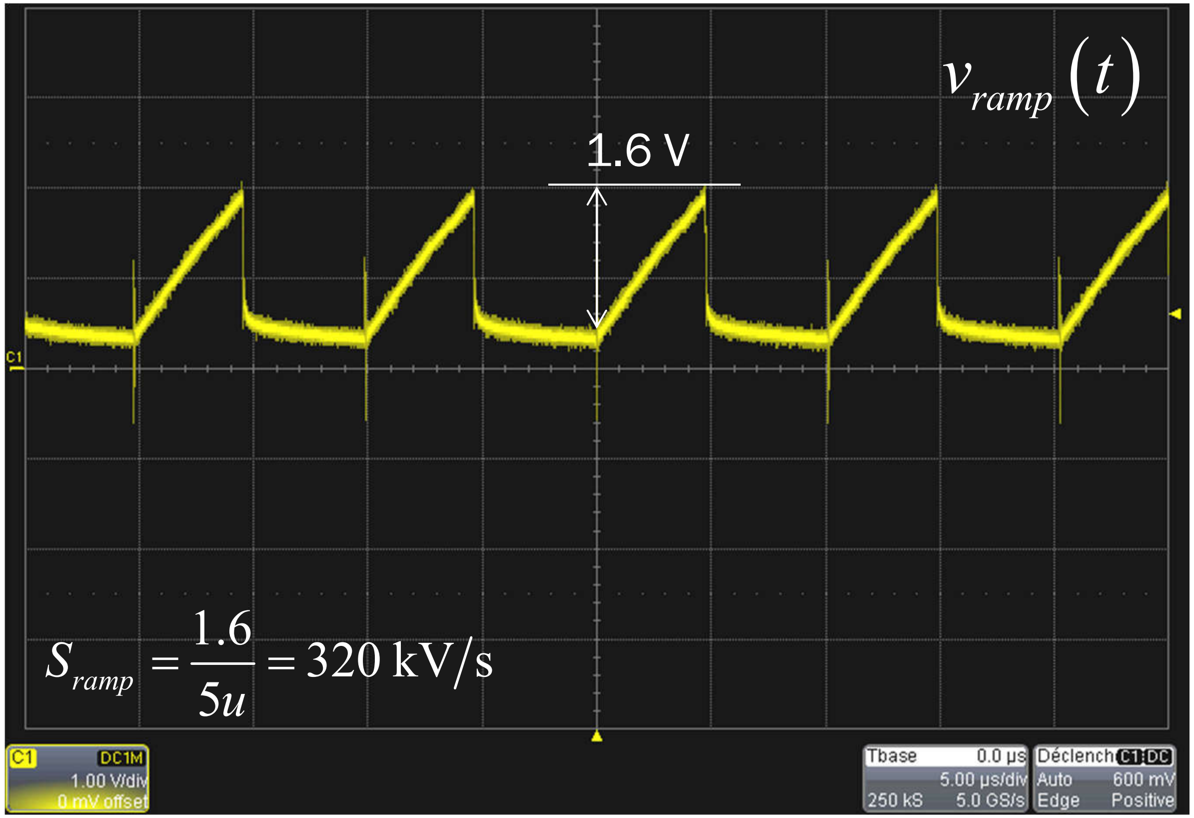
- ☐ The board includes switches to measure open and closed loops



- ☐ The compensation ramp can be turned on and off

Check the Amount of External Ramp

- ❑ The ramp level can be reflected in the SPICE model

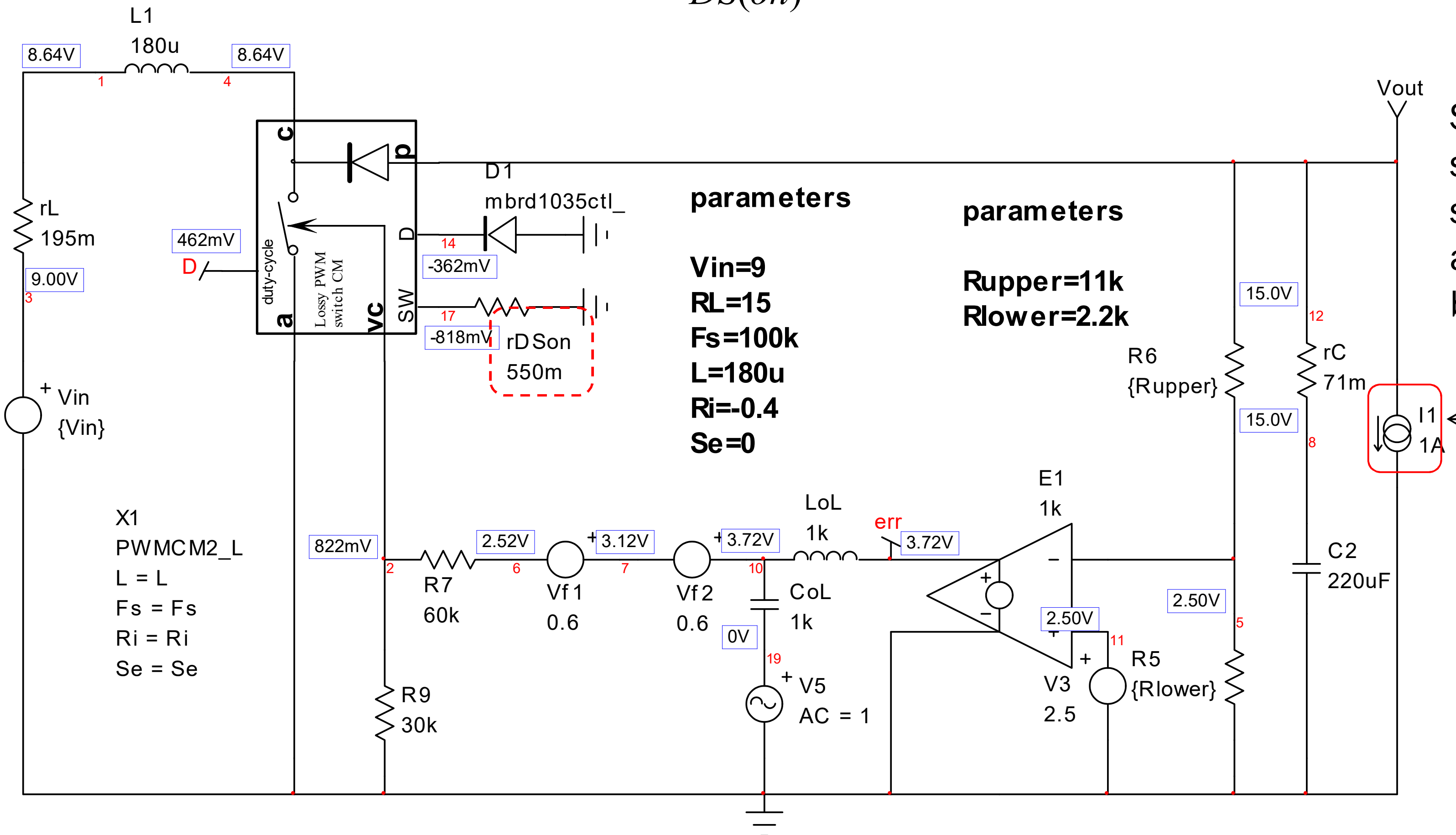


$$S_{ramp} = \frac{1.6}{5u} = 320\ \mathrm{kV/s}$$

- ❑ With 33 kΩ for R_{ramp} and 820 Ω for R_{CS}: $S_e = \frac{R_{CS}}{R_{ramp}} S_{ramp} \approx 8\ \mathrm{kV/s}$

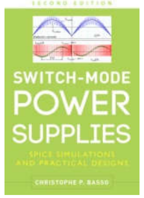
Use a Model which Includes Losses

- ☐ Characterize the MOSFET $r_{DS(on)}$ and all dc paths



Select same source as in the bench.

- ☐ For a low V_{in}, use the lossy current mode model



C. Basso, Switch-Mode Power Supplies: SPICE Simulations and Ptractical Designs, 2nd ed., McGraw-Hill 2014

Watch for the Electronic Load

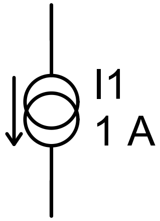
- ☐ The dc gain of the CM CCM boost converter depends on R

$$H_0 = \frac{R}{R_i}\frac{1}{2M + \frac{RT_{sw}}{LM^2}\left(\frac{1}{2} + \frac{S_e}{S_n}\right)} \qquad R = 15\,\Omega \quad \Rightarrow \quad \begin{array}{ll} H_0 = 11\,\text{dB} & \text{with } S_e = 0 \text{ kV/s} \\ H_0 = 10.7\,\text{dB} & \text{with } S_e = 8 \text{ kV/s} \end{array}$$

- ☐ Make sure the load in SPICE matches the bench setup

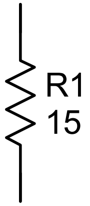
Constant current

use



Constant resistance

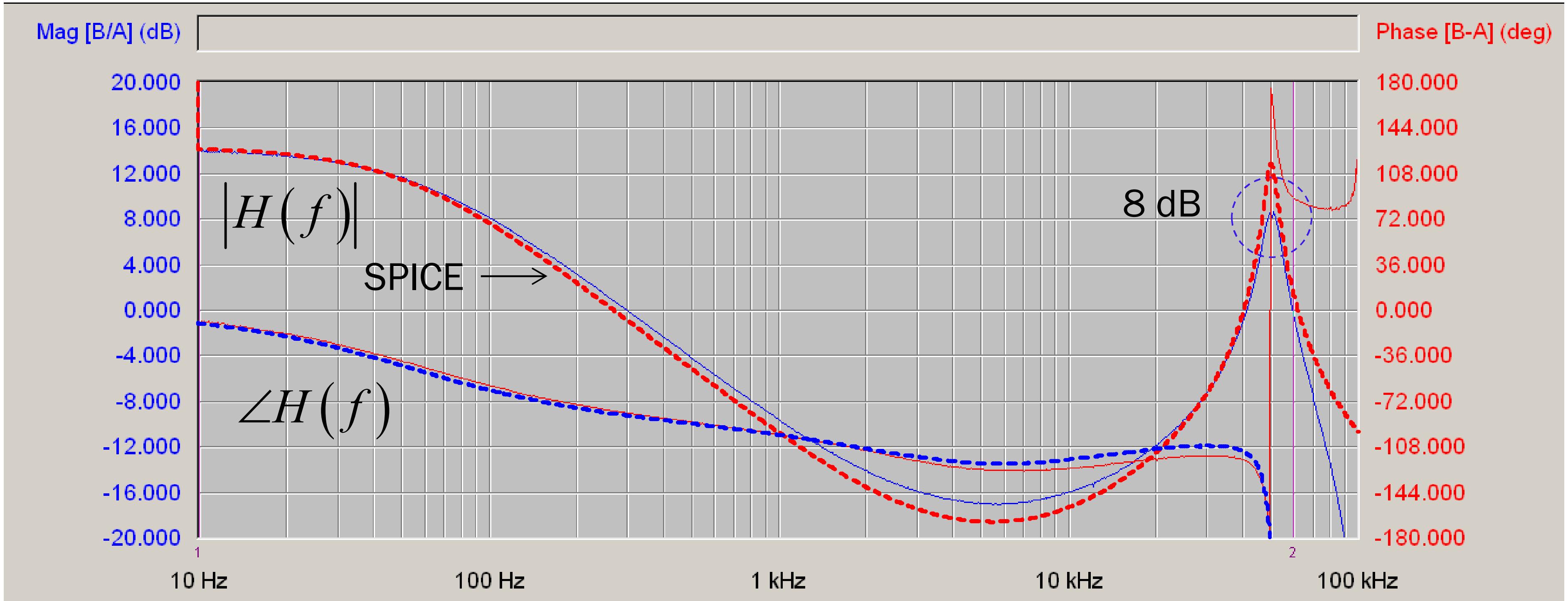
use



❖ H_0 is 14.5 dB with a 1-A source and 11 dB with a 15-Ω resistance

Extract the Power Stage Response

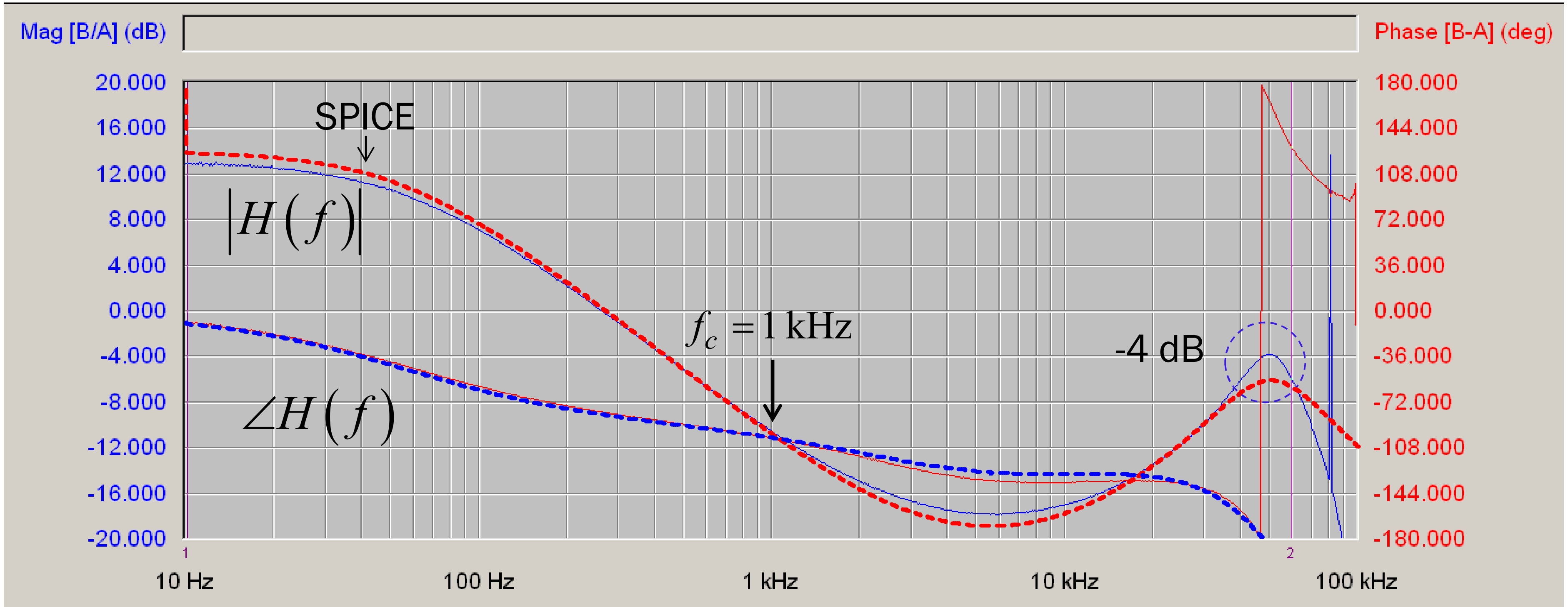
- ☐ The power stage shows peaking as expected at $F_{sw}/2$



- ☐ The measurement was carried with V_{in} = 9 V and I_{out} = 1 A
  - ❖ Load is in constant-current mode
  - ❖ Watch modulation amplitude as you approach $F_{sw}/2$

Turn the Compensation Ramp On

- ☐ The power stage shows reduced peaking at $F_{sw}/2$



- ☐ Extract the data at the selected 1-kHz crossover frequency

$$\left|H(f_c)\right| = -11\,\text{dB} \qquad \angle H(f_c) = -100°$$

Use the Automated SPICE Sheet

- ☐ Paste these data into the automated simulation sheet

parameters

Vref=2.5
Vout=15
Rupper=11k
Ib=(Vout-Vref)/Rupper
Rlower=Vref/Ib

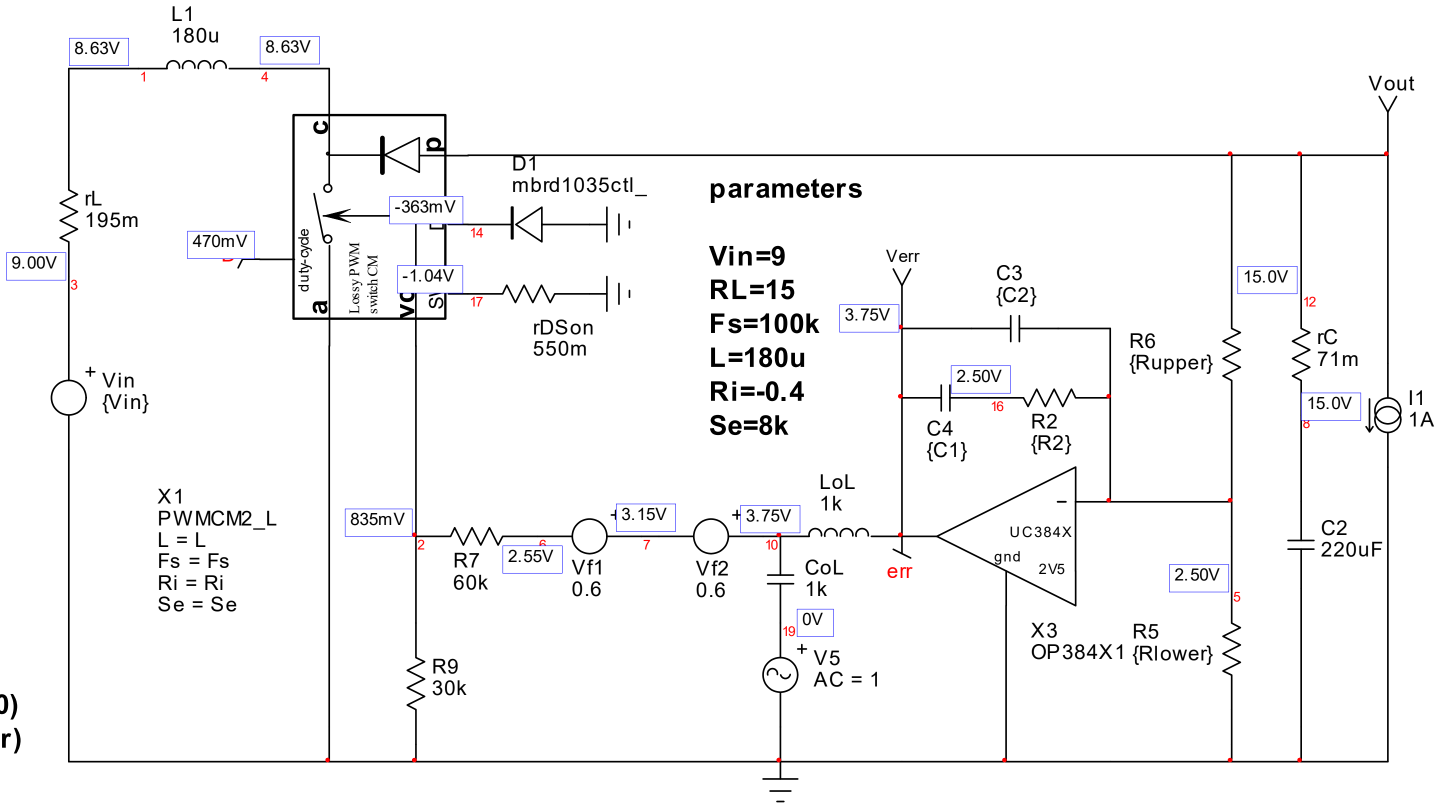
fc=1k
Gfc=-11
pfc=-100
pm=70

boost=pm-(pfc)-90

G=10^(-Gfc/20)
pi=3.14159

G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
C2=1/(2*pi*fc*G*k*Rupper)
C1=C2*(K^2-1)
R2=k/(2*pi*fc*C1)

fp1=1/(2*pi*R2*C2)
fz1=1/(2*pi*R2*C1)



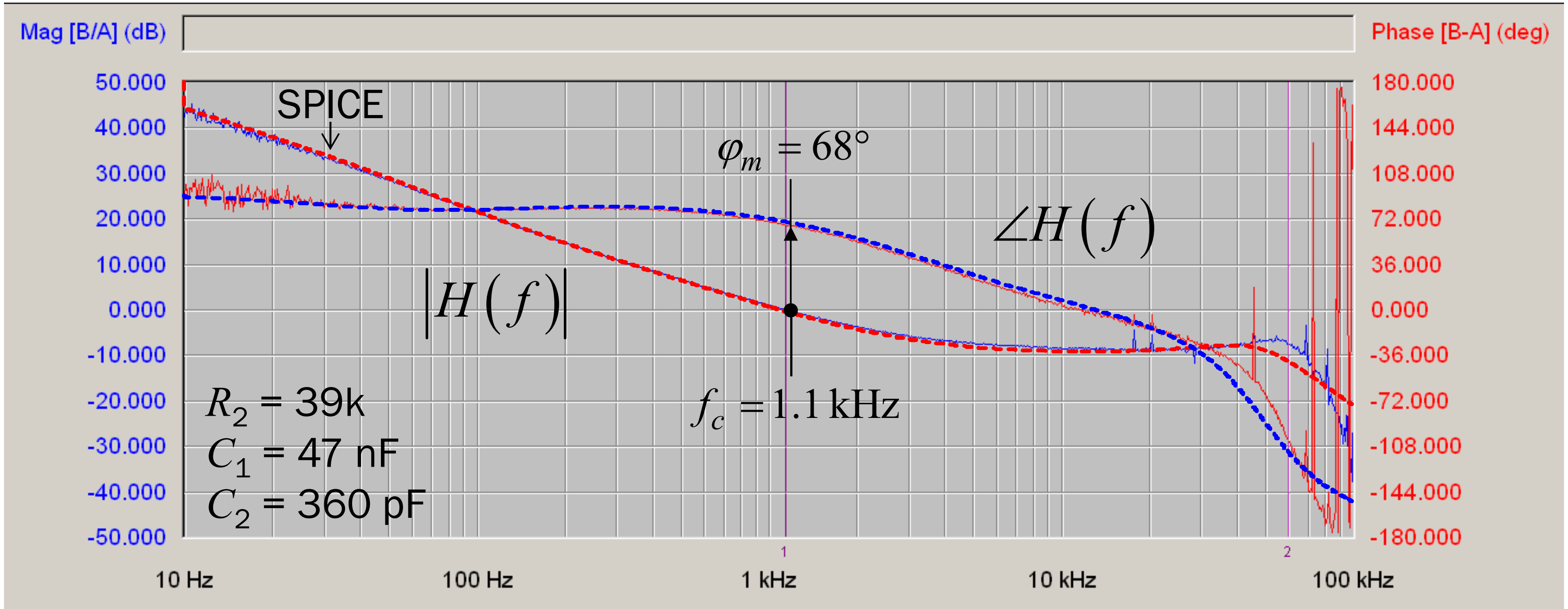
parameters

Vin=9
RL=15
Fs=100k
L=180u
Ri=-0.4
Se=8k

$$C_1 = 47\text{ nF} - C_2 = 360\text{ pF} - R_2 = 39\text{ k}\Omega$$

Verify Loop Gain with the Analyzer

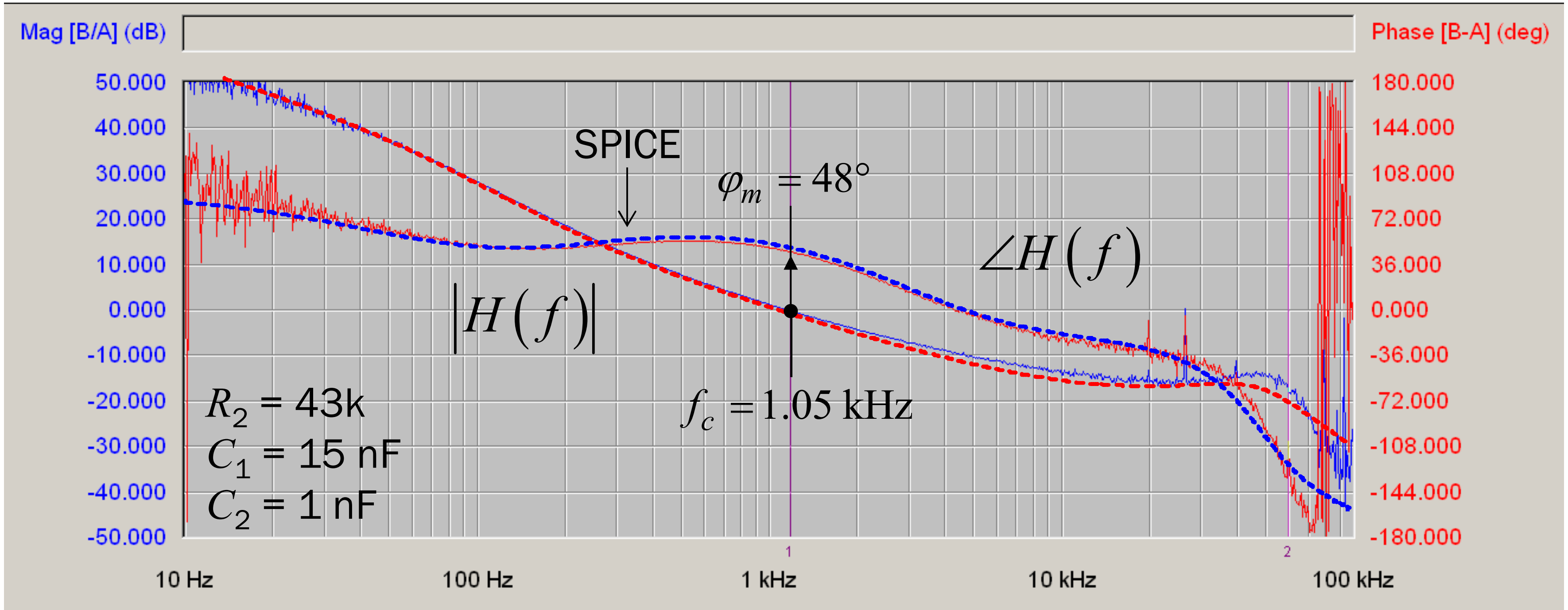
- ❑ The loop gain measurement agrees with the simulation



- ❑ The phase margin is 68° while crossover is 1.1 kHz
- ❖ The model is accurate and can be used for further analyses

Reduced Phase Margin for Experiments

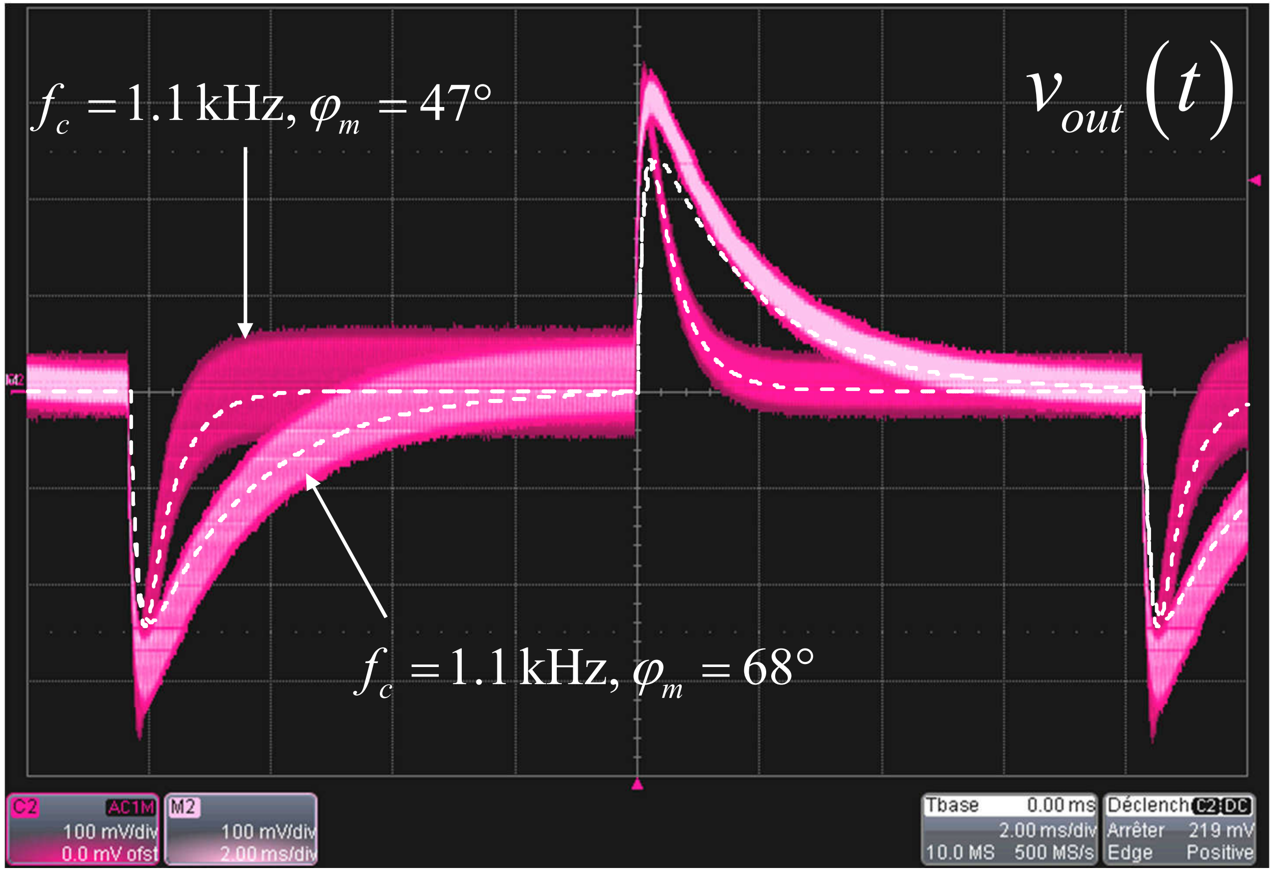
- A different compensation strategy, reducing the phase margin



- Phase margin is purposely reduced to 48° while f_c is constant

Comparing the two Responses

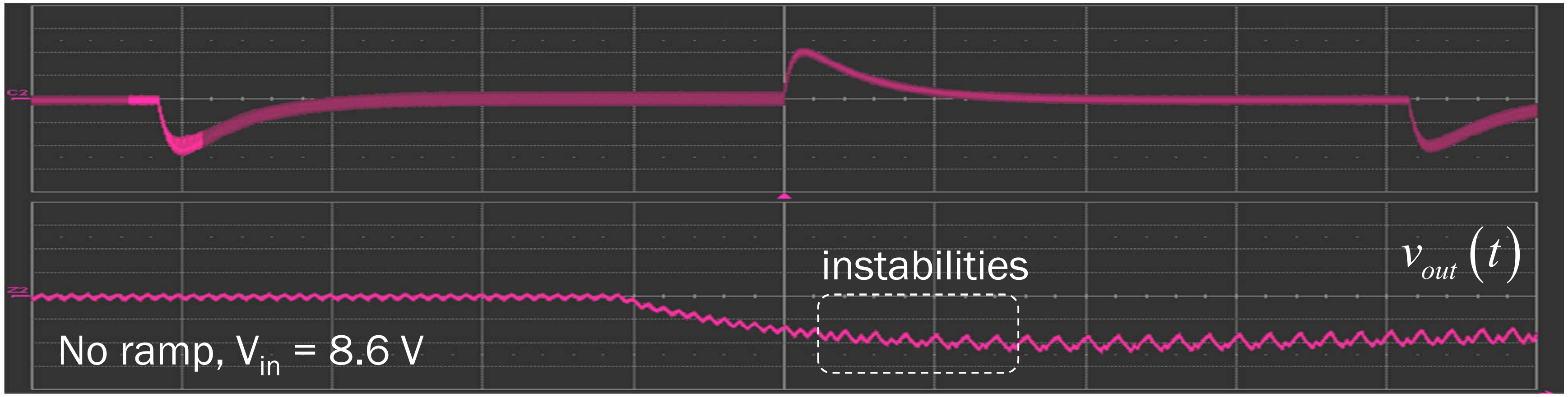
- ☐ The higher phase margin slows the recovery as expected



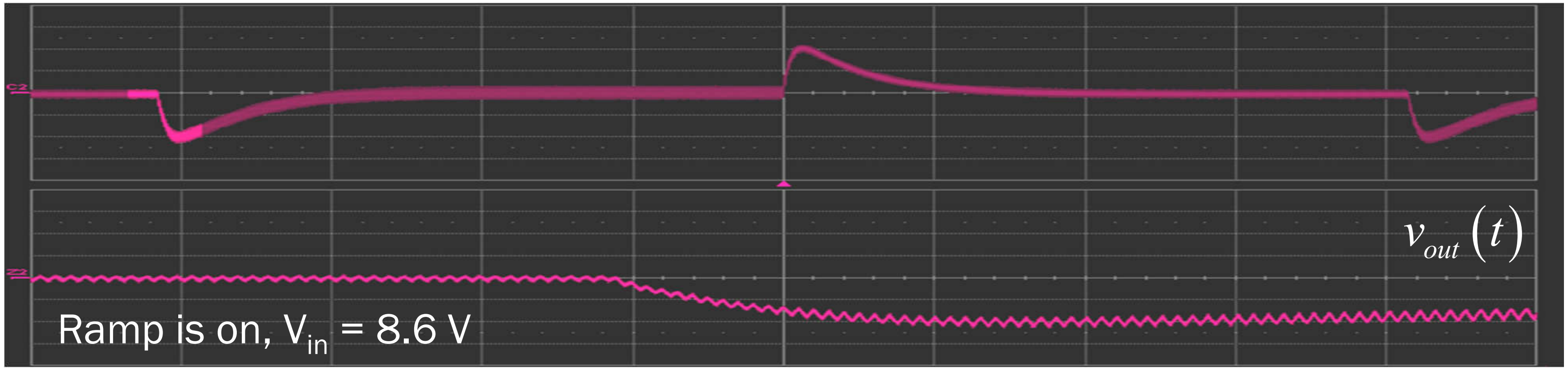
- ☐ Simulation in dash agrees quite well with bench results

Transient Response and Ext. Ramp

- ☐ Added compensation ramp brings stability at the lowest input



No ramp, V_{in} = 8.6 V

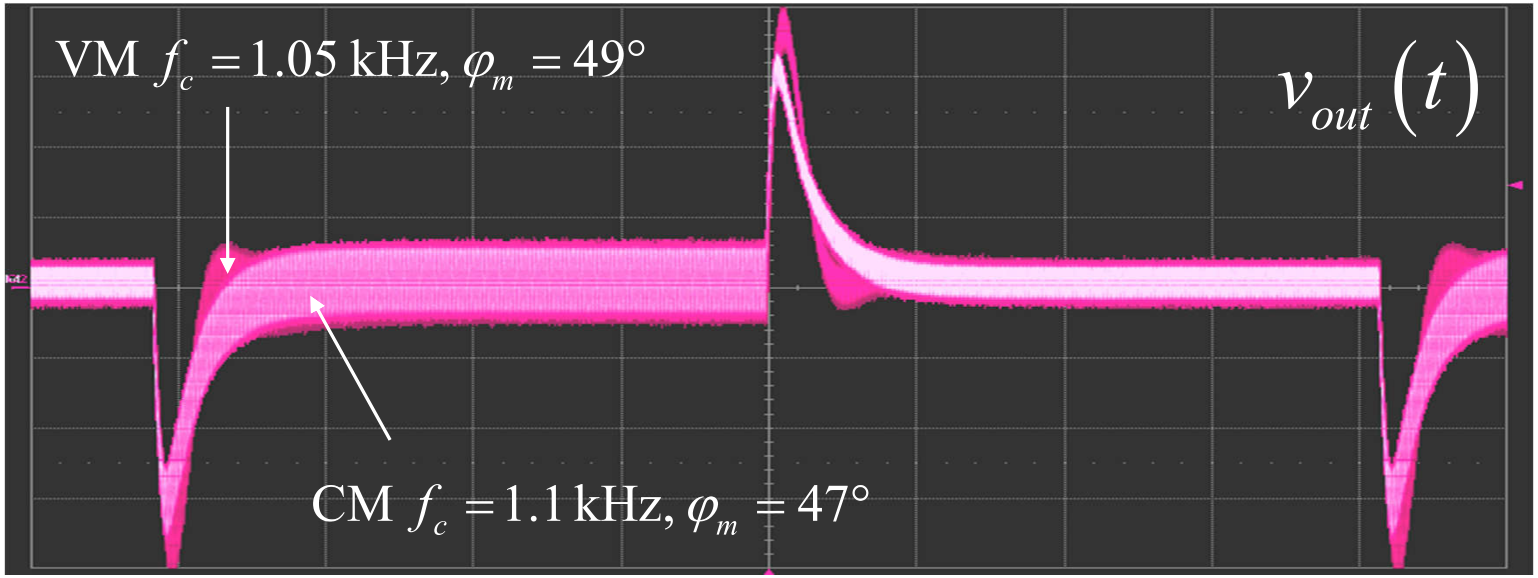


Ramp is on, V_{in} = 8.6 V

Closed loop, 0.5 A to 1 A in 1 µs/A

Comparing VM versus CM

- ☐ This board was purposely designed for a deep CCM operation
  - ❖ In VM, the crossover it too close to the resonant frequency
  - ❖ The gain is limited at f_0 and LC dominates the response



- ➢ Design rules would ask to lower L: RHPZ goes away from f_c
- ➢ Increase the distance between f_0 and crossover
  - ❖ Make output capacitor bigger to reduce resonant frequency

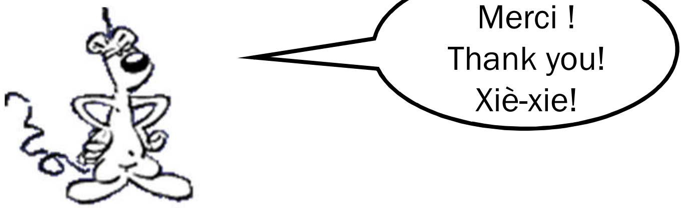
Conclusion

❑ Several blocks shape the converter ac response

❑ The power stage dynamic response is what you need first

❑ You have four ways to obtain it:

- ❖ Use small-signal analysis
- ❖ Build an average model and simulate it
- ❖ Simulate cycle by cycle with a PWL engine
- ❖ Build a hardware prototype and measure



❑ The best is to combine these approaches together:

- ❖ Analytical analysis gets you the insight on who does what?
- ❖ Simulation lets you explore various compensation strategies
- ❖ Hardware measurements confirms you did a good job!